1/10/07

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PLC
Mon 2:00-4:30
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6:00-8:30

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solutions to problems & previous exams

Chap. 21

Coulomb Force

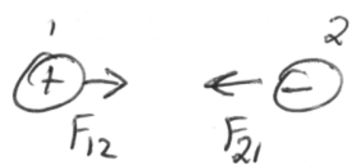
$$\vec{F}_{12} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$\vec{F}_{12}$: force on 1 due to 2

$\hat{r}_{12}$: unit vector, mag 1.00, just gives direction, pts from 2 to 1

$$k = 9.0 \times 10^9 \frac{N\text{-}m^2}{C^2}$$

opposite attract



Likes repel

i.e. equal & opposite $\vec{F}_{12} = -\vec{F}_{21}$

$q_{electron} = -1.6 \times 10^{-19} C$ ← Coulomb unit $q_{proton} = +1.6 \times 10^{-19} C$

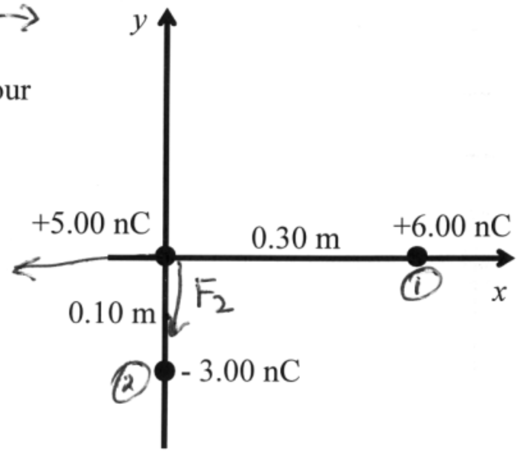
Physics 24 Test-Level Problems for Recitation 1

1. B You are given three charged insulating spheres. Spheres 1 and 2 are found to attract each other. Spheres 2 and 3 are found to repel each other. Which of the following can you conclude? [A] Spheres 1 and 3 carry charges of equal sign, [B] spheres 1 and 3 carry charges of opposite sign, [C] all three spheres carry charges of the same sign, [D] spheres 1 and 3 will repel each other.

(1) (+) → ← (−) (2) and ← (−) (2) (3) (−) →

OR (−) → ← (+) and ← (+) (+) →

2. Three point charges are arranged as shown.
Find the force on the 5.00 nC charge at the origin. Express your answer in unit vector notation.



$$|\vec{F}| = \frac{k|q_1 q_2|}{R^2}$$

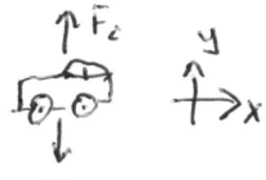
$$|\vec{F}_1| = \frac{(9.0\times10^9\ \frac{N\cdot m^2}{C^2})(5.0\times10^{-9}C)(6.0\times10^{-9}C)}{(0.30m)^2}$$

$$|\vec{F}_1| = 3.0\times10^{-6}N$$

$$|\vec{F}_2| = \left|\frac{(9.0\times10^9\ \frac{N\cdot m^2}{C^2})(5.0\times10^{-9}C)(-3.0\times10^{-9}C)}{(0.10m)^2}\right| = 1.35\times10^{-5}N$$

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 = -(3.0\times10^{-6}N)\hat{i} - (1.35\times10^{-5}N)\hat{j}$$

3. Imagine that space aliens could deposit extra electrons in equal amounts on the earth and on your car, so that both objects have the same negative charge. Assume that the earth's charge is spread uniformly so that the charge acts as if it were concentrated at the center of the earth (radius = 6.8×10^6 m). Your car's tires would act as insulators so that charge would not escape from your car, and your car's mass is 1050 kg. What charge would have to be placed on your car and earth to make the car levitate (i.e., just barely overcome gravity)?



$$\Sigma F_y = 0 = F_c - mg$$

$$\therefore F_c = mg$$

$$\frac{k|q_1 q_2|}{R^2} = mg$$

$$q^2 = \frac{mgR^2}{k}$$

$$q = \left[\frac{(1050kg)(9.8\ m/s^2)(6.8\times10^6 m)^2}{9.0\times10^9\ \frac{N\cdot m^2}{C^2}}\right]^{1/2} = 7{,}270\ C$$

