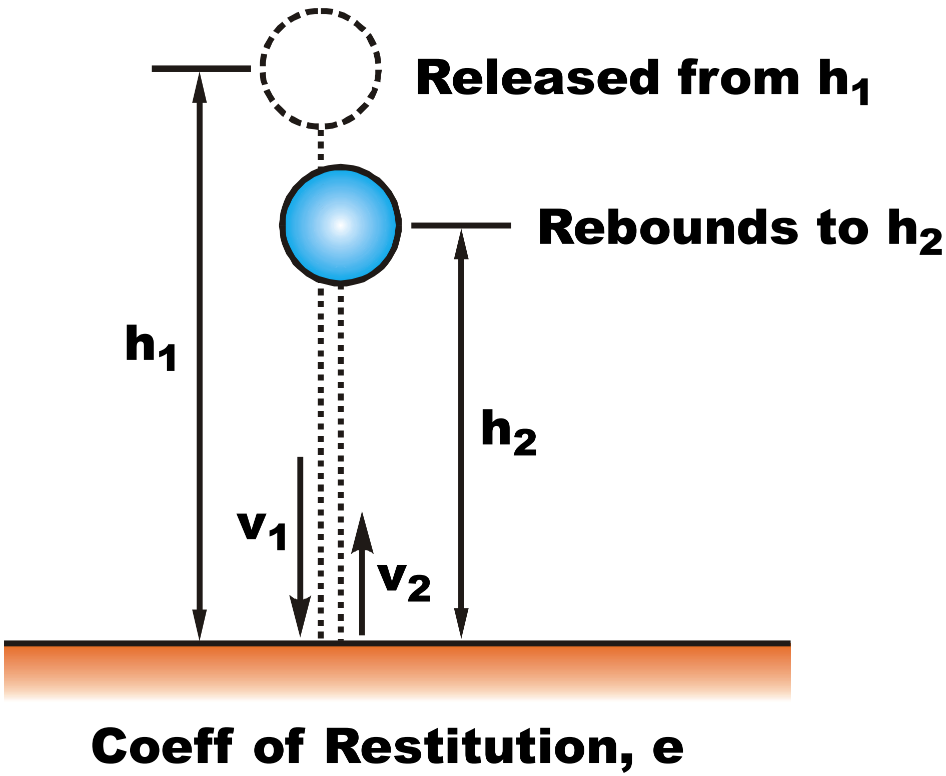
Particle Impact: Ex Prob 3 (Ball on Surface)

A ball is released from some height, h_1, and reaches a velocity v_1 just before striking the ground. The ball rebounds with a velocity v_2 and rises to some height h_2. Write equations that:

(a) Relate v_1, v_2, and e, and (b) Relate h_1, h_2, and e.

