Particle Straight Line (Integration): Ex Prob 1

A rocket takes off vertically with acceleration $a = (6 + 0.02s)$ m/s^2.
Initially, at $t = 0$, $v(0) = 0$ and $s(0) = 0$.
Please determine: Speed v when s = 2 km above the ground.

Discussion: **Acceleration is a function! Of position!** So we must match the acceleration function with one of the defining eqns at right.

Defining Eqns

① $a = \frac{dv}{dt}$

② $v = \frac{ds}{dt}$

③ $a\,ds = v\,dv$

Our accel function and conditions involve a, s and v, but not time. Which defining eqn best fits these? Correct! Use (3) !

Defining equation: $a\,ds = v\,dv$

Sub in our function: $(6 + .02s)\,ds = v\,dv$

Set up integrals and sub in limits from cond's.

$$\int_0^s (6 + .02s)\,ds = \int_0^v v\,dv$$

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$$\int_0^s (6 + .02s)\,ds = \int_0^v v\,dv$$

Integrate….

$$\left(6s + .01s^2\right)\Big|_0^s = \tfrac{1}{2}v^2\Big|_0^v$$

$$\left(6s + .01s^2\right) = \tfrac{1}{2}v^2$$

Algebra…

$$v^2 = \left(12s + .02s^2\right)$$

$$\boxed{v = \sqrt{12s + .02s^2}}$$

At s = 2 km = 2000 m,

At s = 2000 m, solve for speed, v:

$$\boxed{v = 322.5 \text{ m/s}}$$

As a second part of this problem, how would you find the *time* at which the rocket reaches s = 2000 m ?

You may not realize it, but you now have *two* equations to use as starting points….*plus the defining eqns.*

Defining Eqns
① $a = \frac{dv}{dt}$
② $v = \frac{ds}{dt}$
③ $a\,ds = v\,dv$

The initial equation: $a = (6 + 0.02\ s)\ m/s^2$

or the v vs. s equation: $v = \sqrt{12s + .02s^2}$

So, which one can be combined with a defining equation to get time?

Correct! Use equation (2) plus the v vs. s equation….

$$v = \frac{ds}{dt} = \sqrt{12s + .02s^2}$$

Finding the time, t, at which the rocket reaches s = 2 km altitude:

Defining eqn plus v vs. s eqn:

$$v = \frac{ds}{dt} = \sqrt{12s + .02s^2}$$

Separate variables:

$$dt = \frac{ds}{\sqrt{12s + .02s^2}}$$

The integration is difficult. You must use a table or MathCad or other solver. I want you to know how to set up a problem like this—even on an exam.

Integrate:

(Hard!)

(Yuk!)

$$\int_0^t dt = \int_0^{2000} \frac{ds}{\sqrt{12s + .02s^2}}$$

t = 19.28 sec

Again, the procedure: Match your given equation with one of the three defining equations. Separate variables. Then integrate. Voila!