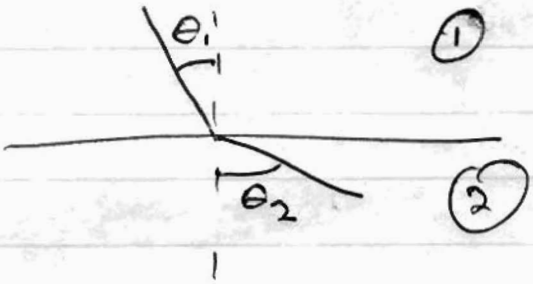
4/12/07

Snell's Law - Refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Small angles $\sin\theta \cong \theta$

$n_1\theta_1 \cong n_2\theta_2$

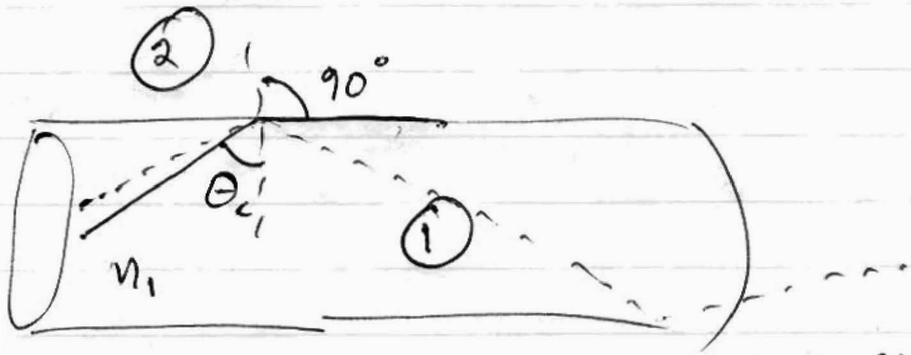
or $\frac{\theta_1}{\theta_2} \simeq \frac{n_2}{n_1}$

Figure ←

Obviously: $n_2 < n_1$

Since: $\theta_2 > \theta_1$

Total internal reflection (fiber optics)



$\theta > \theta_c$ total internal reflection

$$n_1 \sin\theta_c = n_2 \sin 90°$$

$$n_1 \sin\theta_c = n_2$$

Index of refraction

$n \geq 1.0$ [air, vacuum]

$$v_n = \frac{c}{n}$$

$\lambda_n f = v_n$ (frequency doesn't change)

$f = f_0$ (freq in air)

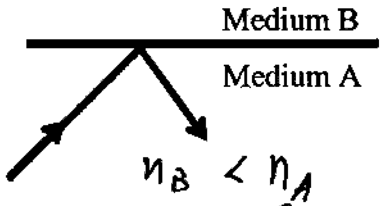
$\therefore \lambda_n = \frac{\lambda_0 (air)}{n}$ wavelength gets shorter

Physics 24 Test-Level Problems for Recitation 25

B 1. Light traveling in medium A undergoes total internal reflection at the boundary with medium B. What can be said about the speed of light in medium A, v_A, compared to the speed of light in medium B, v_B?

[A] $v_A = c$ [B] $v_A < v_B$

[C] $v_A > v_B$ [D] $v_A = v_B$



$n_B < n_A$

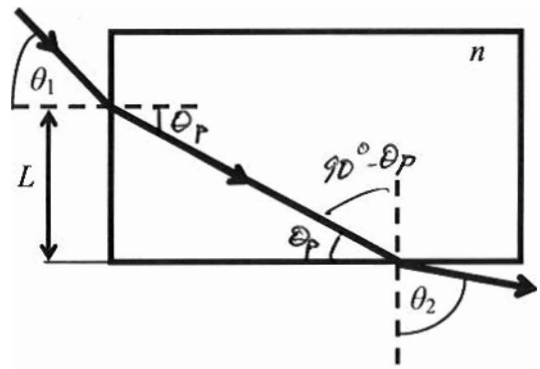
$v_n = \frac{c}{n}$ $\therefore$ $v_B > v_A$

2. A light ray in air enters a rectangular block of plastic at an angle of $\theta_1 = 45.0°$ and emerges at an angle of $\theta_2 = 76.0°$.

(a) Determine the index of refraction of the plastic.

(b) If the ray enters the plastic at a distance $L = 50.0$ cm from the bottom edge, how far does the light travel in the plastic ***and*** how long does it take the light to travel through the plastic?

