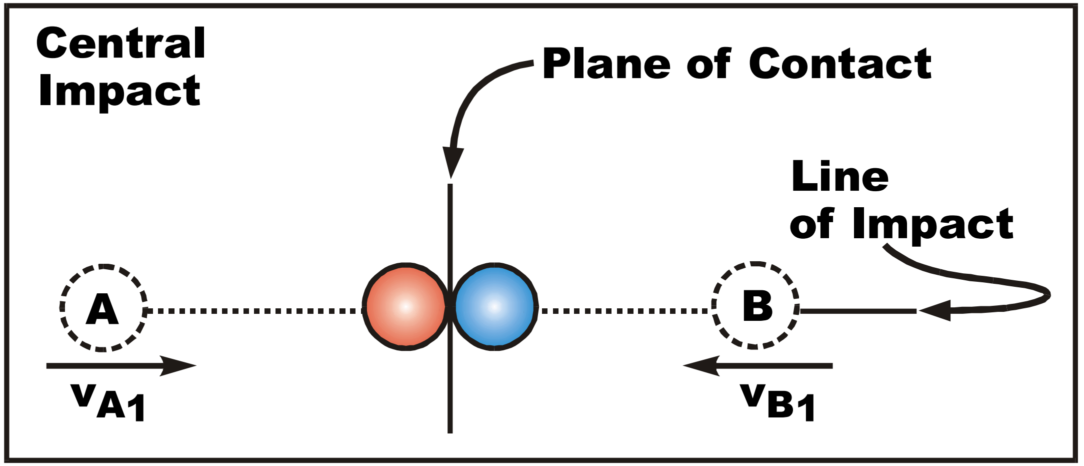
Impact for Particles

The focus of this class is particle impact. We will not discuss impact for rigid bodies.

Terminology:

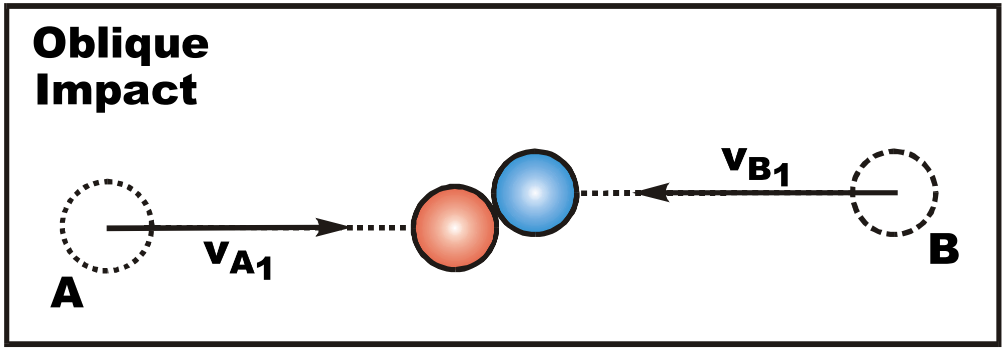
1. Central Impact:

The incident and departure velocities of the two particles are collinear. (After impact, A and B move along same line.)



2. Oblique Impact:

(See next page)