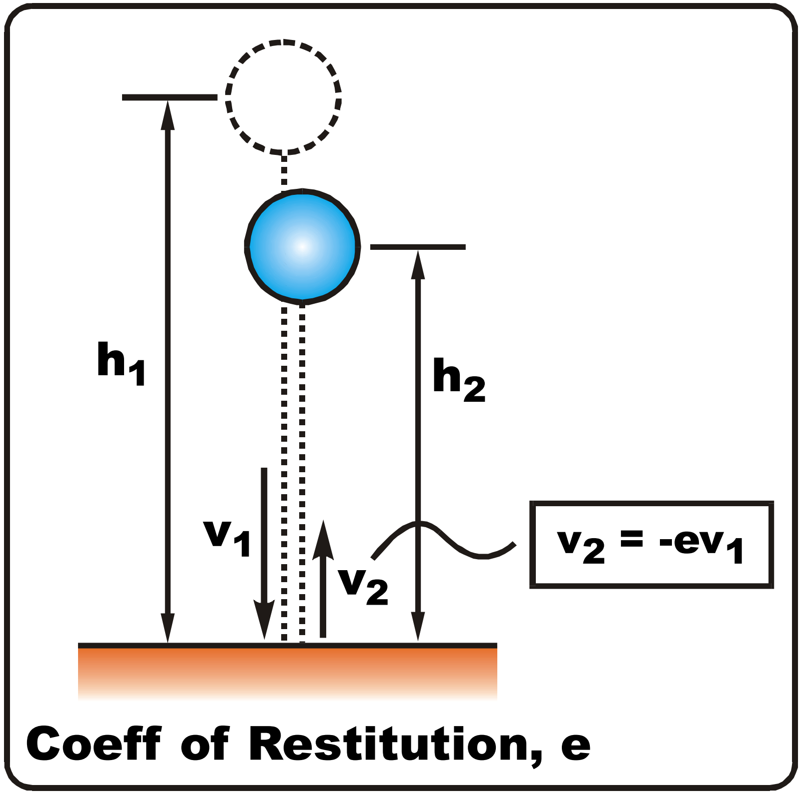


Think about this. The ball rebounds to h_2 (lower than h_1) because energy is lost in the impact with the ground. With each successive bounce, the rebound heights are lower and lower, decaying relatively quickly. How can we calculate these heights? That's what this problem is about.

First relate v_1, v_2, and e.

What impact equations can be used when a particle strikes a solid surface like this? We cannot use conservation of momentum because we cannot quantify the velocity of the floor nor the "mass" of the region of the floor involved in the impact.

Consider the e equation between the ball and the floor:



$$e = \frac{v_{Ball2} - \cancelto{0}{v_{Floor2}}}{\cancelto{0}{v_{Floor1}} - v_{Ball1}} = \frac{v_{Ball2}}{-v_{Ball1}} = \frac{v_2}{-v_1}$$

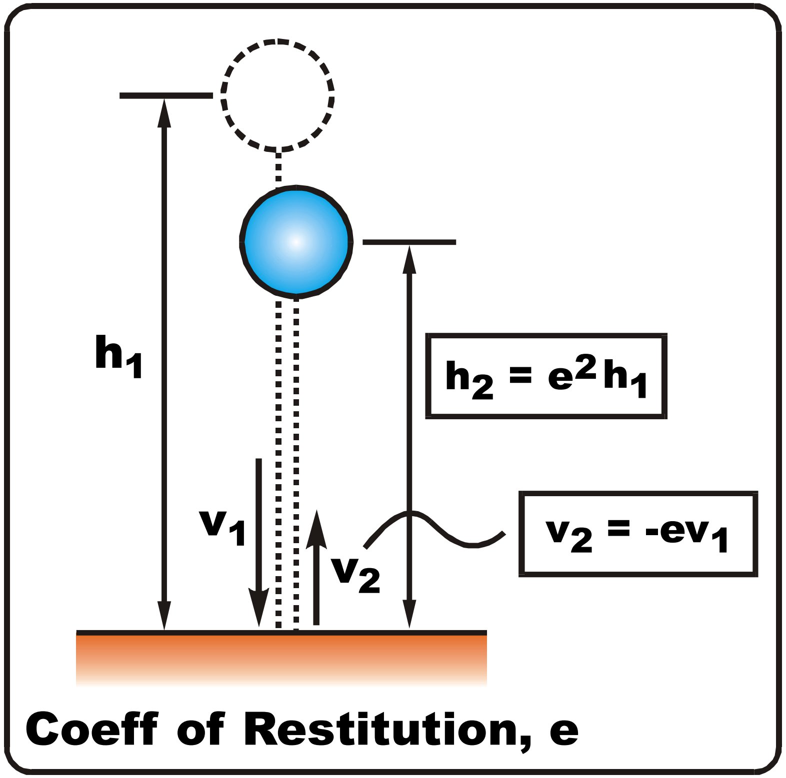
Key result: $\boxed{v_2 = -ev_1}$

Why the neg sign? Because v_2 acts in the opposite direction of v_1.

Note also: This is a simplified model of impact. Impact can be more complicated than this ev_1 model!

Now relate h_1, h_2, and e.