21.8 Two small aluminum spheres, each having mass 0.0250 kg, are separated by 80.0 cm. a) How many electrons does each sphere contain? (The atomic mass of aluminum is 26.982 g/mol, and its atomic number is 13.) b) How many electrons would have to be removed from one sphere and added to the other to cause an attractive force between the spheres of magnitude 1.00×10^4 N (roughly one ton)? Assume that the spheres may be treated as point charges. c) What fraction of all the electrons in each sphere does this represent?

a) $\dfrac{25.0\,g}{27.0\,g/mol} = 0.9265\,mol \times 6.02\times10^{23}\,\dfrac{atoms}{mol}$

$= 5.58\times10^{23}\,atoms \times 13\,\dfrac{electrons}{atom} =$

$\boxed{7.25\times10^{24}\text{ electrons}}$

b) Conductor

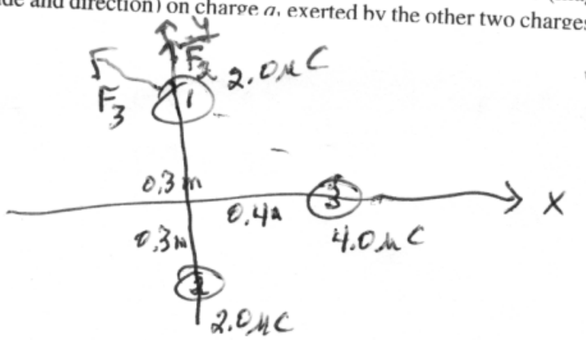
$F = 1.00\times10^4 N = \dfrac{k|q_1 q_2|}{r^2} = \dfrac{(9.0\times10^9\,\frac{N\text{-}m^2}{C^2})\,q^2}{(0.80m)^2}$

$\boxed{q = 8.43\times10^{-4} C}$

$n_e = \dfrac{8.43\times10^{-4} C}{1.60\times10^{-19} C/elect} = \boxed{5.26\times10^{15}\text{ electrons}}$

c) $\text{fraction} = \dfrac{5.26\times10^{15}\,el}{7.25\times10^{24}\,el} = \boxed{7.26\times10^{-10}}$

21.16 In Example 21.4 (Section 21.3), what is the net force (magnitude and direction) on charge q_1 exerted by the other two charges?



$|\vec{F}_3| = \dfrac{(9.0\times10^9\,\frac{N\text{-}m^2}{C^2})(2.0\times10^{-6}C)(4.0\times10^{-6}C)}{(0.5m)^2}$

$= 0.288\,N$

$|\vec{F}_2| = \dfrac{(9.0\times10^9\,\frac{N\text{-}m^2}{C^2})(2.0\times10^{-6}C)(2.0\times10^{-6}C)}{(0.6m)^2}$

$= 0.100 N$

Vectors

$\vec{F}_2 = 0\,\hat{i} + (0.100N)\,\hat{j}$

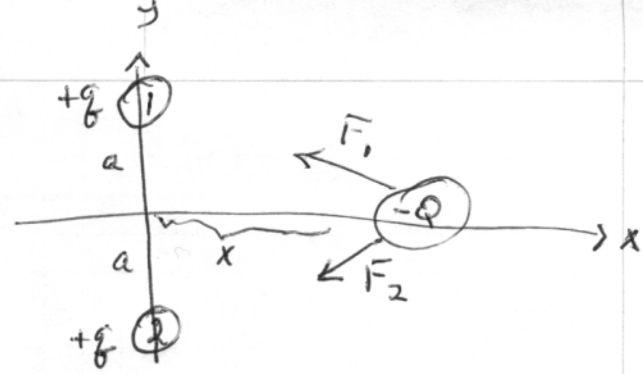
$\vec{F}_3 = -\frac{4}{5}(0.288N)\,\hat{i} + \frac{3}{5}(0.288N)\,\hat{j}$

$\boxed{\vec{F}_2 + \vec{F}_3 = (-0.230N)\,\hat{i} + (0.273N)\,\hat{j}}$

$|\vec{F}_2 + \vec{F}_3| = 0.357$

$\theta = 180^\circ - \tan^{-1}\left(\dfrac{0.273}{0.230}\right) = 130^\circ$

21.22 Two positive point charges q are placed on the y-axis at $y = a$ and $y = -a$. A negative point charge $-Q$ is located at some point on the $+x$-axis. a) In a free-body diagram, show the forces that act on the charge $-Q$. b) Find the x- and y-components of the net force that the two positive charges exert on $-Q$. (Your answer should only involve k, q, Q, a, and the coordinate x of the third charge.) c) What is the net force on the charge $-Q$ when it is at the origin $(x = 0)$? d) Graph the x-component of the net force on the charge $-Q$ as a function of x for values of x between $-4a$ and $+4a$.