Terminology (cont'd): Oblique Impact

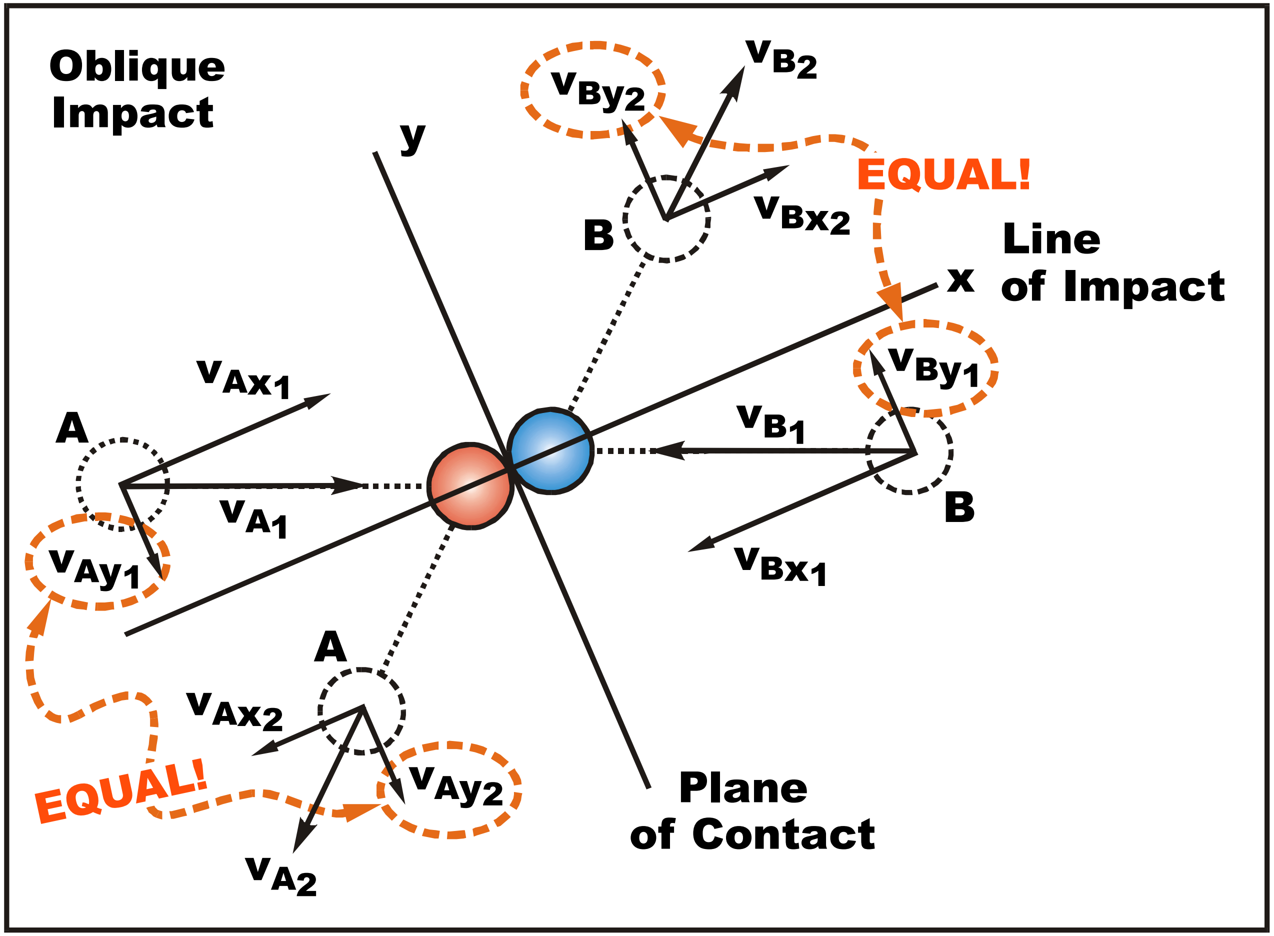
2. Oblique Impact: Initial velocities of A and B are NOT collinear. Particles A and B strike a glancing blow and their departure velocities (at least one, A or B) are at angles to their initial velocities.

Important!!

You must draw and label the **plane of contact** and **line of impact**.