Write a work-energy equation for the rebound and the fall, and divide the latter into the former:



$$\frac{\text{Rebound}}{\text{Fall}} = \frac{\frac{1}{2}mv_2^2}{\frac{1}{2}mv_1^2} = \frac{mgh_2}{mgh_1}$$

Canceling like terms gives:

$$\frac{v_2^2}{v_1^2} = \frac{h_2}{h_1} = \frac{(-ev_1)^2}{v_1^2}$$

Sub in

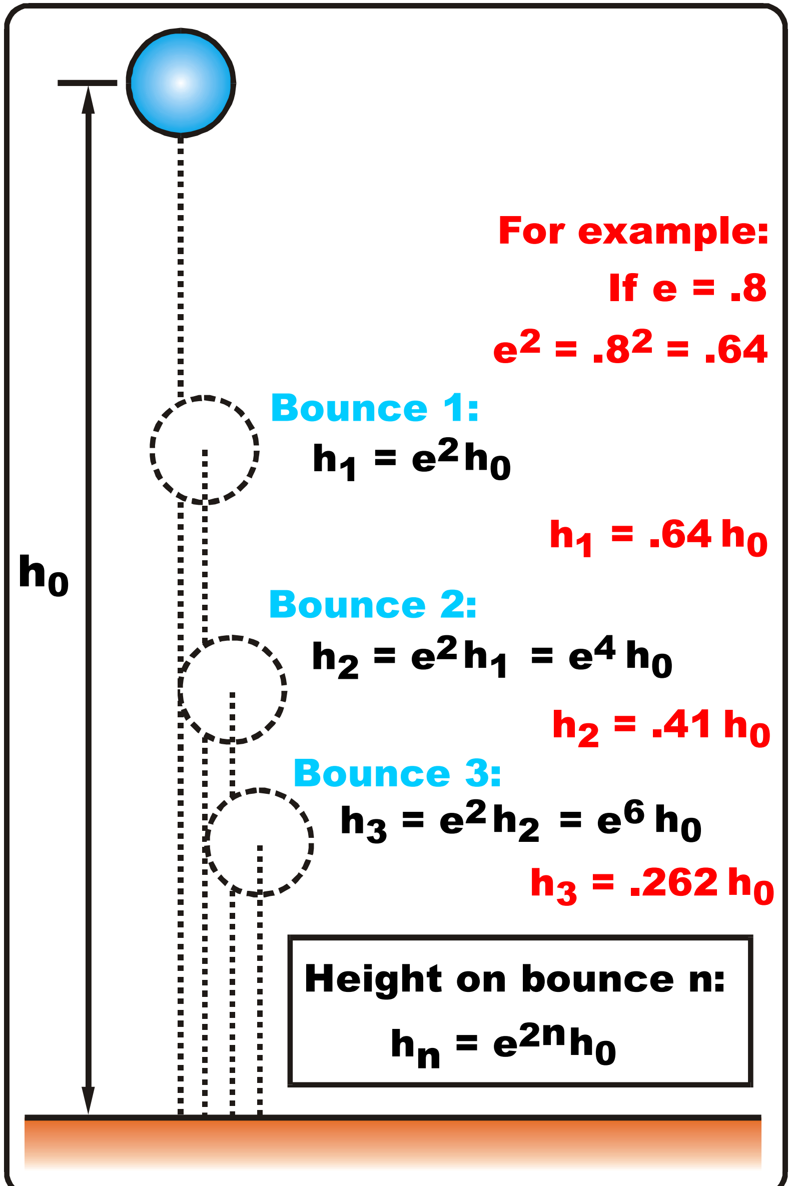
$$\boxed{v_2 = -ev_1}$$

$$\boxed{\frac{h_2}{h_1} = e^2}$$

Two other ways to write the same h_1, h_2, and e equation:

$$\boxed{e = \sqrt{\frac{h_2}{h_1}}} \qquad \boxed{h_2 = e^2 h_1}$$

Think about these equations….



1. First let's discuss: $\boxed{h_2 = e^2 h_1}$

Bounce height, h, is a measure of (potential) energy.

If $h_1 = e^2 h_0$, then on the first bounce e^2 of the original energy is retained.