(c) What is the largest value of θ_1 for which total internal reflection occurs at the bottom surface of the plastic?



a) $1.00 \sin 45° = n_P \sin \theta_P$

$n_P \sin(90° - \theta_P) = 1.00 \sin 76°$

$n_P \sin \theta_P = 0.7071$

$n_P \cos \theta_P = 0.970$

$\div$ $\tan \theta_P = 0.7288$ $\therefore$ $\boxed{\theta_P = 36.08°}$

$n_P = \frac{1.00 \sin 45°}{\sin 36.08°} = \boxed{1.20 = n_P}$

b) $\sin \theta_P = \frac{L}{Hypo} \Rightarrow Hypo = \frac{0.50\,m}{\sin 36.08°} = \boxed{0.849\,m}$

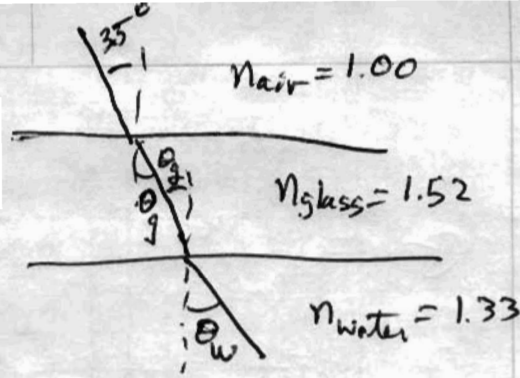
$v_n = \frac{c}{n} = \frac{3.0 \times 10^8\,m/s}{1.20} = 2.5 \times 10^8\,m/s$

$t = \frac{Hypo}{v_n} = \frac{0.849}{2.5 \times 10^8\,m/s} = \boxed{3.396\,ns}$

c) $1.20 \sin \theta_R = 1.00 \sin 90° \Rightarrow \theta_R = 56.44°$

$1.00 \sin \theta_1 = 1.20 \sin 33.56°$ $\therefore$ $\boxed{\theta_1 = 41.55°}$

33.10 A horizontal, parallel-sided plate of glass having a refractive index of 1.52 is in contact with the surface of water in a tank. A ray coming from above in air makes an angle of incidence of 35.0° with the normal to the top surface of the glass. a) What angle does the ray refracted into the water make with the normal to the surface? b) What is the dependence of this angle on the refractive index of the glass?



a) $1.00 \sin 35° = 1.52 \sin \theta_g$

$\theta_g = 22.17°$

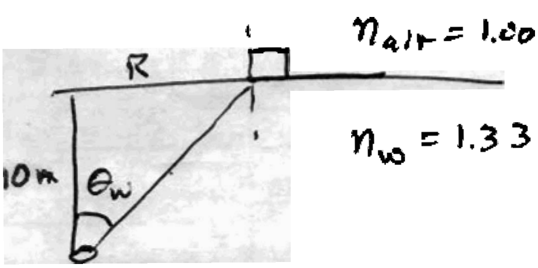
$1.52 \sin 22.17° = 1.33 \sin \theta_w \Rightarrow \boxed{\theta_w = 25.55°}$

b) $1.00 \sin 35° = 1.52 \sin \theta_g = 1.33 \sin \theta_w$

$\therefore \quad 1.00 \sin 35° = 1.33 \sin \theta_w \Rightarrow \theta_w = 25.55°$

n_g has no influence as long as the glass surfaces are parallel to one another

33.16 At the very end of Wagner's series of operas *Ring of the Nibelung*, Brünnhilde takes the golden ring from the finger of dead Siegfried and throws it into the Rhine, where it sinks to the bottom of the river. Assuming that the ring is small enough compared to the depth of the river to be treated as a point and that the Rhine is 10.0 m deep where the ring goes in, what is the area of the largest circle at the surface of the water over which light from the ring could escape from the water?



$1.33 \sin \theta_w = 1.00 \sin 90°$

$\boxed{\theta_w = 48.75°}$

$\tan \theta_w = \dfrac{R}{10.0\text{m}} \Rightarrow R = 11.40\text{m}$

$A = \pi R^2 = \pi (11.40\text{m})^2 = \boxed{408.6\text{ m}^2}$

33.34 In a physics lab, light with wavelength 490 nm travels in air from a laser to a photocell in 17.0 ns. When a slab of glass 0.840 m thick is placed in the light beam, with the beam incident along the normal to the parallel faces of the slab, it takes the light 21.2 ns to travel from the laser to the photocell. What is the wavelength of the light in the glass?