Assume **no friction impulse** along the plane of contact, thus:

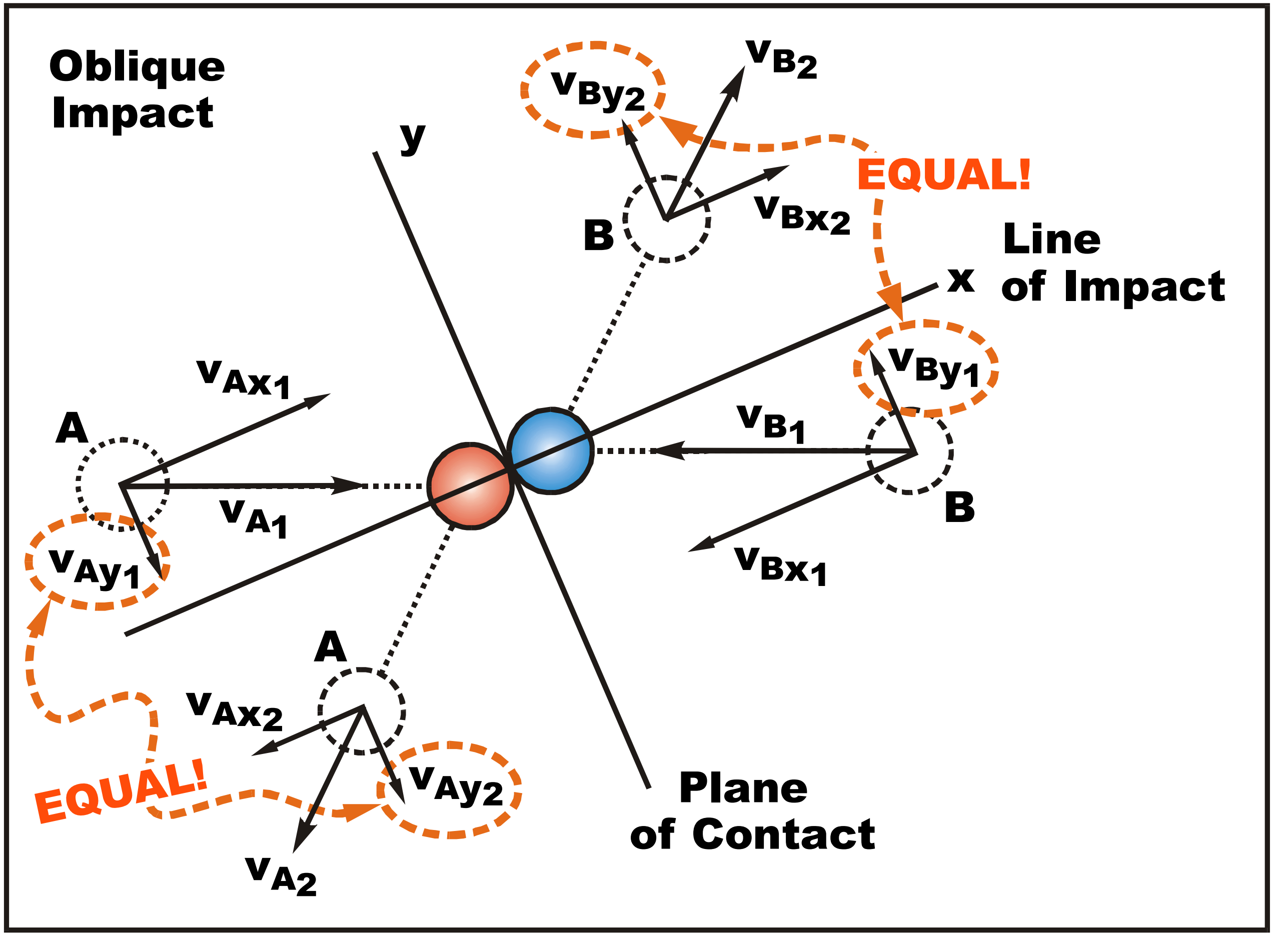
$$v_{Ay2} = v_{Ay1}$$
$$v_{By2} = v_{By1}$$

