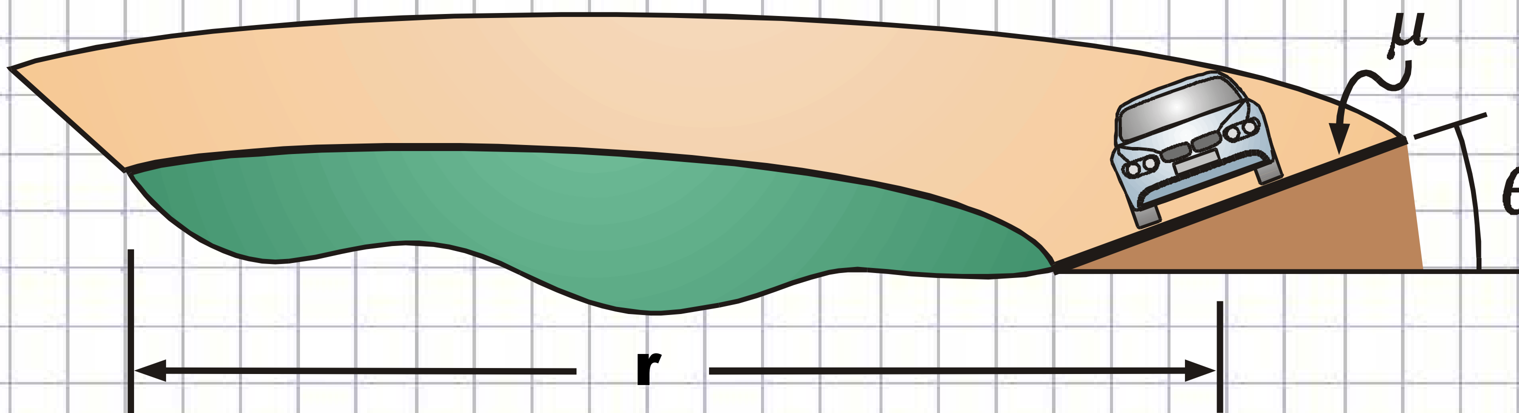
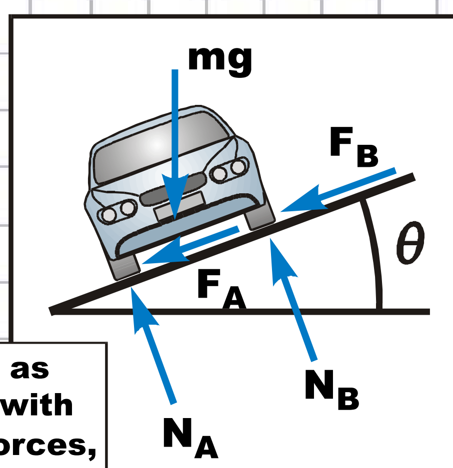
Particle F=ma (n-t): Example Problem 5

An automobile drives around a banked curve on a rural road. (a) What direction does friction act? (b) Obtain an equation that relates car speed v, bank angle θ, curve radius r, and friction μ.

