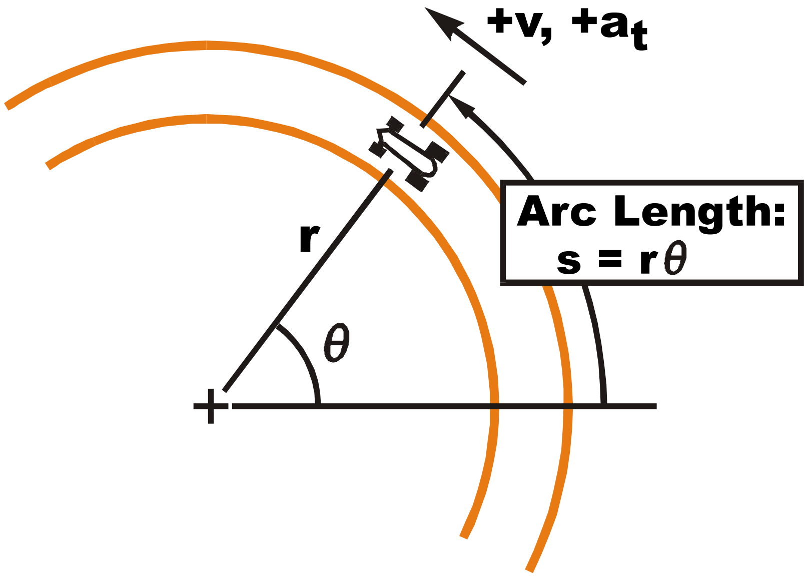
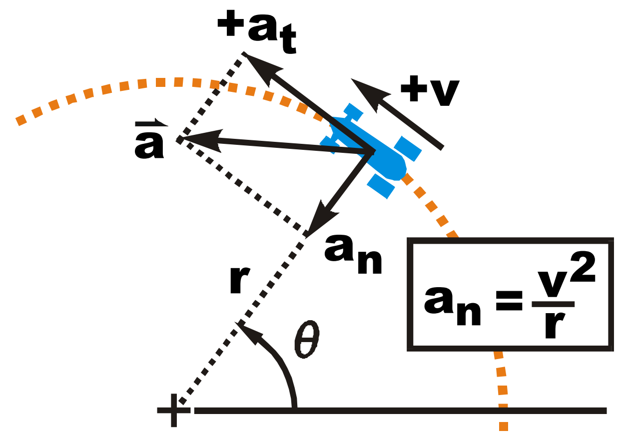
Particle Kinematics: Circular Motion





Magnitude of the $\vec{a}$ vector:
(the "total acceleration")

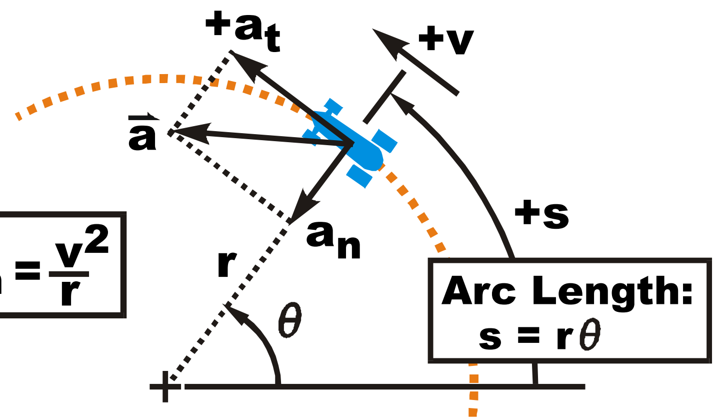
$$|\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

Circular Motion Problems

- **Particle moves along a circular path.**
- **All the same cases as straight line problems.**
- **Velocity acts tangent to path.**
- **Position, s, is along the curve.**
- **Acceleration has both a_t and a_n components.**
- **DO NOT forget a_n.**
- **A problem will often ask for "magnitude of accel"**

Circular Motion

$$a_n = \frac{v^2}{r}$$



Magnitude of the $\vec{a}$ vector:
(the "total acceleration")

$$|\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

If a function is given, match it with a defining equation and integrate.

