Potential Energy (J) (Pt. Charges)

\[ U = \frac{h e_1 e_2}{r^2} \; ; \; \text{Scalar, units N-m = J} \]

\[ W_{k1} + W_{k2} = W_{k1} + W_{c} \; \text{K} = \frac{1}{2} m V^2 \; \text{Work} = \Delta U + \Delta K \]

**Potential (Volts)**

\[ V = \frac{U}{q} \]

\[ AV = V_f - V_i = -\int_C \vec{E} \cdot dl \]

If \( \vec{E} \) constant, such as between \( \parallel \) plates\n
\[ V_f - V_i = AV = -E d \]

\[ \begin{array}{c}
\text{High potential} \\
\uparrow \\
x=d
\end{array} \]

**Relationship to electric field**

\[ \vec{E} = -\frac{dV}{dx} \hat{i} - \frac{dV}{dy} \hat{j} - \frac{dV}{dz} \hat{k} \]

\[ \hat{E} = -\frac{dV}{d\hat{r}} \]

\[ V = \frac{ke_1}{r} \; \frac{dV}{d\hat{r}} = -\frac{ke_1}{r^2} \]

\[ \Rightarrow |\vec{E}| = \frac{ke_1}{r^2} \]

Outside a sphere, \( E \) same as if all charge was at the center (Gauss' Law) \Rightarrow same thing for the potential
Physics 24 Test-Level Problems for Recitation 6

1. An electron is released from rest in a uniform electric field. The electron then moves under the influence of the electric field. Which of the following is true for the electron?
   [A] Its potential energy increases and it moves toward higher electric potential,
   [B] its potential energy decreases and it moves toward higher electric potential,
   [C] its potential energy increases and it moves toward lower electric potential,
   [D] its potential energy decreases and it moves toward lower electric potential.

2. A uniformly charged insulating sphere of radius \( R = 1.00 \text{ m} \) and total charge \( Q = 4.60 \mu \text{C} \) is held centered at the origin. A point charge \( q = 1.20 \mu \text{C} \) with mass \( 2.80 \times 10^{-4} \text{ kg} \) is placed on the \( x \)-axis a distance of \( 2.00 \text{ m} \) from the origin.

   a) What is the electric potential energy of the system?

   \[
   U = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r} = \frac{9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{2.0 \text{ m}} \left( 4.60 \times 10^{-6} \text{ C} \right) \left( 1.20 \times 10^{-6} \text{ C} \right)
   \]

   \[
   U = \frac{2.48 \times 10^{-2} \text{ N}\cdot\text{m}}{2.0 \text{ m}} = 2.48 \times 10^{-2} \text{ J}
   \]

   b) If the point charge \( q \) is released from rest what is its speed when it is a distance of \( 50.0 \text{ m} \) from the origin?

   \[
   \frac{1}{2} m v^2 + U_i = K_f + U_f
   \]

   \[
   U_f = \frac{9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{50.0 \text{ m}} \left( 4.60 \times 10^{-6} \text{ C} \right) \left( 1.20 \times 10^{-6} \text{ C} \right)
   \]

   \[
   U_f = 9.94 \times 10^{-4} \text{ J}
   \]

   \[
   2.48 \times 10^{-2} \text{ J} - 9.94 \times 10^{-4} \text{ J} = \frac{1}{2} \left( 2.80 \times 10^{-4} \text{ kg} \right) v^2
   \]

   \[
   v = 13.0 \text{ m/s}
   \]
23.6 Three equal 1.20-μC point charges are placed at the corners of an equilateral triangle whose sides are 0.500 m. What is the potential energy of the system? (Take as zero the potential energy of the three charges when they are infinitely far apart.)

\[ U = \frac{kQ^2}{r_{12}} \]

\[ U_{tot} = U_{12} + U_{13} + U_{23} = 3 \left( 9.0 \times 10^9 \text{N} \cdot \text{m} \cdot \text{C}^{-2} \right) \left( 1.20 \times 10^{-6} \text{C} \right)^2 \]

\[ U_{tot} = 0.0778 \text{ N} \cdot \text{m} = 0.0778 \text{ J} \]

23.10 A proton, an alpha particle, an electron, and a neutron are at rest at the corners of a square whose side length is 5.00 \times 10^{-10} m. electron and neutron at opposite corners. How much work must be done to move the particles far from each other?

