49. Let $H_0$ be a Hamiltonian with a degenerate spectrum of energies $\{\varepsilon_n^{(0)}\}$. Let the degeneracy of level $n$ be denoted by $N_n$, and let $S_n^{(0)}$ denote the $N_n$ dimensional eigenspace of $H_0$ associated with energy $\varepsilon_n^{(0)}$. A weak perturbation $H^{(1)}$ is applied. Briefly, but clearly describe the basic technique of degenerate perturbation theory, and describe what information one generally obtains by applying that technique.
50. A two dimensional isotropic harmonic oscillator, expressed in appropriate dimensionless variables

\[ H^{(0)} = \sum_{i=x,y} \frac{\hbar \omega}{2} (\hat{q}_i^2 + \hat{p}_i^2) = \sum_{i=x,y} \hat{H}_i^{(0)} \]

is subject to a perturbation

\[ H^{(1)} = \lambda \hbar \omega \hat{q}_x \hat{q}_y. \]

(a) Find the unperturbed energy levels and the degeneracies of the unperturbed Hamiltonian, and express the eigenstates as appropriate direct product states formed from the 1d eigenstates of the implicitly defined operators \( H_x^{(0)} \) and \( H_y^{(0)} \).

(b) Find the first order energy level splitting, if any, of the lowest three (possibly degenerate) unperturbed energy levels of this system. Plot the corresponding energies as a function of the parameter \( \lambda \).

(c) For the first excited state, find a new basis of eigenstates for that unperturbed level that are not connected by the perturbation, and show that at least some of them are not direct product states.
51. A system with Hamiltonian $H^{(0)}$ is subject to a perturbation $H^{(1)}$, which in a certain ONB can be represented by the following matrices

$$[H^{(0)}] = \begin{pmatrix} \varepsilon_0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_0 & 0 & 0 \\ 0 & 0 & 0 & 2\varepsilon_0 & 0 \\ 0 & 0 & 0 & 0 & 2\varepsilon_0 \end{pmatrix} \quad [H^{(1)}] = \begin{pmatrix} 0 & \Delta & 0 & 3\Delta & 0 \\ \Delta & 0 & 0 & 0 & 0 \\ 0 & 0 & 4\Delta & 0 & 0 \\ 3\Delta & 0 & 0 & 0 & 2i\Delta \\ 0 & 0 & 0 & -2i\Delta & 0 \end{pmatrix}$$

(a) Find the new energies of this system, correct to first order in the perturbation.

(b) Find a new basis of eigenstates of $H^{(0)}$ such that states of the same unperturbed energy are not connected by $H^{(1)}$. 

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52. Briefly but clearly state the four postulates (feel free to look these up) of the general formulation of quantum mechanics as they apply to arbitrary quantum mechanical systems (e.g., to the universe itself) and explain the differences (where they arise) between those postulates and the postulates of Schrödinger's wave mechanics for a single particle.

Does the universe have a wave function, a state vector, both, neither? Explain. (no wrong answer here, just interested in what you think.)