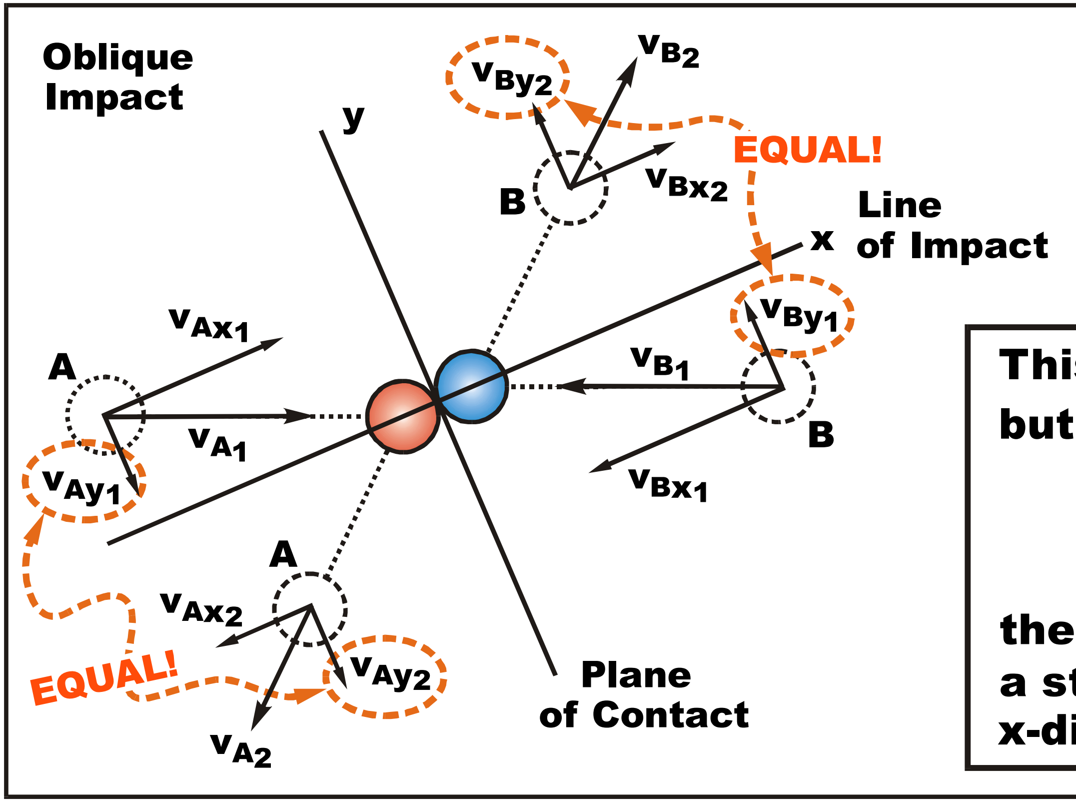


Terminology (cont'd): Oblique Impact

Labeling the plane of contact vs. the line of impact: Some texts label these tangential (t) and normal (n), but these confuse me. I always have to think about which is which. So, I usually just draw the picture and label them x and y. You may do either, whichever makes sense to you.

The point is this: Draw them and label them with something! Then, resolve the vectors into components along these axes.





Resolving vectors along the plane of contact greatly aids solving the problem!

This looks complicated, but once you see that

$$v_{Ay2} = v_{Ay1}$$

$$v_{By2} = v_{By1}$$

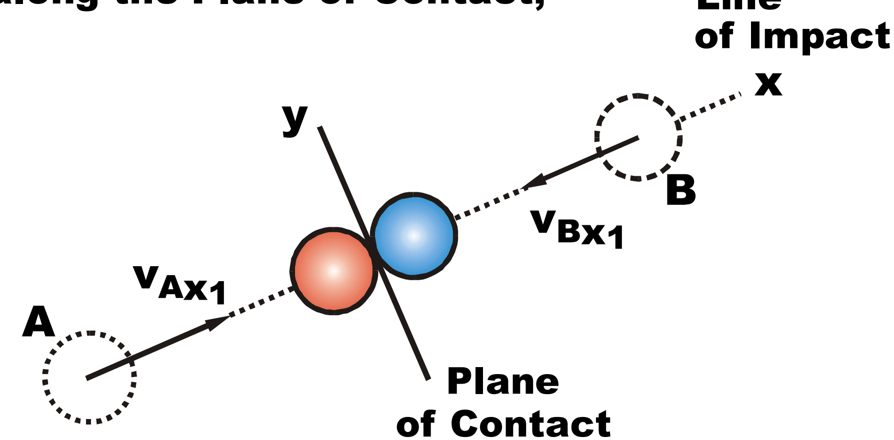
then the problem becomes a straight line one in the x-direction.