Each successive bounce height is e^2 of the previous bounce.

Example: Let e = .8. This seems fairly high, doesn't it?
But $.8^2 = .64 = 64\%$

On the first bounce, h_1 = 64% of h_0. Then, h_2 = 64% of 64% of h_0.

And so on….

Think about these eqns….

2. How would you measure e ?

Use this equation: $\boxed{e = \sqrt{\dfrac{h_2}{h_1}}}$

In class we often do the demonstration shown at right.

Get a tennis ball and a yard stick.

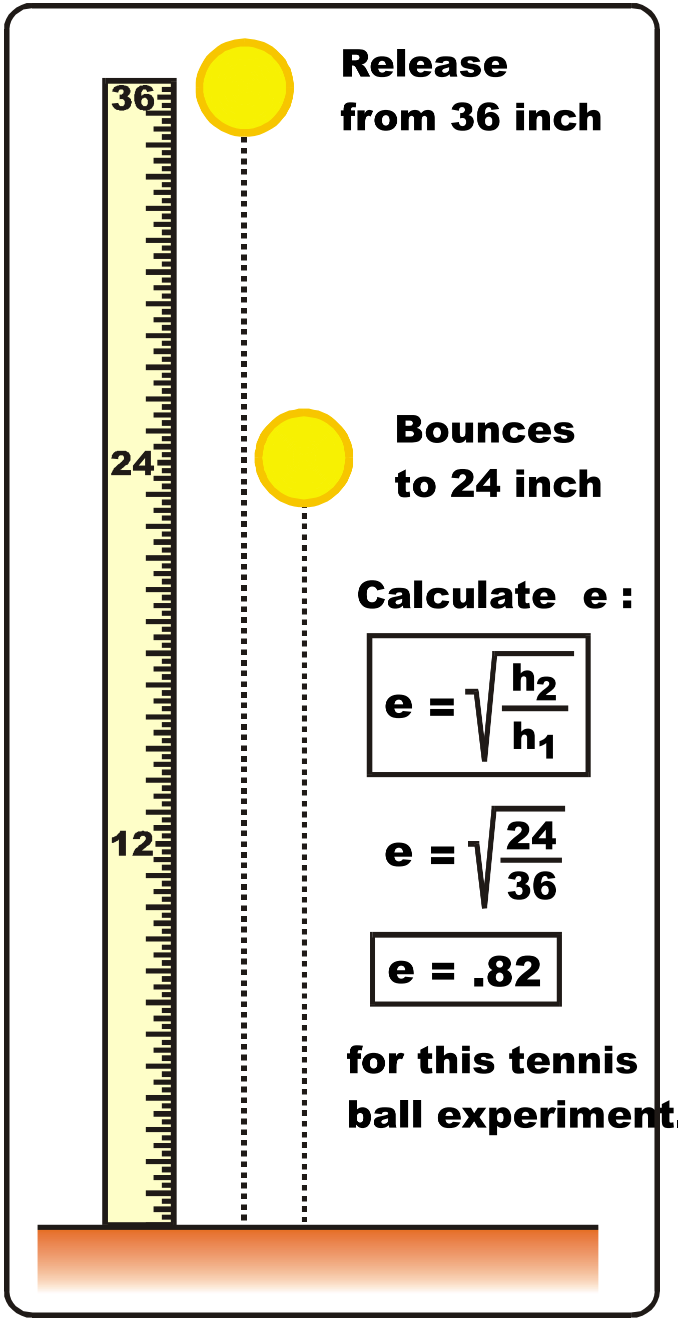
Drop the tennis ball from 36 inch and see how high it bounces.

Let's say it bounces to 24 inch.

(I think this is too high…it's more like 20 inches, I think…)

Then, $e = (24/36)^{1/2} = 0.82$ for this example.

Try this with other balls on various surfaces.



$$\boxed{e = \sqrt{\frac{h_2}{h_1}}}$$

$$e = \sqrt{\frac{24}{36}}$$

$$\boxed{e = .82}$$

for this tennis ball experiment.