$x = vt = (3.0 \times 10^8 \text{ m/s})(17.0 \times 10^{-9}\text{s}) = 5.10\text{m}$

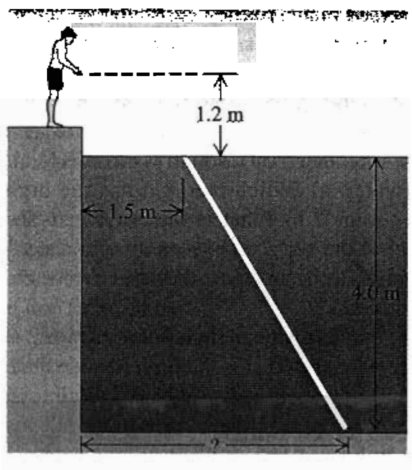
$\boxed{v_n = \dfrac{c}{n} \ \& \ \lambda_n = \dfrac{\lambda_0}{n}, \ f_n = f_0}$

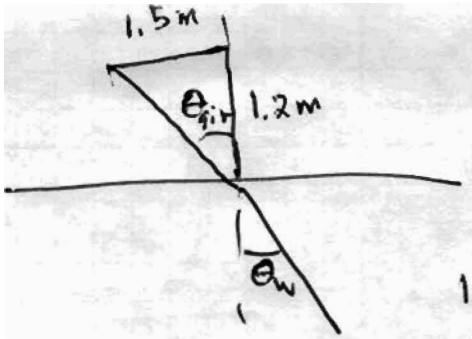
$t = \dfrac{\overset{air}{5.10\text{m} - 0.840\text{m}}}{3.0 \times 10^8 \text{ m/s}} + \dfrac{0.840}{v_n} = 21.2 \times 10^{-9}\text{s}$

$\therefore \boxed{v_n = 1.20 \times 10^8 \text{ m/s}} \ \& \ n = \dfrac{3.00 \times 10^8 \text{ m/s}}{1.20 \times 10^8 \text{ m/s}} = \boxed{2.50}$

$\lambda_n = \dfrac{490\text{nm}}{2.50} = \boxed{196 \text{ nm}}$

33.40 After a long day of driving you take a late-night swim in a motel swimming pool. When you go to your room, you realize that you have lost your room key in the pool. You borrow a powerful flashlight and walk around the pool, shining the light into it. The light shines on the key, which is lying on the bottom of the pool, when the flashlight is held 1.2 m above the water surface and is directed at the surface a horizontal distance of 1.5 m from the edge (see Fig. 33.39). If the water here is 4.0 m deep, how far is the key from the edge of the pool?

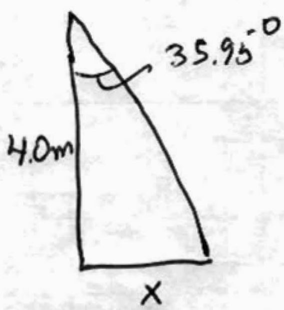




$$\theta_{air} = \tan^{-1}\left(\frac{1.5\text{ m}}{1.2\text{m}}\right) = 51.34^\circ$$

$$1.00 \sin 51.34^\circ = 1.33 \sin \theta_w$$

$$\boxed{\theta_w = 35.95^\circ}$$



$$\tan 35.95^\circ = \frac{x}{4.0\text{m}}$$

$$\boxed{x = 2.90\text{m}}$$

$\therefore$ Distance $= (1.50 + 2.90)\text{m} = \boxed{4.40\text{m}}$

Physics 24 **Special Homework Assignment #8**

An optical fiber consists of a glass core with index of refraction $n_g = 1.52$ surrounded by a coating with index of refraction $n_c = 1.25$. The fiber is submerged in water ($n_w = 1.33$) and light enters the end of the cable from the water at an angle θ as shown. The light strikes the surface between the glass and the coating at the critical angle θ_c so that the light is refracted along the boundary between the glass and the coating. Determine the angle θ.

$$1.52 \sin\theta_c = 1.25 \sin 90^\circ$$

$$\boxed{\theta_c = 55.32^\circ}$$

$$1.52 \sin(90^\circ - 55.32^\circ) = 1.33 \sin\theta_w$$

$$\boxed{\theta_w = 40.56^\circ}$$