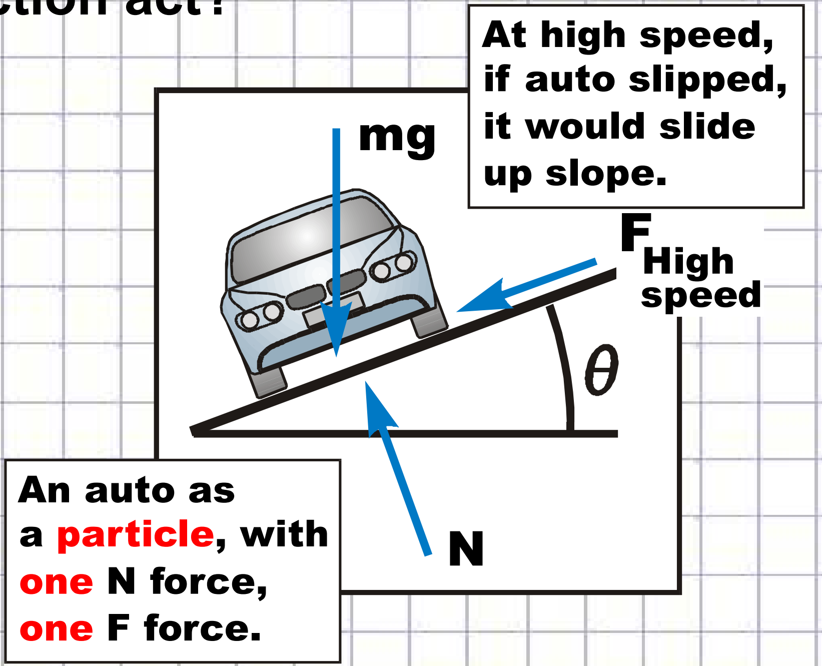


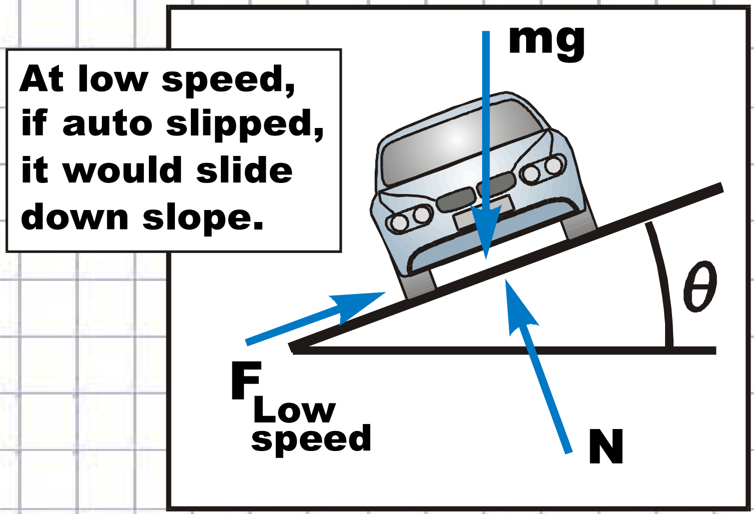


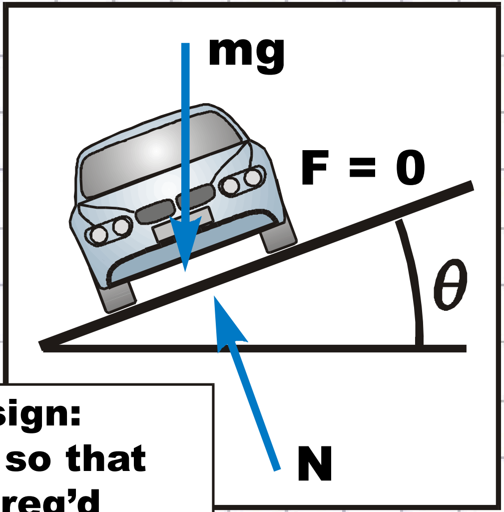
An auto as a body, with two N forces, two F forces.

Again, what direction does friction act?

Answer: "It depends!"

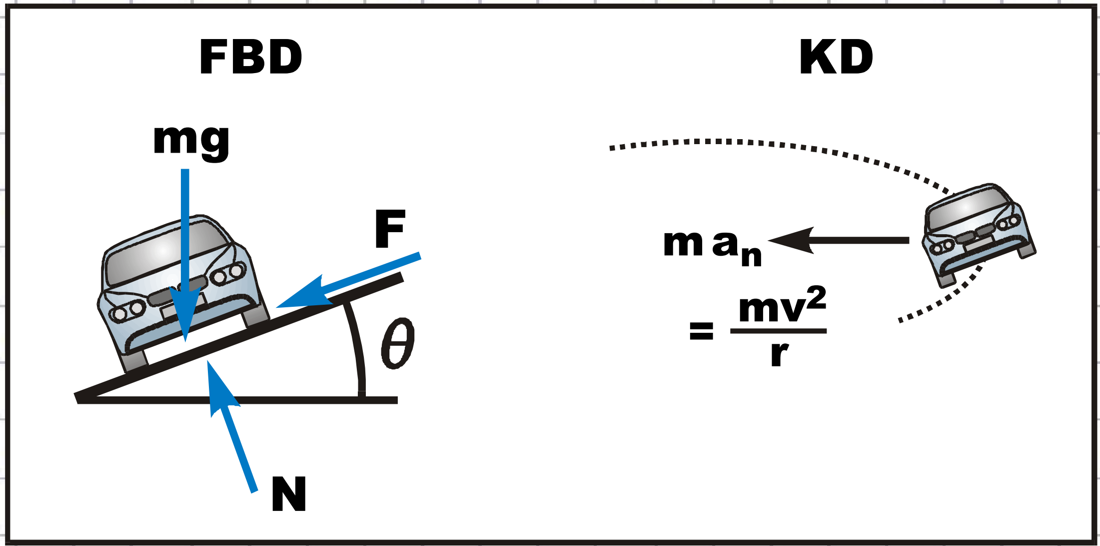






Optimal Design:
Hwy banked so that zero friction req'd to keep auto on curve.