① $a_t = \dfrac{dv}{dt}$

② $v = \dfrac{ds}{dt}$

③ $a_t\,ds = v\,dv$

If a_t = constant, use these:

① $v = v_0 + a_t t$

② $s = s_0 + v_0 t + \frac{1}{2} a_t t^2$

③ $v^2 = v_0^2 + 2a_t(s - s_0)$

Motion along the (circular) curve is given by a v or a_t function.

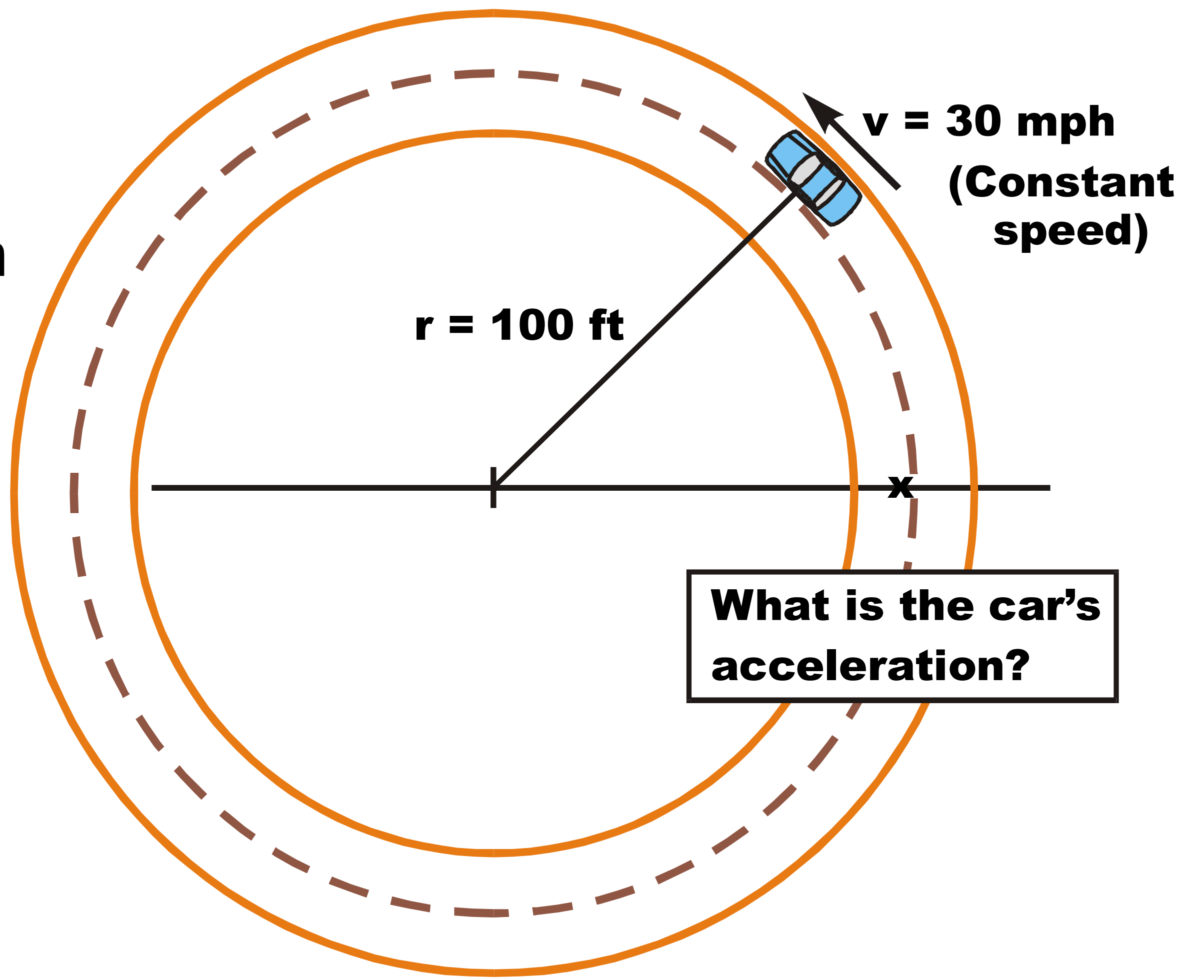
Cases: (1) a_t = constant
(2) $a_t = f(t)$
(3) $a_t = f(v)$
(4) $a_t = f(s)$
(5) $v = f(s)$
...etc.

