3/1/07

Magnetic Flux thru a loop

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

$\vec{A}$ vector is $\perp$ to loop's plane

Force on a current-carrying wire in a B-field

$$\vec{F} = I\vec{L} \times \vec{B}$$

$$|\vec{F}| = ILB\sin\theta$$

Torque on a current-carrying loop (i.e. motors)

$$\vec{\tau} = \vec{r} \times \vec{F} \quad \text{(o.k. for st. wires)}$$

Better method

Magnetic moment

$$\vec{\mu} = NI\vec{A}$$

$\vec{A}$ ← Area vector

I ← current

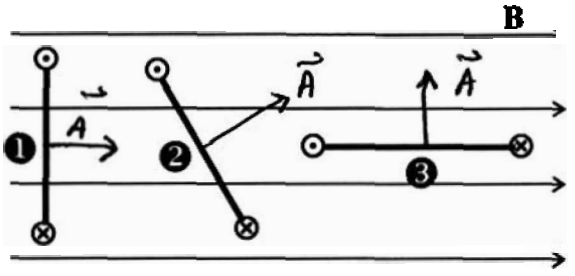
N ← # of loops of wire

$$\vec{\tau} = \vec{\mu} \times \vec{B} = NI\vec{A} \times \vec{B}$$

$$|\vec{\tau}| = NIAB\sin\theta$$

Physics 24 Test-Level Problems for Recitation 16

A = 1 1. The diagram shows a side view of three current loops in a uniform magnetic field. All three loops are identical and each carries the same current. For which loop is the torque zero?



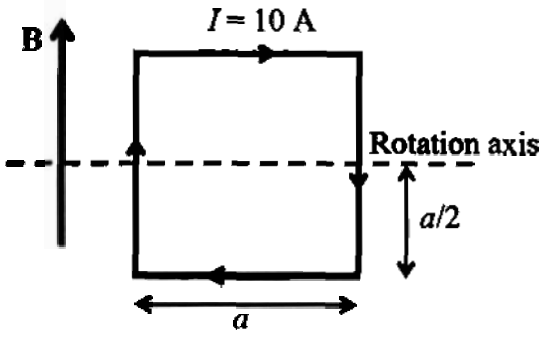
[A] ❶ [B] ❷

[C] ❸ [D] None of these

$\vec{\tau} = \vec{\mu} \times \vec{B} = I\vec{A} \times \vec{B}$

$\tau = IAB \sin\theta$

2. A square conducting coil with 50 turns and sides of length $a = 20.0$ cm is located in a region of uniform magnetic field pointing toward the top of the page. The current in the coil is $I = 10.0$ A, and the magnitude of the magnetic field is $B = 100$ mT.



(a) Calculate the ***magnitude*** and ***direction*** of the magnetic force on each side of the coil (that is find the magnetic force on the top, bottom, left, and right sides of the coil).

(b) If the coil is allowed to rotate about a horizontal axis that bisects it (see figure) what is the maximum torque exerted on the coil? Indicate on the figure above the direction of rotation for the coil.

a) $\vec{F} = I\vec{L} \times \vec{B}$ $F = ILB \sin\theta$

$F_{top} = N(10A)aB = N10aB$ out of plane $N10aB = (50)(10A)(0.2m)(0.1T) = 10.0N$

$F_{sides} = 0$ $\theta = 0°$ or $180°$

$F_{Bott} = N10aB$ Into plane $NIaB = 10.0N$

b) Top rotates out of plane & bottom rotates into the plane

$\vec{\tau} = \vec{r} \times \vec{F}$

$\tau = N\frac{a}{2}10aB + N\frac{a}{2}10aB = N(10a^2B) = (50)(10A)(0.2m)^2(0.1T) = 2.0\ N\text{-}m$

-OR-

$\vec{\tau} = \vec{\mu} \times \vec{B} = NI\vec{A} \times \vec{B} = NIAB \sin\theta$

