Particle Kinematics: x-y Coordinates

Current unit: Particle Kinematics

Last class: Curvilinear, Circular, Projectile Motion

Today:

(a) Math Preliminaries: Dot notation, chain rule

(b) "Path Known" x-y Problem

(c) "Parametric Equations" x-y Problem

Math Preliminaries:

1. Dot Notation

Dot Notation

$$\dot{x} = \frac{dx}{dt} = v_x \qquad \dot{y} = \frac{dy}{dt} = v_y$$

$$\ddot{x} = \frac{d^2x}{dt^2} = a_x \qquad \ddot{y} = \frac{d^2y}{dt^2} = a_y$$

A convenient, shorthand notation for time derivatives.

Important!

Do not confuse dot and prime !

$\dot{y}$ **and** $\ddot{y}$ **are time derivatives.**

y' **and** y'' **are x derivatives used in the** ρ **equation.**

$$y' = f'(x)$$

$$y'' = f''(x)$$

$$\boxed{\rho = \left| \frac{\left[1 + (y')^2\right]^{3/2}}{y''} \right|}$$

Math Preliminaries (cont'd)

2. Chain Rule: Used to introduce time derivatives into a $y = f(x)$ function which does not contain time (t) terms.

Chain Rule

Given a function: $f(y) = f(x)$
Introduce time derivatives by:

Take y deriv's of f(y) terms,
then multiply by $\frac{dy}{dt} = \dot{y}$.

$$\frac{d\,f(y)}{dy}\underbrace{\frac{dy}{dt}}_{=\,\dot{y}} = \frac{d\,f(x)}{dx}\underbrace{\frac{dx}{dt}}_{=\,\dot{x}}$$

Similarly, take x deriv's of f(x) terms,
then multiply by $\frac{dx}{dt} = \dot{x}$.

See the course web page for practice chain rule problems. Learn this well! We'll use the chain rule often in dynamics.

2. Chain Rule (Applied to an example problem):

Chain Rule:

$$\frac{df(y)}{dy}\,\frac{dy}{dt} = \frac{df(x)}{dx}\,\frac{dx}{dt}$$

or

$$\frac{df(y)}{dy}\,\dot{y} = \frac{df(x)}{dx}\,\dot{x}$$

Product Rule:

$$d(uv) = u\,dv + v\,du$$

Dot Notation

$$\dot{x} = \frac{dx}{dt} = v_x$$

$$\ddot{x} = \frac{d^2x}{dt^2} = a_x$$

$$\dot{y} = \frac{dy}{dt} = v_y$$

$$\ddot{y} = \frac{d^2y}{dt^2} = a_y$$

Chain Rule Example Problem:

Take first and second time deriv's of this y = f(x) function:

Orig function: $y^2 = x + 5$

Apply chain rule: $2y\,\dot{y} = 1\cdot\dot{x} + 0$

Simplify: $2y\,\dot{y} = \dot{x}$

$$\frac{d}{dt}\dot{x} = \ddot{x}$$

A **product** of two differentiable terms.. $2y$ and $\dot{y}$

Product Rule: $d(uv) = u\,dv + v\,du$

$$2\dot{y}\,\dot{y} + 2y\,\ddot{y} = \ddot{x}$$

Apply chain rule to 2y...

$$\frac{d}{dt}\dot{y} = \ddot{y}$$

See the course web page for practice chain rule problems. Learn this well! We'll use the chain rule often in dynamics.

3. Chain Rule Example (Why is it useful in dynamics?):

Chain Rule:

$$\boxed{\frac{df(y)}{dy}\frac{dy}{dt} = \frac{df(x)}{dx}\frac{dx}{dt}}$$

or

$$\boxed{\frac{df(y)}{dy}\dot{y} = \frac{df(x)}{dx}\dot{x}}$$

Product Rule:

$$\boxed{d(uv) = u\,dv + v\,du}$$

Dot Notation

$$\dot{x} = \frac{dx}{dt} = v_x$$

$$\ddot{x} = \frac{d^2x}{dt^2} = a_x$$

$$\dot{y} = \frac{dy}{dt} = v_y$$

$$\ddot{y} = \frac{d^2y}{dt^2} = a_y$$

Why the chain rule is useful in dynamics:

Path function: $\boxed{y^2 = x + 5}$

Relate velocity components: $2y\dot{y} = \dot{x} \Rightarrow \boxed{2y\,v_y = v_x}$

Relate accel components: $\underbrace{2\dot{y}\,\dot{y}}_{v_y^2} + 2y\,\underbrace{\ddot{y}}_{a_y} = \ddot{x} \Rightarrow \boxed{2v_y^2 + 2y\,a_y = a_x}$

When do you use the chain rule?

When given a path function [y = f(x)] plus some derivative information [(v_y and a_y) or v_x and a_x)], use the chain rule to get a velocity and an accel equation. Plug in the comp's you have to get the ones you seek.