Analogous to straight line motion problems, except accel is now the a_t component, and distance s is along a curve.

Circular Motion: Simple Example (thought problem….)

A car moves along a circular track with **constant speed**: v = 30 mph

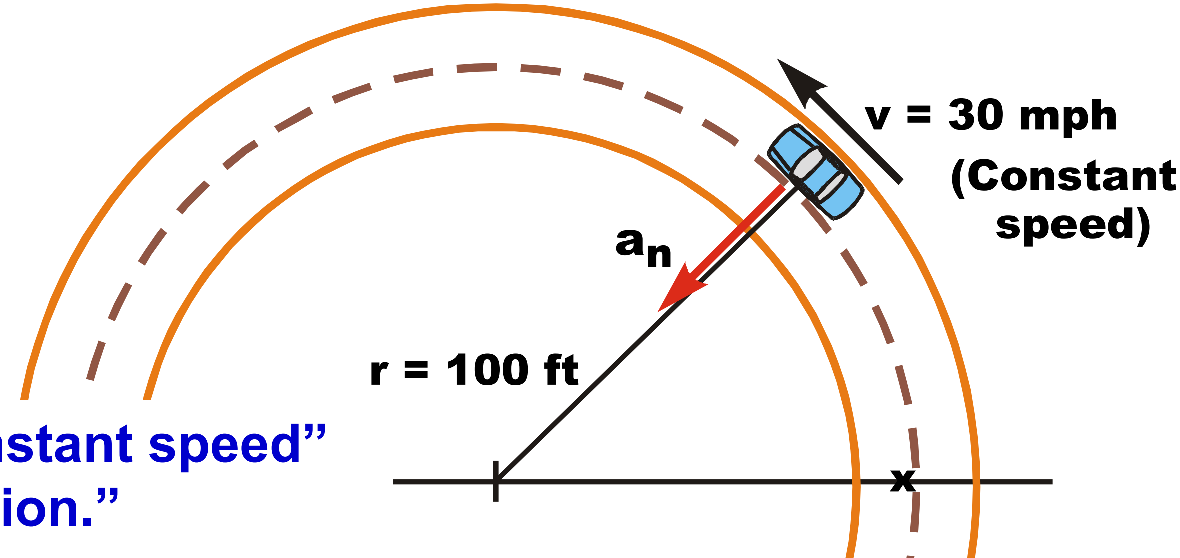
What is the car's acceleration?



Circular Motion: Simple Example (answers…)

A car moves along a circular track with constant speed: v = 30 mph

What is the car's acceleration?



What is the car's acceleration?

Most students see "constant speed" and say "zero acceleration."

Yes, the *speed* is constant, so a_t is zero. But, the velocity (vector) is *not* constant. The velocity vector is changing directions. There is a normal acceleration (a_n) that points toward the center that continuously changes the v vector direction.