$\tau_{MAX} = NIAB = 50(10A)(0.2m)(0.2m)(0.1T) = 2.0\ N\text{-}m$

27.30 Crossed $\vec{E}$ and B Fields. A particle with initial velocity $\vec{v}_0 = (5.85 \times 10^3\ \text{m/s})\hat{j}$ enters a region of uniform electric and magnetic fields. The magnetic field in the region is $\vec{B} = -(1.35\ \text{T})\hat{k}$. Calculate the magnitude and direction of the electric field in the region if the particle is to pass through undeflected, for a particle of charge a) +0.640 nC; b) −0.320 nC. You can ignore the weight of the particle.

$$\vec{F} = m\vec{a} = q\left[\vec{E} + \vec{v} \times \vec{B}\right] = 0$$

$$\therefore \quad \vec{E} = -(\vec{v} \times \vec{B})$$

$$\vec{E} = -\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5.85\times10^3 & 0 \\ 0 & 0 & -1.35 \end{vmatrix} = (5.85\times10^3)(-1.35)\hat{i}$$

$$= \boxed{7{,}900\ \frac{N}{C}\ \hat{i}}$$

27.39 A thin, 50.0-cm long metal bar with mass 750 g rests on, but is not attached to, two metallic supports in a uniform 0.450-T magnetic field, as shown in Fig. 27.45. A battery and a 25.0-Ω resistor in series are connected to the supports. a) What is the largest voltage the battery can have without breaking the circuit at the supports? b) The battery voltage has the maximum value calculated in part (a). If the resistor suddenly gets partially short-circuited, decreasing its resistance to 2.0 Ω, find the initial acceleration of the bar.

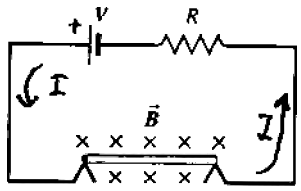
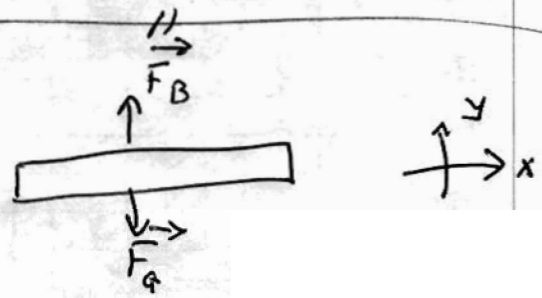


Figure 27.45 Exercise 27.39.



$$\vec{F}_B = I\vec{L} \times \vec{B}$$

a) $\Sigma F_y = F_B - F_G = ma_y = 0$

$$0 = ILB - mg \Rightarrow I = \frac{mg}{LB} = \frac{(0.75\ kg)(9.8\ m/s^2)}{(0.5m)(0.45T)} = \boxed{32.7A}$$

$$V = IR = (32.7A)(25.0\Omega) = \boxed{817\ V}$$

b) $V = IR \Rightarrow I = V/R = \frac{817V}{2.0\Omega} = \boxed{408.3A}$

$$ma = F_B - F_g = ILB - mg$$

$$a = \frac{ILB}{m} - g = \frac{(408.3A)(0.5m)(0.45T)}{0.75kg} - 9.8\ m/s^2$$

$$\boxed{a = 112.7\ m/s^2}$$

Section 27.7 Force and Torque on a Current Loop

27.42 The plane of a 5.0 cm × 8.0 cm rectangular loop of wire is parallel to a 0.19-T magnetic field. The loop carries a current of 6.2 A. a) What torque acts on the loop? b) What is the magnetic moment of the loop? c) What is the maximum torque that can be obtained with the same total length of wire carrying the same current in this magnetic field?

b) $\vec{\mu} = NI\vec{A}$

$|\ = (1.0)(6.2A)(0.05m)(0.08m)$

$|\vec{\mu}| = 0.0248\ A\text{-}m$

a) $\vec{\tau} = \vec{\mu} \times \vec{B}$

$\tau = \mu B \sin\theta = (0.0248\ A\text{-}m)(0.19T)\sin 90^\circ = \boxed{0.00471\ N\text{-}m}$

c) Try a circular loop

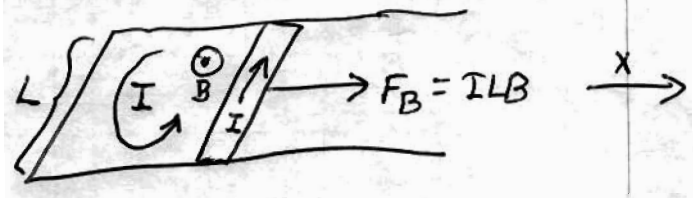
$C = 26cm$ $\quad C = 2\pi r \Rightarrow r = \frac{26cm}{2\pi} = 4.138\ cm$