Once you recognize that along the Plane of Contact,

$$v_{Ay2} = v_{Ay1}$$

$$v_{By2} = v_{By1}$$

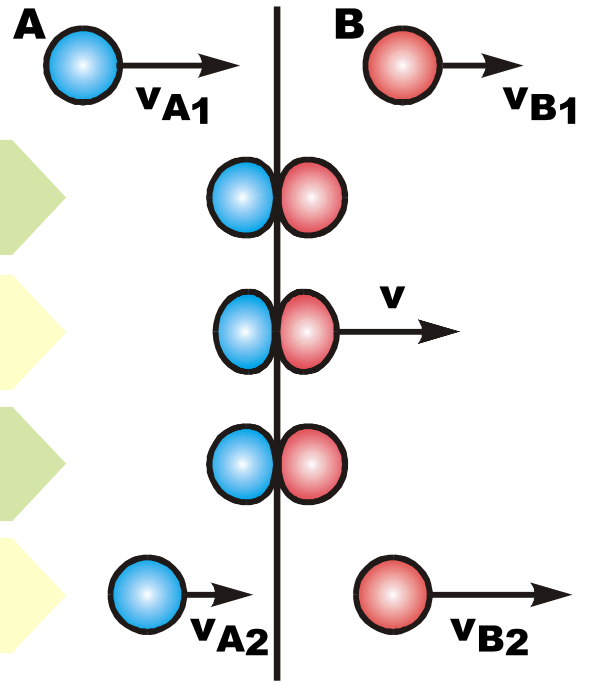
all that remains is a simple straight line problem in the x-. direction only



Details of Impact; Coefficient of Restitution

What happens during impact? An Up-Close View:

Description of Phase of Impact	A Close View of Impact of Particles A and B
Before impact, relative velocity $v_{B1/A1} = v_{B1} - v_{A1}$	
Deformation phase: Kinetic energy stored as deformation	
Max deformation: A and B momentarily share same velocity, v.	
Restitution phase: Energy stored as deformation converted back to KE.	
After impact: If e < 1, some KE lost. Rel Velocity: $v_{B2/A2} = v_{B2} - v_{A2}$	



Coefficient of Restitution, e:

$$e = \frac{-v_{B2/A2}}{v_{B1/A1}} = \frac{-(v_{B2} - v_{A2})}{(v_{B1} - v_{A1})} = \frac{(v_{B2} - v_{A2})}{(v_{A1} - v_{B1})}$$

Details of Impact; Coefficient of Restitution

Coefficient of Restitution, e: (Abbreviated as "COR")

COR, e, is a measure of the energy stored in deformation during impact which is recovered back to kinetic energy.

More precisely…. (see your text for a derivation…)

$$e = \frac{\text{Relative Departure Velocity}}{\text{Relative Incident Velocity}} = \frac{-v_{B2/A2}}{v_{B1/A1}} = \frac{-(v_{B2} - v_{A2})}{(v_{B1} - v_{A1})} = \frac{(v_{B2} - v_{A2})}{(v_{A1} - v_{B1})}$$

Cases: $e = 1$ "Perfectly Elastic"

Rel Departure Velocity = Rel Incident Velocity

$e = 0$ "Perfectly Plastic" (Particles stick together…)

Rel Departure Velocity = 0

$0 < e < 1$ Range for e is between zero and one.