$$v = 30 \text{ mph}\left[\frac{88 \text{ fps}}{60 \text{ mph}}\right]$$

$$v = 44 \text{ fps}$$

$$a_n = \frac{v^2}{r} = \frac{44^2}{100} = 19.4 \text{ fps}^2$$

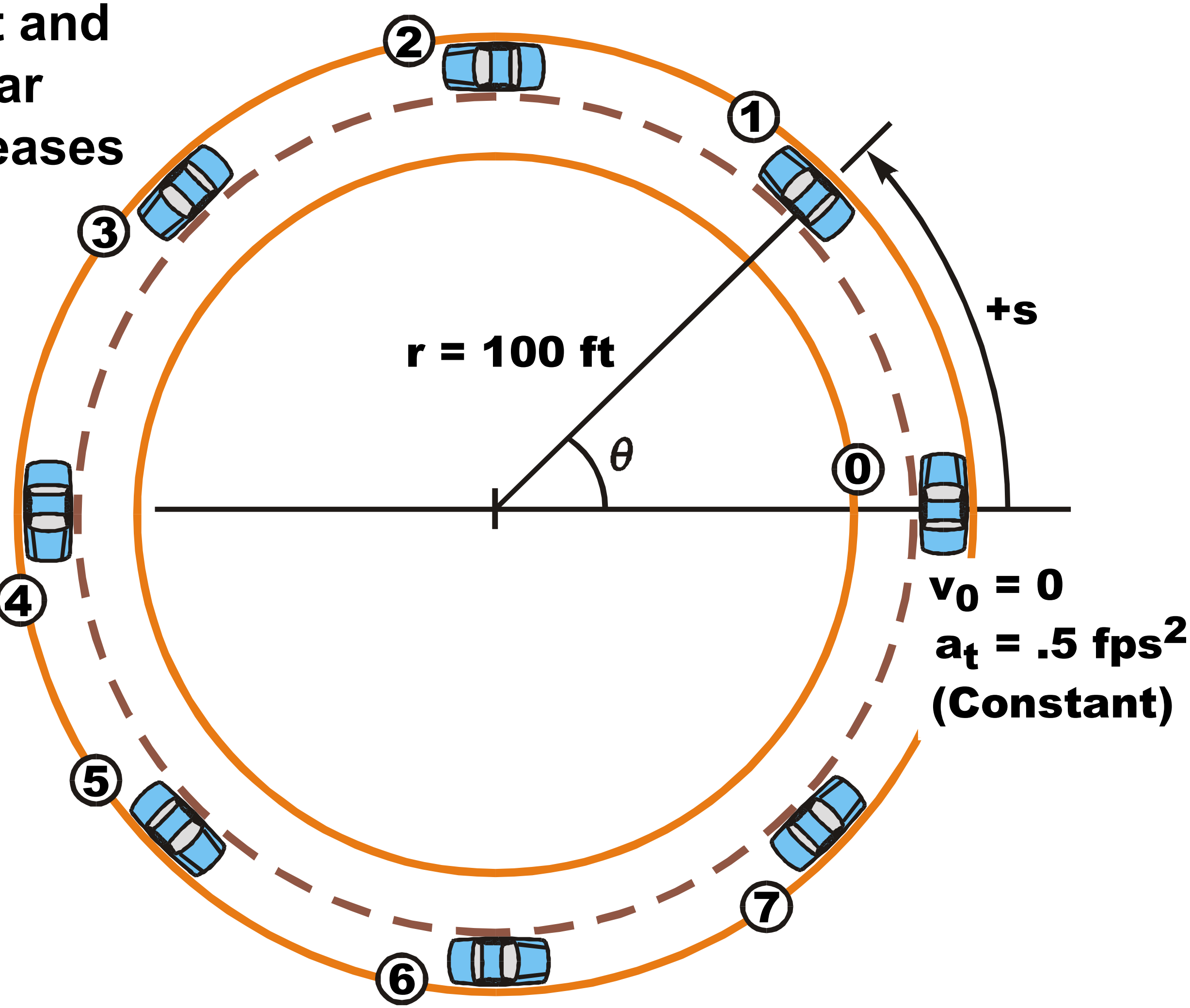
a_n points toward circle center

Circular Motion: Simple Example 2

A car starts from rest and moves along a circular track. Its *speed* increases at: $a_t = 0.5\ fps^2$

Find v and accel vectors for the car at various positions (1) through (7).

Can you guess what the velocity and acceleration vectors will look like?



