Particle Kinematics: Intro to Curvilinear Motion

Current unit: Particle Kinematics

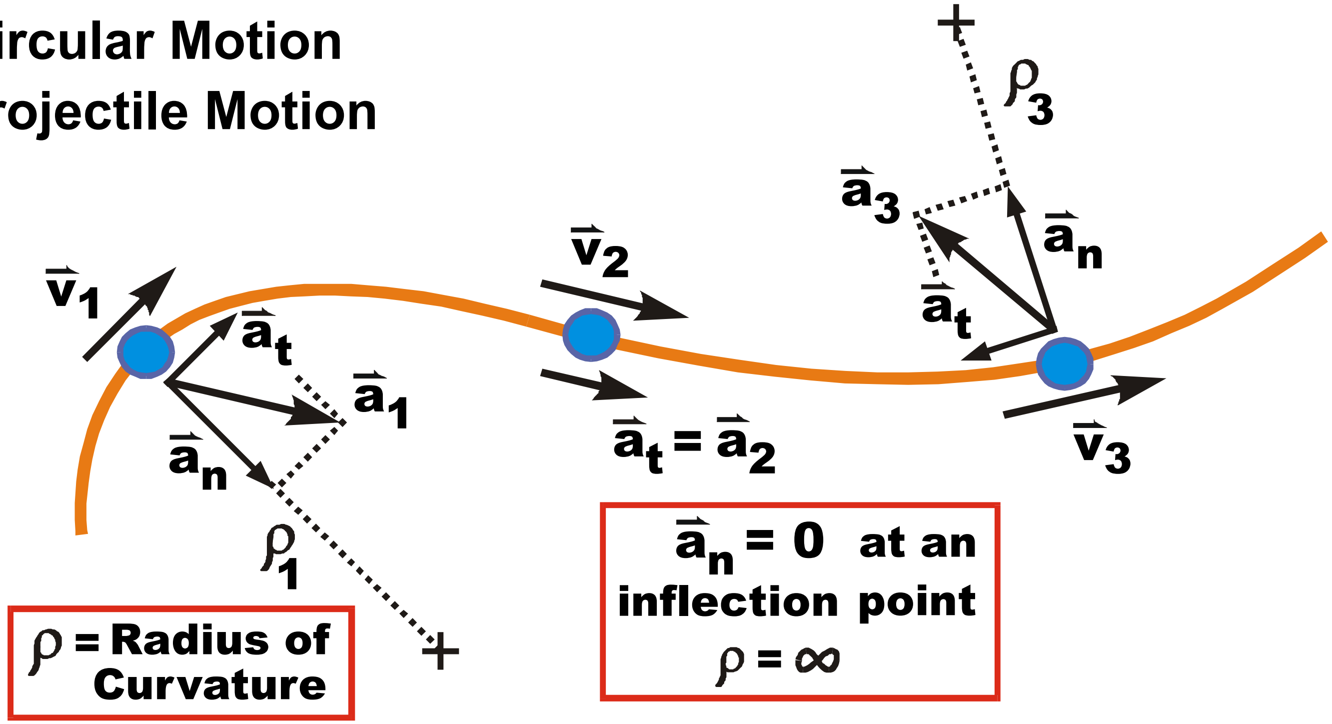
Last two classes: Straight Line Motion

Today:

(a) Intro to Curvilinear Motion

(b) Circular Motion

(c) Projectile Motion



Introduction to Curvilinear Motion

Key Principle #1:

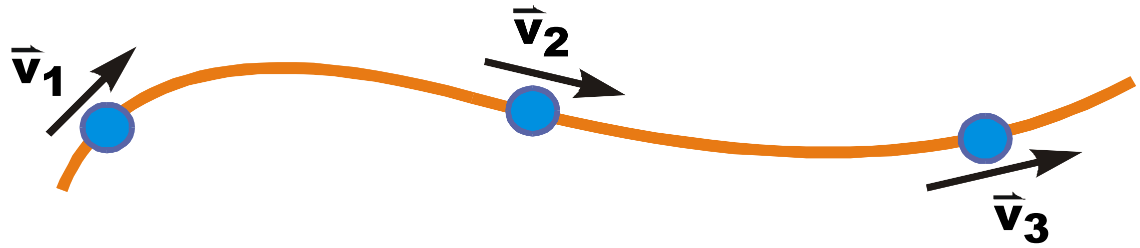
Velocity is always tangent to the path.

Knowing this…

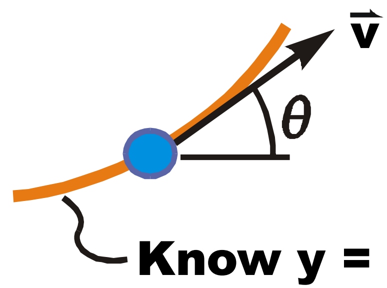
Can use path slope to get velocity angle.

or,

Can use velocity components to get path slope.