Typical values: .5 to .8 for balls

More on Coefficient of Restitution (COR)

Kinetic Energy recovered after impact is approx e^2 .
A COR (e) of 0.8 sounds high. But the KE after impact is $(.8)^2$ = 64% of the original KE, meaning 36% of KE was lost!

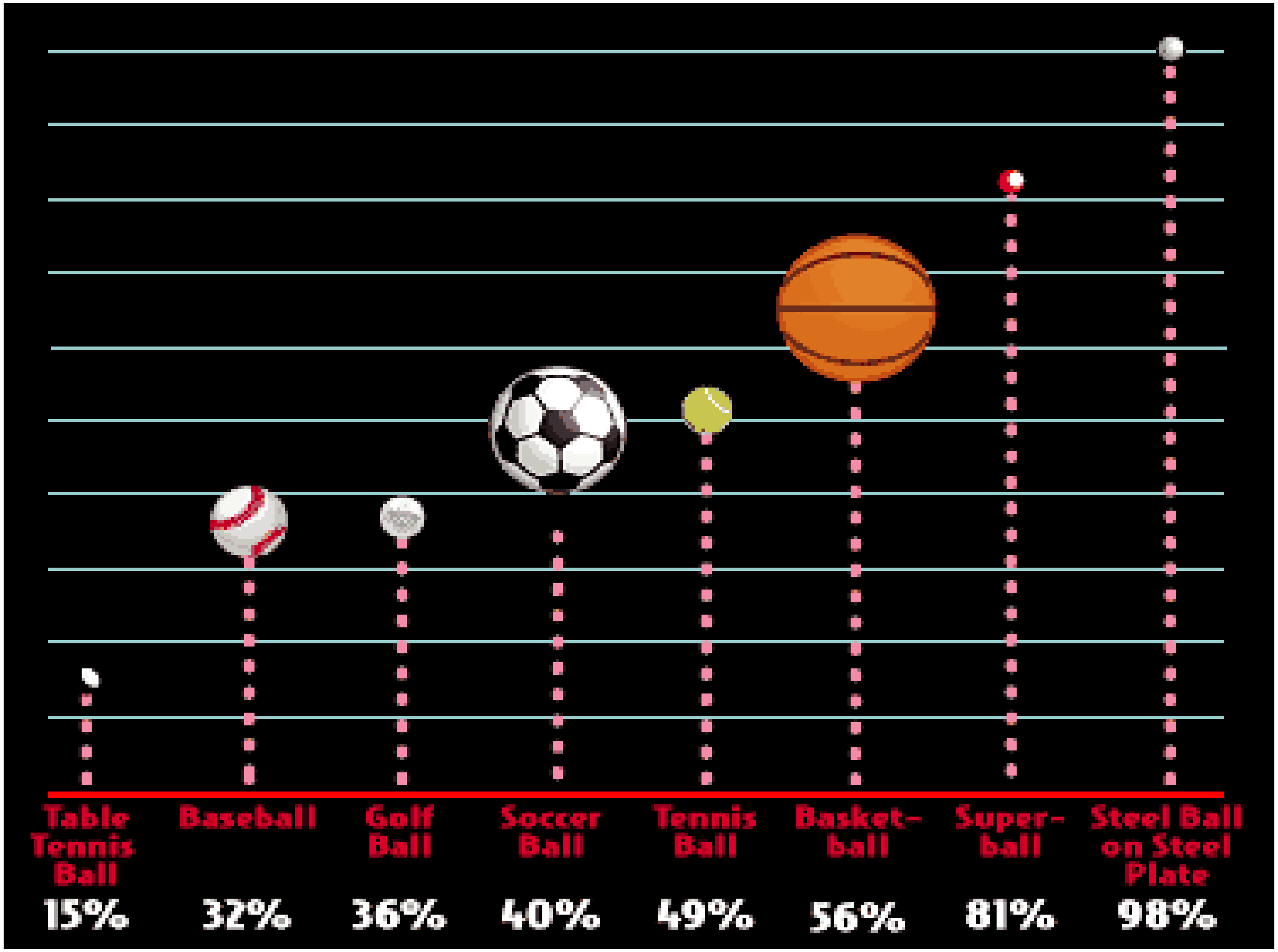
Applications:

Golf Drivers: The USGA limits the COR of a driver's face to be no greater than COR = 0.83. They have on-site testing facilities to test compliance (if requested).