Circular Motion: Simple Example 2

A car starts from rest and moves along a circular track. Its *speed* increases at: $a_t = 0.5 \text{ fps}^2$

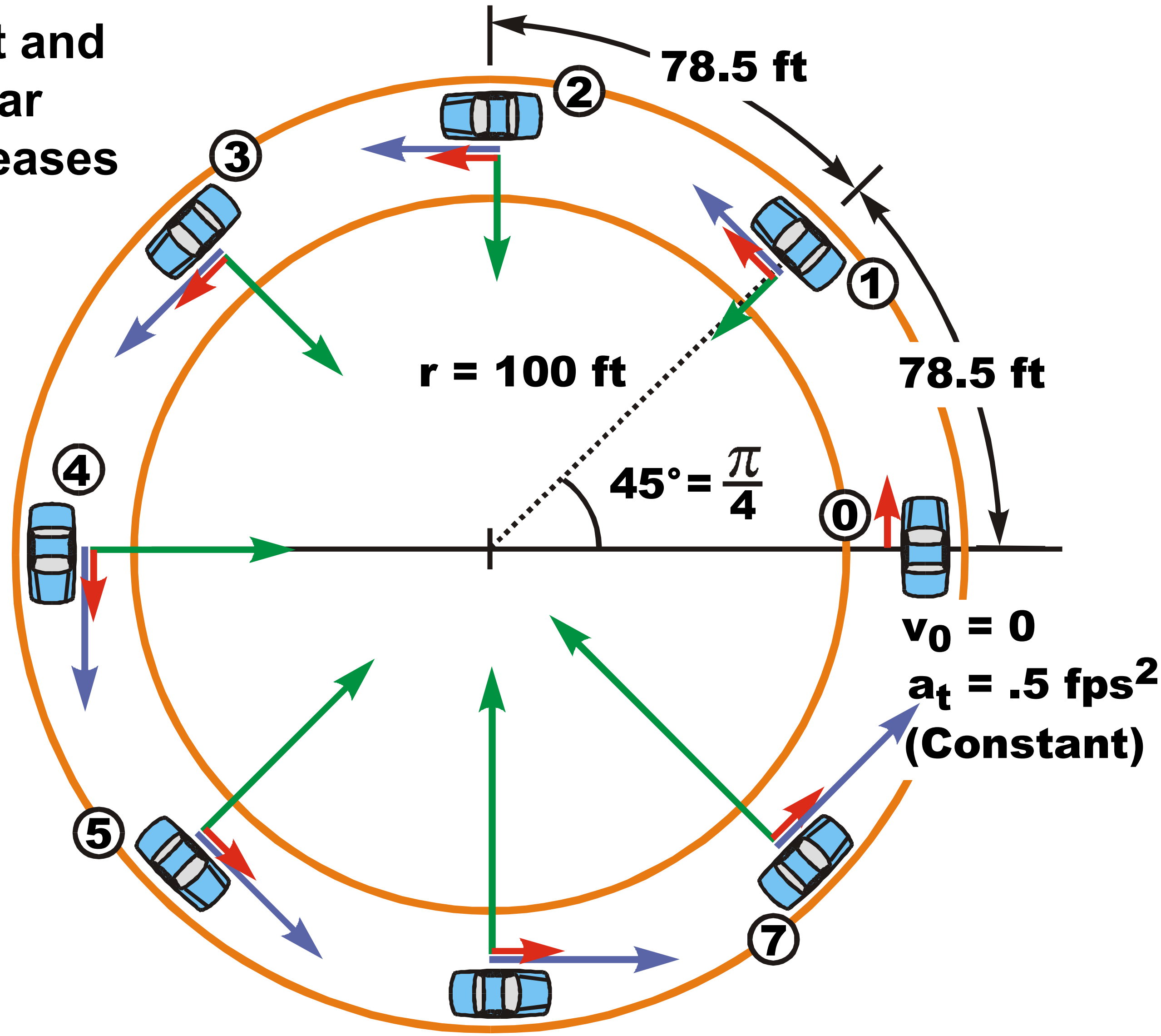
Find v and accel at various positions.