$A = \pi r^2 = \pi(0.04138m)^2 = 0.00538\ m^2$

$\tau_{\substack{Circle \\ MAX}} = NIAB\sin 90^\circ = (6.2A)(0.00538m^2)(0.19T)$

$= \boxed{0.00634\ N\text{-}m}$

27.66 An Electromagnetic Rail Gun. A conducting bar with mass m and length L slides over horizontal rails that are connected to a voltage source. The voltage source maintains a constant current I in the rails and bar, and a constant, uniform, vertical magnetic field $\vec{B}$ fills the region between the rails (see Fig. 27.54). a) Find the magnitude and direction of the net force on the conducting bar. Ignore friction, air resistance, and electrical resistance. b) If the bar has mass m, find the distance d that the bar must move along the rails from rest to attain speed v. c) It has been suggested that rail guns based on this principle could accelerate payloads into earth orbit or beyond. Find the distance the bar must travel along the rails if it is to reach the escape speed for the earth (11.2 km/s). Let $B = 0.50$ T, $I = 2.0 \times 10^3$ A, $m = 25$ kg, and $L = 50$ cm. For simplicity asssume the net force on the object is equal to the magnetic force, as in parts (a) and (b) even though gravity plays an important role in an actual launch in space.



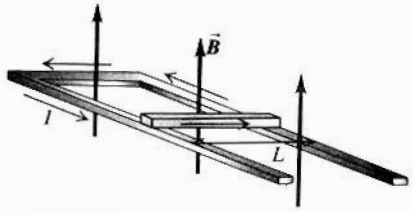


Figure 27.54 Problem 27.66.

a) $F_B = ILB$

b) $ma = ILB \Rightarrow a = \frac{ILB}{m}$

$v^2 - \cancel{v_0^2} = 2a\Delta x$

$v^2 = \frac{2ILB}{m}\Delta x$

$v = \left(\frac{2ILB\Delta x}{m}\right)^{1/2}$

or $\Delta x = \frac{mv^2}{2ILB} = \frac{(25kg)(11.2\times10^3 m/s)^2}{2(2{,}000A)(0.5m)(0.5T)}$

$\Delta x = \boxed{3.136\times10^6 m = 3{,}136\ km = 1{,}940\ miles}$

27.76 The rectangular loop shown in Fig. 27.60 is pivoted about the y-axis and carries a current of 15.0 A in the direction indicated. a) If the loop is in a uniform magnetic field with magnitude 0.48 T in the $+x$-direction, find the magnitude and direction of the torque required to hold the loop in the position shown. b) Repeat part (a) for the case in which the field is in the $-z$-direction. c) For each of the above magnetic fields, what torque would be required if the loop were pivoted about an axis through its center, parallel to the y-axis?

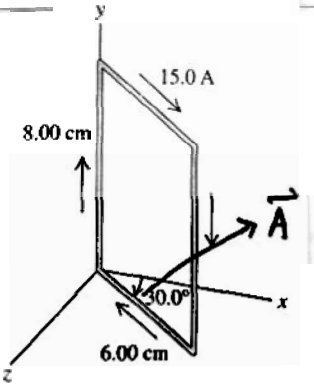


Figure 27.60 Problem 27.76.

a) $\tau = \vec{\mu} \times \vec{B} = NI\vec{A} \times \vec{B}$

$\tau = NIAB \sin\theta$

$\tau = (15A)(0.08m)(0.06m)(0.48T) \sin 60^\circ$

$\tau = 0.0299\ N\text{-}m$ in neg y-direction

To hold in place, need

$0.0299 N\text{-}m$ in $+y$ direction

b) $\tau = (15A)(0.08m)(0.06m)(0.48T) \sin 30^\circ$

$\tau = 0.0173 N\text{-}m$ in positive y-direction

c) Same answers