Read the directions carefully.
Write neatly in pencil and show all your work
(you will only get credit for what you put on paper).
You may use your homework solutions.
If you get stuck, free feel to ask me for help.

LEAD: Thursdays, 5:00 - 7:00 PM
CSF G5D
Consider the DE \( \frac{xy}{dx} = \frac{e^{-3y}}{4-x} \)

a. Give the order of the DE. Determine if the DE is (non)linear, (non)autonomous, and (non)separable. State why.

1st order, nonlinear - nonlinear function of \( y \), nonautonomous - \( x \) is in the equation separable - can be rewritten as \( \frac{dy}{dx} = g(x)h(y) \).

b. What method(s) can you use to solve this DE?

\[ \text{SoV - separable} \]
\[ \text{IR - nonlinear} \]

c. Solve the DE.

SoV \( \Rightarrow \int ye^{3y} dy = \int \frac{dx}{x(4-x)} \)

LHS: \( u = y \quad dv = e^{3y} dy \) \( \Rightarrow \int ye^{3y} dy = \frac{1}{3} ye^{3y} - \frac{1}{3} \int e^{3y} dy \)

\[ du = dy \quad v = \frac{1}{3} e^{3y} \quad \frac{1}{3} ye^{3y} - \frac{1}{3} e^{3y} \]

RHS: \( \frac{1}{x(4-x)} = A + B \) \( \Rightarrow \int \frac{1}{x} \left( \frac{1}{4-x} \right) dx = 1 \ln \left| \frac{x}{4-x} \right| + C = \ln \left| \frac{x}{4-x} \right| + C \)

\[ \frac{1}{4} \int \left( \frac{1}{x} + \frac{1}{4-x} \right) dx = \frac{1}{4} \left( \ln \left| x \right| - \ln \left| 4-x \right| \right) + C = \frac{1}{4} \ln \left| \frac{x}{4-x} \right| + C \]

So \( 3e^{3y} (y - \frac{1}{3}) = \frac{1}{4} \ln \left| \frac{x}{4-x} \right| + C \)