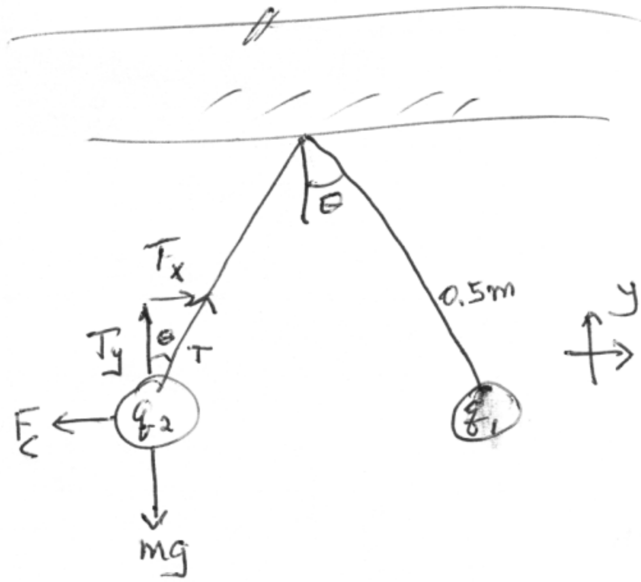
$$|\vec{F}| = \frac{kqQ}{a^2+x^2}$$

$$\vec{F}_1 = -\frac{kqQ}{(a^2+x^2)} \cdot \frac{x}{\sqrt{a^2+x^2}}\,\hat{i} + \frac{kqQ}{(a^2+x^2)} \cdot \frac{a}{\sqrt{a^2+x^2}}\,\hat{j}$$

$$\vec{F}_2 = -\frac{kqQ}{(a^2+x^2)} \cdot \frac{x}{\sqrt{a^2+x^2}}\,\hat{i} - \frac{kqQ}{(a^2+x^2)} \cdot \frac{a}{\sqrt{a^2+x^2}}\,\hat{j}$$

$$\vec{F}_1 + \vec{F}_2 = -\frac{2kqQx}{(a^2+x^2)^{3/2}}\,\hat{i}$$; @ $x = 0$ $\vec{F}_1 + \vec{F}_2 = 0$

21.72 Two identical spheres are each attached to silk threads of length $L = 0.500$ m and hung from a common point (Fig. 21.36). Each sphere has mass $m = 8.00$ g. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. One sphere is given positive charge q_1, and the other a different positive charge q_2; this causes the spheres to separate so that when the spheres are in equilibrium, each thread makes an angle $\theta = 20.0°$ with the vertical. a) Draw a free-body diagram for each sphere when in equilibrium, and label all the forces that act on each sphere. b) Determine the magnitude of the electrostatic force that acts on each sphere, and determine the tension in each thread. c) Based on the information you have been given, what can you say about the magnitudes of q_1 and q_2? Explain your answers. d) A small wire is now connected between the spheres, allowing charge to be transferred from one sphere to the other until the two spheres have equal charges; the wire is then removed. Each thread now makes an angle of 30.0° with the vertical. Determine the original charges. (*Hint:* The total charge on the pair of spheres is conserved.)



$$\Sigma F_x = 0 = T\sin\theta - F_c$$
$$\Sigma F_y = 0 = T\cos\theta - mg$$

$$\therefore \tan\theta = \frac{F_c}{mg} \Rightarrow F_c = (8.0\times10^{-3}\,kg)(9.8\,\tfrac{m}{s^2})\tan 20°$$

$F_c = 0.0285N$ $\qquad T = \frac{F_c}{\sin\theta} = 0.0834N$

72 con't $\sin\theta = \frac{r/2}{0.5m}$ $\frac{r}{2} = 0.5m \sin 20°$ $r = 0.342m$

$$0.0285N = \frac{\left(9.0\times10^{9}\ \frac{N\text{-}m^2}{C^2}\right)|q_1 q_2|}{(0.342m)^2}$$

