Impulse-Momentum Equation for Particles

Linear, not Angular, Momentum: In this section, we deal with linear momentum (mv) of particles only. Another section of your book talks about linear (mv_G) and angular ($I_G\omega$) momentum of rigid bodies.

An Integrated Form of F=ma: The impulse-momentum (I-M) equation is a reformulation—an integrated form, like the work-energy equation—of the equation of motion, F=ma.

Particle Impulse-Momentum Equation

Important! This is a Vector Equation!

$$m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2$$

Initial Linear Momentum $= m\vec{v}_1$

"Impulse" $= \int \vec{F}dt$

Final Linear Momentum $= m\vec{v}_2$

Derivation of the Impulse-Momentum Equation

Derivation of the Impulse-Momentum Equation

Typical Eqn of Motion: $\vec{F} = m\vec{a}$

Sub in $\vec{a} = \frac{d\vec{v}}{dt}$ **:** $\vec{F} = m\frac{d\vec{v}}{dt}$

If mass is changing:
(e.g. for a rocket...)
(Newton wrote it this way...)

$$\boxed{\vec{F} = \frac{d}{dt}(m\vec{v})}$$

Net Force = time rate of change of momentum

Separate variables: $\vec{F}\,dt = m\,d\vec{v}$

Set up integrals: $\int_{t_1}^{t_2} \vec{F}\,dt = \int_{v_1}^{v_2} m\,d\vec{v}$

Integrate: $\int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$

Usual form:

$$\boxed{m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2}$$

"Impulse" vs. "Impulsive Force"

Particle Impulse-Momentum Equation

$$m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2$$

Initial Momentum ($m\vec{v}_1$)

"Impulse" $= \int \vec{F}dt$

Final Momentum ($m\vec{v}_2$)

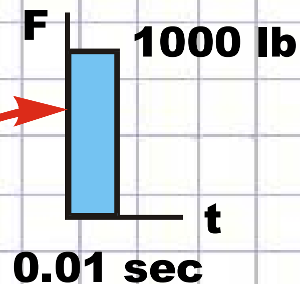
Impulse
= Area under Force-time curve
= Any force acting over any time....
e.g. 100 lb for two weeks....

$$\text{Impulse} = \int \vec{F}dt$$

Impulsive Force:
A relatively large force which acts over a very short period of time, like in an impact, e.g. bat-on-ball, soccer kick, hammer-on-nail, etc.

e.g. 1000 lb acting in a 0.01 sec impact

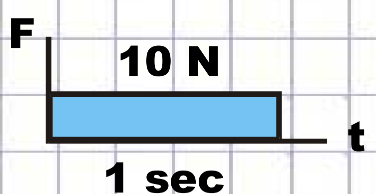
Impulsive Force



Area under F-t curve
= 10 lb-sec
= Impulse

or 10 N acting for 1 second...

Non-Impulsive Force



Area under F-t curve
= 10 N-sec
= Impulse

Applications of the Impulse-Momentum Equation

For any problems involving F, v, t: The impulse momentum equation may be used for *any problems* involving the variables force F, velocity v, and time t. The IM equation is not directly helpful for determining acceleration, a, or displacement, s.

Helpful for impulsive forces: The IM equation is most helpful for problems involving impulsive forces. Impulsive forces are relatively large forces that act over relatively short periods of time, for example during impact. If one knows the velocities, and hence momenta, of a particle before and after the action of an impulse, then one can easily determine the impulse. If the time of impulse is known, then one can calculate the average force F_{avg} that acts during the impulse.

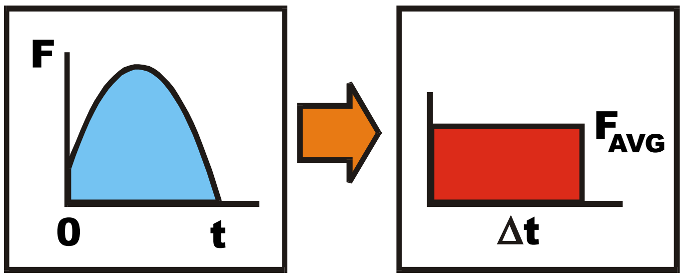
For problems involving graph of F vs. t: Some problems give a graph of Force vs. time. The area under this curve is impulse. Important: You may need to find t_{start} for motion!

Various Forms of the Impulse-Momentum Equation

Various Forms of the I-M Equation

Force During Impact:

Actual Force | **Often Modeled as**



Impulse = Area under curve...

$$= \int \vec{F}\,dt \qquad = \vec{F}_{AVG}\,\Delta t$$

General Form:

$$m\vec{v}_1 + \int \vec{F}\,dt = m\vec{v}_2$$

If force F is constant:

$$m\vec{v}_1 + \vec{F}_{AVG}\,\Delta t = m\vec{v}_2$$

Know vectors $\vec{v}_1$ and $\vec{v}_2$:

$$\int \vec{F}\,dt = m\vec{v}_2 - m\vec{v}_1$$
$$= m(\vec{v}_2 - \vec{v}_1)$$

If force F is constant:

$$\vec{F}_{AVG}\,\Delta t = m\vec{v}_2 - m\vec{v}_1$$
$$= m(\vec{v}_2 - \vec{v}_1)$$

Impulse produces change in momentum

I-M Equation Compared to the Work-Energy Equation

Comparison	Impulse-Momentum Eqn	Work-Energy Eqn
Both derived from F=ma	First write: $\vec{F} = m\frac{d\vec{v}}{dt}$ Integrate: $\int_{t_1}^{t_2} \vec{F}\,dt = \int_{v_1}^{v_2} m\,d\vec{v}$	Work: $dU = F\cdot ds = m\,a\,ds$ Eqn of Motion: $F = ma$ Kinematics: $a\,ds = v\,dv$ Integrate: $\int_1^2 dU = \int_{v_1}^{v_2} mv\,dv$
Equation	$m\vec{v}_1 + \int \vec{F}dt = m\vec{v}_2$	$\frac{1}{2}mv_1^2 + \sum U_{1\text{-}2} = \frac{1}{2}mv_2^2$ or $\frac{1}{2}mv_1^2 + \int Fds = \frac{1}{2}mv_2^2$
Equation Explained	Impulse $\int \vec{F}\,dt$ produces a change in momentum, $\Delta m\vec{v}$	Work $\sum U_{1\text{-}2}$ or $\int Fds$ effects a change in KE, $\Delta\frac{1}{2}mv^2$
Vector or Scalar?	A vector equation!	A scalar equation!
Applications	Problems involving v, F, t. Not for problems needing accel, a, or displacement, s.	Problems involving v, F, s. Not for problems needing accel, a, or time, t.