Notice:

(1) v grows. (blue arrows)

(2) a_t = constant (red arrows)

(3) a_n grows (green arrows)



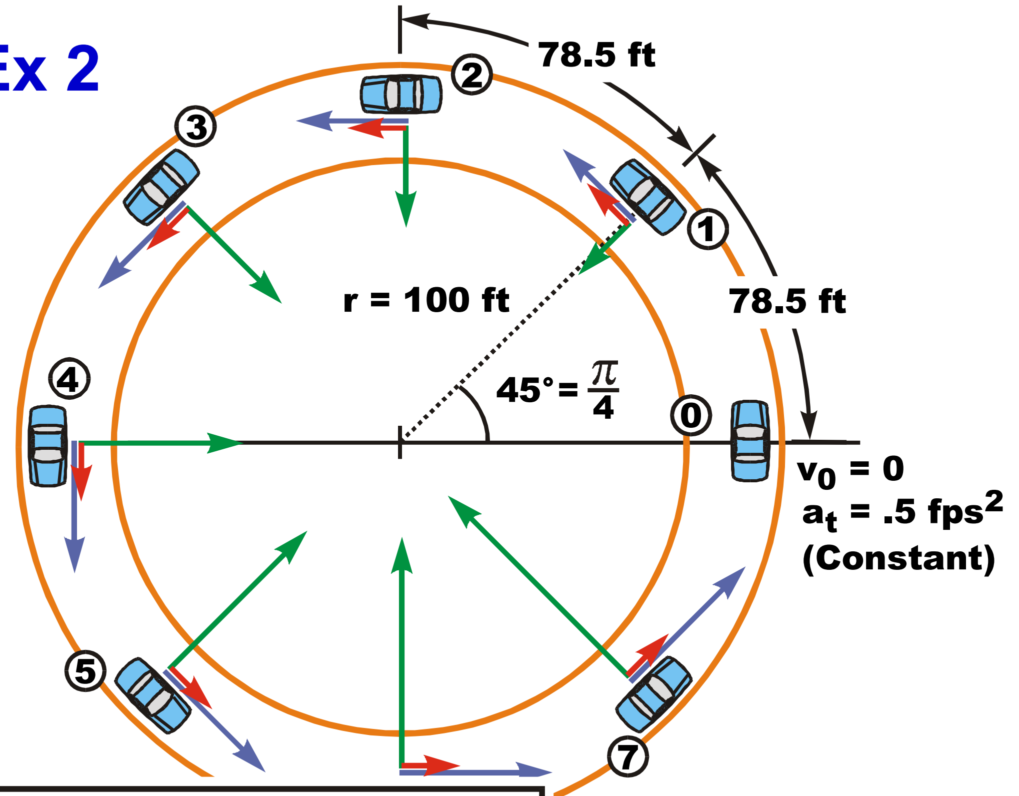
Circular Motion: Simple Ex 2

Sample calculations…

45° Arc Length: $s = r\theta$

$$s = (100)\left(\frac{\pi}{4}\right) = 25\pi$$

$$s = 78.5 \text{ ft}$$



Sample Calculations:
Circular Motion, a_t = Const

From ① to ② :

$$v_2^2 = v_1^2 + 2a_t(s_2 - s_1)$$

$$= 8.86^2 + 2(.5)(78.5)$$

$$v_2 = 12.5 \text{ fps}$$

$$a_{n_2} = \frac{v^2}{r} = \frac{12.5^2}{100}$$

$$a_{n_2} = 1.57 \text{ fps}^2$$

From ⓪ to ① :

$$v_1^2 = v_0^2 + 2a_t(s_1 - s_0)$$

$$= 0 + 2(.5)(78.5)$$

$$v_1 = 8.86 \text{ fps}$$

$$a_{n_1} = \frac{v^2}{r} = \frac{8.86^2}{100}$$

$$a_{n_1} = .785 \text{ fps}^2$$