High school and college metal baseball bats: Using metal bats saves money because wood bats break. But metal bats have a higher COR (batted balls have a 5-10% higher velocity off of a metal bat) than wooden bats. Batting and slugging averages are inflated because of this. Pitchers are also subject to injury.

Cool high speed videos of balls on bats:
http://www.kettering.edu/~drussell/bats-new/ball-bat-0.html
Also excellent information at this site about bats and balls. Highly recommended.

Various Balls: Rebound Height (function of energy recovered from bounce collision)



Equations for Impact Problems

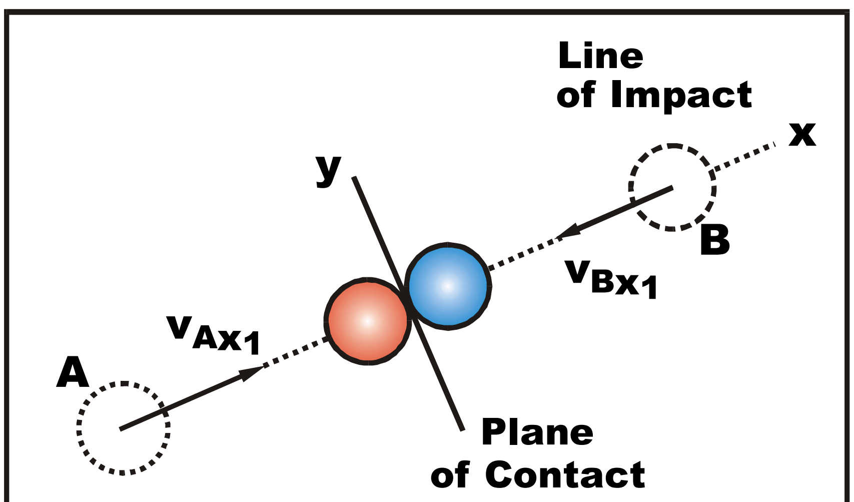
(Let x be the Line of Impact, y be the Plane of Contact)

Along the Plane of Contact:
(Assumes no friction impulse along this plane....)

$$v_{Ay_2} = v_{Ay_1}$$
$$v_{By_2} = v_{By_1}$$

Along the Line of Impact (Conservation of Momentum:

$$(+\nearrow) \quad m_A v_{Ax_1} + m_B v_{Bx_1} = m_A v_{Ax_2} + m_B v_{Bx_2}$$



Also along the Line of Impact:

$$(+\nearrow) \quad e = \frac{(v_{Bx_2} - v_{Ax_2})}{(v_{Ax_1} - v_{Bx_1})}$$

Use a consistent sign convention for v's in these equations.

Along the Line of Impact (x-direction…)

Write TWO equations to solve for TWO unknowns (v_{Ax2}, v_{Bx2}):

Equation 1: Conservation of Momentum

$$\overset{+}{\nearrow} \quad m_A v_{Ax_1} + m_B v_{Bx_1} = m_A v_{Ax_2} + m_B v_{Bx_2}$$

Equation 2: "e Equation", Coeff of Restitution Equation

$$\overset{+}{\nearrow} \quad e = \frac{(v_{Bx_2} - v_{Ax_2})}{(v_{Ax_1} - v_{Bx_1})}$$

We know: v_{Ax1}, v_{Bx1}

Solve for: v_{Ax2}, v_{Bx2}

using these two equations.