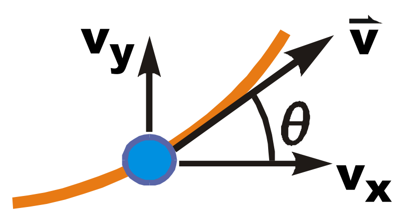
If know y = f(x) path,
can use it to get v direction (angle):



$$\vec{v} = [v \angle \theta]$$

$$\theta = \tan^{-1} y'$$

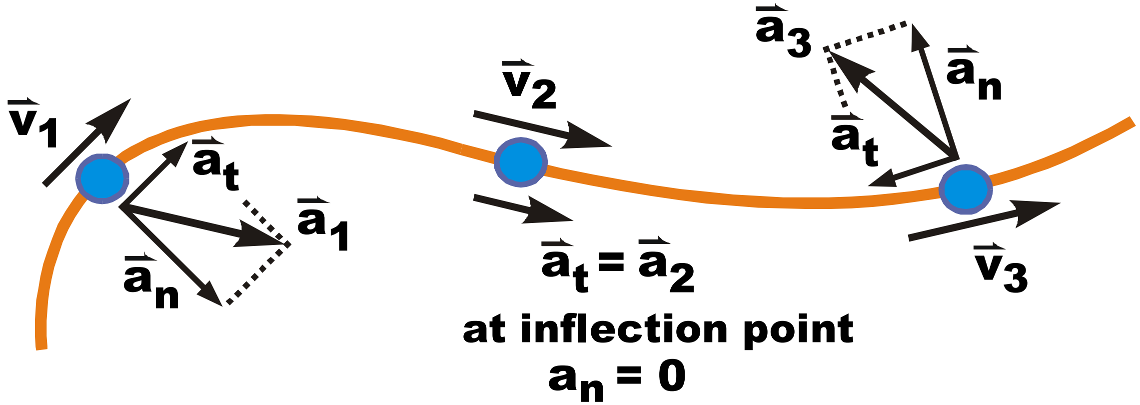
If know v_x and v_y (as in projectile motion),
can use these to get $|\vec{v}|$ and the angle, θ.



$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

$$\theta = \tan^{-1} \frac{v_y}{v_x}$$

Introduction to Curvilinear Motion



Key Principle #2:

Acceleration always acts toward the concave side of the curve.

The accel vectors $\vec{a}_1$ and $\vec{a}_3$ act toward the concave side of the curve.

If we resolve the $\vec{a}$'s into a_n and a_t components:

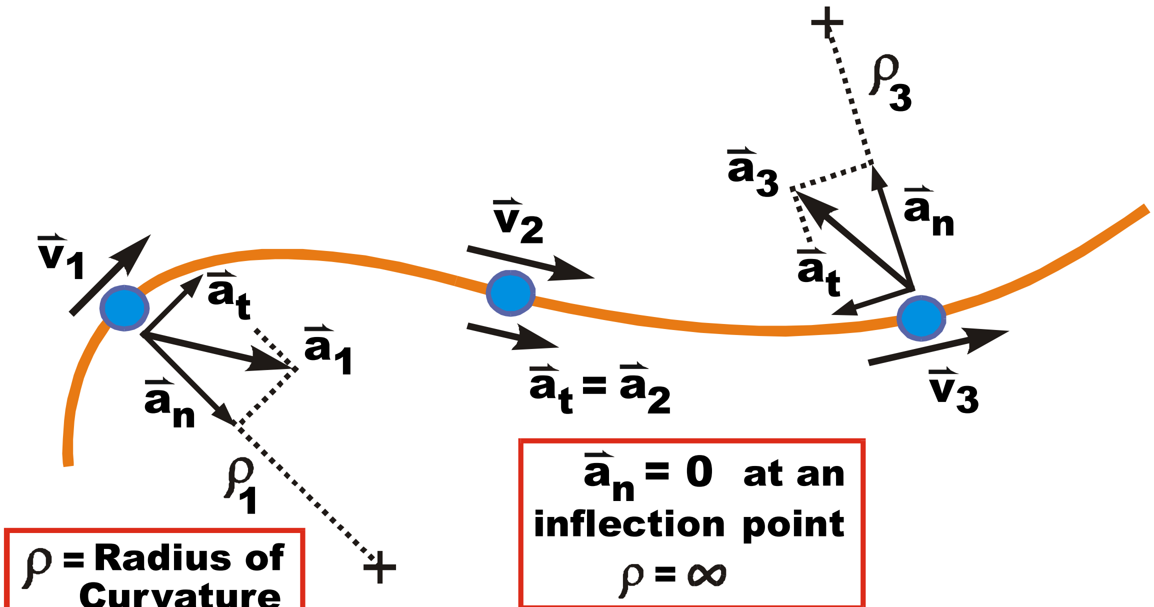
- a_t components act tangent to the curve
- a_n components act $\perp$ to $\vec{v}$, toward the concave side of the curve.

Significance of the a_t and a_n components?

- a_t components change the length (speed) of $\vec{v}$.
- a_n components change the direction of $\vec{v}$.

Intro to Curvilinear Motion

Key Principle #3: a_n acts toward the center of curvature and may be calc'd from $a_n = v^2/\rho$



$$\rho = \left| \frac{\left[1 + (y')^2\right]^{3/2}}{y''} \right|$$

ρ = "radius of curvature" of a general curve, $y = f(x)$ where, $y' = f'(x)$ and $y'' = f''(x)$

Normal Accel, a_n

$$a_n = \frac{v^2}{\rho}$$

a_n acts $\perp$ to $\vec{v}$, toward center of curvature. Changes $\vec{v}$ direction.