$\therefore$ $q_1 q_2 = 3.70\times10^{-13}C^2$

Charges now equal

$$F_c = mg\tan 30° = 0.0453N = \frac{\left(9.0\times10^{9}\ \frac{N\text{-}m^2}{C^2}\right)q^2}{(0.50m)^2}$$

$\therefore$ $q_1 = q_2 = 1.12\times10^{-6}C$

$q_1 q_2 = 3.70\times10^{-13}C^2$

$q_1 + q_2 = 2.24\times10^{-6}C$

$q_1(2.24\times10^{-6} - q_1) = 3.70\times10^{-13}$

$-q_1^2 + 2.24\times10^{-6}q_1 - 3.70\times10^{-13} = 0$

$$q_1 = \frac{-2.24\times10^{-6} \pm \sqrt{(2.24\times10^{-6})^2 - 4(3.70\times10^{-13})}}{-2.0}$$

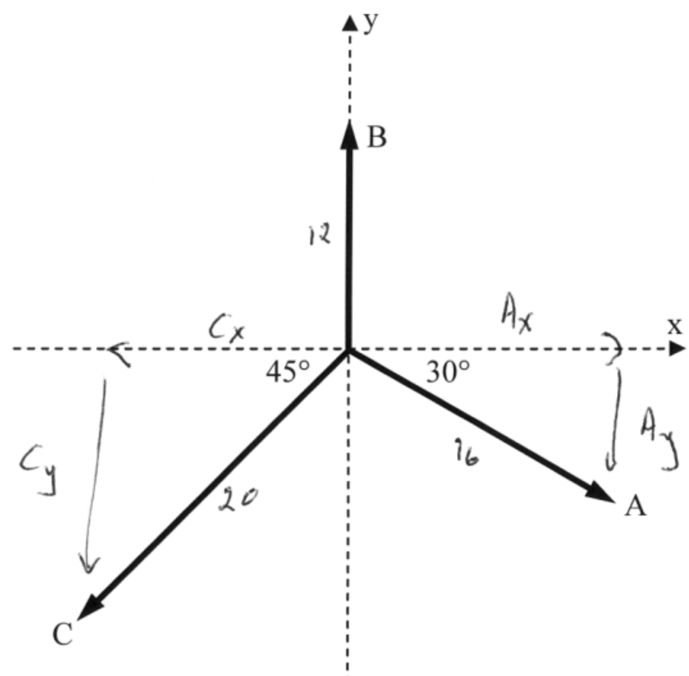
$q_1 = (1.12\times10^{-6} \pm 0.94\times10^{-6})C$

$\therefore$ $q_1 = 2.06\times10^{-6}C$ & $q_2 = 1.80\times10^{-7}C$

or $q_1 = 1.80\times10^{-7}C$ $q_2 = 2.06\times10^{-6}C$

Physics 24 **Special Homework Assignment #1**

The magnitudes of the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are 16, 12, and 20, respectively. Calculate $\vec{A}+\vec{B}+\vec{C}$ and $\vec{A}+\vec{B}-\vec{C}$. Express your answers both as a magnitude and direction, and in unit vector notation.

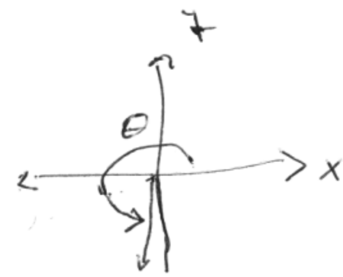


$\vec{A} = 13.86\,\hat{i} - 8.00\,\hat{j}$

$\vec{B} = 0\,\hat{i} + 12.00\,\hat{j}$

$\vec{C} = -14.14\,\hat{i} - 14.14\,\hat{j}$

$\vec{A}+\vec{B}+\vec{C} = -0.28\,\hat{i} - 10.14\,\hat{j}$



$|\vec{A}+\vec{B}+\vec{C}| = 10.14$

$\theta = 270° - \tan^{-1}\frac{0.28}{10.14}$

$\theta = 268.4°$ CCW from +x axis

$\vec{A}+\vec{B}-\vec{C} = 28.0\,\hat{i} + 18.1\,\hat{j}$ ⇒ Mag 33.3 @ 32.9°