33. Consider a one-dimensional harmonic oscillator with $H = \frac{1}{2} \hbar \omega (\hat{q}^2 + \hat{p}^2)$. Find and solve the equations of motion for the mean values $\langle \hat{q}(t) \rangle$ and $\langle \hat{p}(t) \rangle$, expressing the solution in terms of the initial values $q_0 = \langle \hat{q}(0) \rangle$ and $p_0 = \langle \hat{p}(0) \rangle$. 
34. Compute, using any correct symmetry arguments you wish, the following expectation values taken with respect to the eigenstates of the harmonic oscillator Hamiltonian: \( \langle n|q^3|n \rangle \), \( \langle n|p^3|n \rangle \), \( \langle n|q^4|n \rangle \), \( \langle n|p^4|n \rangle \).
35. We have evaluated the Harmonic oscillator ground state wave function $\phi_0(q)$ in the position representation.

(a) Take an appropriate Fourier transform of $\phi_0(q)$ to find $\phi_0(p)$ in the (dimensionless) momentum representation.

(b) By considering the expression for $a^+|n\rangle$ in the momentum representation, determine the form of the wave functions $\phi_n(p) = \langle p|n\rangle$ and show that they can be expressed in terms of the Hermite polynomials.
36. The destruction operator \( a = (\hat{q} + i\hat{p})/\sqrt{2} \) is not Hermitian and is not normal. It does, however, have eigenstates (often referred to as "coherent states"). Let \( |\alpha\rangle \) denote the square-normalized eigenstate of \( a \) with eigenvalue \( \alpha \) (so that \( a|\alpha\rangle = \alpha|\alpha\rangle \)).

(a) Solve the eigenvalue equation for the state \( |\alpha\rangle \) directly in the \( q \)-representation by solving an appropriate first-order differential equation for the wave function \( \phi_\alpha(q) = \langle q|\alpha\rangle \). For what values of \( \alpha \) will square-normalizeable solutions exist? Show that eigenstates with different eigenvalues \( \alpha \) and \( \alpha' \), say, are never orthogonal.

(b) Show, working in the \( q \)-representation, that the operator \( a^+ \) has no non-trivial normalizeable eigenkets.