\[ U_e = \frac{k(1.6 \times 10^{-19} \text{C}) (1.6 \times 10^{-19} \text{C})}{5 \times 10^{-10} \text{m}} \]

\[ + k \left( \frac{1.6 \times 10^{-19} \text{C}}{\sqrt{2}} \right) \left( \frac{3.2 \times 10^{-19} \text{C}}{5 \times 10^{-10} \text{m}} \right) \]

\[ + k \left( -1.6 \times 10^{-19} \text{C} \right) \left( 3.2 \times 10^{-19} \text{C} \right) = -7.31 \times 10^{-19} \text{ J} \]

\[ W_{tot} = \Delta U = U_e - U_c = 0.00 - (-7.31 \times 10^{-19} \text{ J}) = 7.31 \times 10^{-19} \text{ J} \]

A takes this much energy to pull them apart.
23.18 Two stationary point charges +3.00 nC and +2.00 nC are separated by a distance of 50.0 cm. An electron is released from rest a point midway between the two charges and moves along the line connecting the two charges. What is the speed of the electron when it is 10.0 cm from the +3.00-nC charge?

\[ U_i = \frac{kq_1q_2}{r} = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(-1.6 \times 10^{-19} \text{ C})(2.0 \times 10^{-9} \text{ C} + 3.0 \times 10^{-9} \text{ C})}{0.25 \text{ m}} \]

\[ U_f = -2.88 \times 10^{-19} \text{ J} \]

\[ U_f = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(-1.60 \times 10^{-19} \text{ C})}{0.40 \text{ m}} \left[ \frac{2.0 \times 10^{-9} \text{ C}}{0.40 \text{ m}} + \frac{3.0 \times 10^{-9} \text{ C}}{0.10 \text{ m}} \right] \]

\[ U_f = -5.04 \times 10^{-17} \text{ J} \]

\[ U_i + k_e = U_f + k_f \]

\[ \frac{1}{2} m_e v_f^2 = U_i - U_f = 2.16 \times 10^{-17} \text{ J} \]

\[ v_f = \frac{1}{2} \sqrt{\frac{2(2.16 \times 10^{-17} \text{ J})(2.0 \text{ m})}{9.11 \times 10^{-31} \text{ kg}}} = \frac{6.89 \times 10^6 \text{ m/s}}{1} \]

23.22 Two positive point charges, each of magnitude q, are fixed on the y-axis at the points y = +a and y = -a. Take the potential to be zero at an infinite distance from the charges. a) Show the positions of the charges in a diagram. b) What is the potential \( V_0 \) at the origin? c) Show that the potential at any point on the x-axis is

\[ V = \frac{1}{4\pi\varepsilon_0} \frac{2q}{\sqrt{a^2 + x^2}} \]

d) Graph the potential on the x-axis as a function of x over the range from \( x = -4a \) to \( x = 4a \). e) What is the potential when \( x \gg a \)? Explain why this result is obtained.

b) \( V(x=0, y=0) = \frac{2kq}{a} \)

c) \( V(x=x, y=0) = \frac{kq}{(x^2+a^2)^{3/2}} + \frac{kq}{(x^2+a^2)^{1/2}} = \frac{2kq}{(x^2+a^2)^{3/2}} \)

e) When \( x \gg a \)

\[ V \approx \frac{2kq}{(x^2)^{3/2}} = \frac{kq}{x} \]

i.e. potential for a point charge of size +2q
An alpha particle with kinetic energy 11.0 MeV makes a head-on collision with a lead nucleus at rest. What is the distance of closest approach of the two particles? (Assume that the lead nucleus remains stationary and that it may be treated as a point charge. The atomic number of lead is 82. The alpha particle is a helium nucleus, with atomic number 2.)

\[ K_i + U_i = K_f + U_f \]

\[(11.0 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV}) = \left(9.0 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{(2.0)(82.0)}{(1.6 \times 10^{-19} \text{ C})^2} \]

\[ R = 2.15 \times 10^{-14} \text{ m} = 21.5 \text{ fm} \]

\[ U = \frac{q_i}{q} U \]

\[ V = \frac{U}{q} = \frac{(11.0 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{2.0(1.6 \times 10^{-19} \text{ C})} \]

\[ V = 5.50 \times 10^6 \frac{\text{J}}{\text{C}} = 5.50 \times 10^6 V = 5.50 \text{ MeV} \]