Obtain an equation that relates car speed v, bank angle θ, curve radius r, and friction μ.



Eqns of Motion from FBD

$$\xleftarrow{+} \quad N(\sin\theta) + F(\cos\theta) = \frac{mv^2}{r}$$

$$+\uparrow \quad N(\cos\theta) - F(\sin\theta) - mg = 0$$

Assume on verge of slip.

Sub in

$$F = \mu N$$

① $N(\sin\theta) + \mu N(\cos\theta) = \frac{mv^2}{r}$

② $N(\cos\theta) - \mu N(\sin\theta) = mg$

Divide: $\frac{①}{②}$

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{gr}$$

Car on banked curve, high speed case, slip impending UP-slope.

(N and m cancel and drop out...)

If you know θ, μ and r, and need to find the car speed v, this is an easy problem.

If you know v, r and μ and need to find θ, then you have to iterate or use an equation solver to get θ.

High Speed Case (Car on Verge of Slipping Upslope)

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{gr}$$

If you know θ, μ and r, and need to find the car speed v, this is an easy problem.

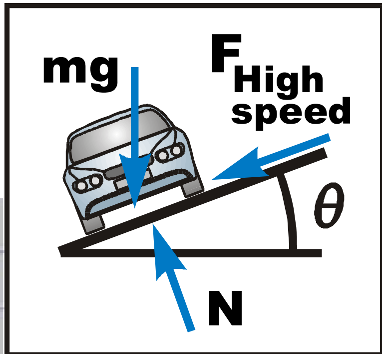
For example, let θ = 20, μ = .3, r = 200 ft, g = 32.2 fps^2; find the speed v at which the car is on the verge of slipping upslope.

High Speed Case (Car on Verge of Slipping Upslope)

$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{gr}$$

If you know v, r and μ and need to find θ, then you have to iterate or use an equation solver to get θ. Let μ = .3, r = 100 m, g = 9.81 m/s^2, and v = 20 m/s; find the angle θ at which the car is on the verge of slipping upslope.

High Speed Case (Car on Verge of Slipping Upslope)



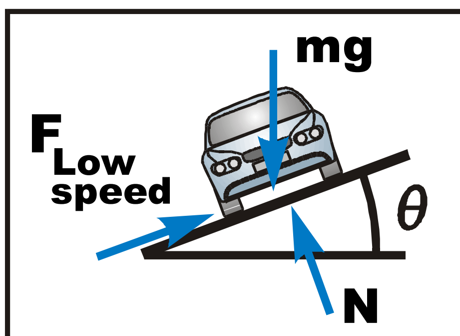
$$\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = \frac{v^2}{gr}$$

Car on banked curve, high speed case, slip impending UP-slope.

Low Speed Case (Car on Verge of Slipping Downslope)

(Signs reverse on μ terms...)

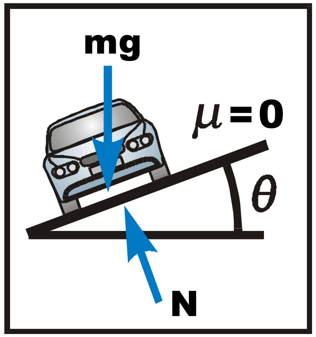
$$\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} = \frac{v^2}{gr}$$



Car on banked curve, low speed case, slip impending DOWN-slope.

Zero Friction Case (set $\mu = 0$):

Car on banked curve, zero friction case. (Highway design case.)



$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{v^2}{g\,r}$$

This is the highway design case. For a given speed, v, find the bank angle, θ, at which zero friction is needed to keep the car on the curve.

Curve Not Banked ($\theta = 0$) Case:

Car on level curve ($\theta = 0$) case.

$$\frac{0 - \mu\cdot 1}{1 + \mu\cdot 0} = \mu = \frac{v^2}{g\,r}$$

Simplified...

$$\mu = \frac{v^2}{g\,r}$$

Solving for v_{max}

$$v_{max} = \sqrt{\mu\, g\, r}$$