Particle Straight Line (Integration): Ex Prob 2

A particle experiences a rapid deceleration: $a = -2v^3$ m/s^2.
Initially, at t = 0, v(0) = 8 m/s and s(0) = 10 m.
Please determine: Speed v and position s at t = 4 sec.

Discussion: Acceleration is a function! Of velocity! So we must match the acceleration function with one of the defining eqns at right.

Our accel function and conditions involve a, v, t *and* s. Which defining eqn(s) best fit these? Answer: You can use either (1) or (3).

Defining Eqns

① $a = \frac{dv}{dt}$

② $v = \frac{ds}{dt}$

③ $a\,ds = v\,dv$

If you use (3) now, you can integrate to get an equation relating v and s, but you cannot solve for v or s because you do not have a final position (s) or speed (v).

We'll use (1) now. After integration we will have a v vs. t function. At t = 4 sec (given), we can find v.

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③ $a\,ds = v\,dv$

Defining equation: $$a = \frac{dv}{dt} = -2v^3$$

Separate variables: $$dt = \frac{dv}{-2v^3} = -\frac{1}{2}v^{-3}\,dv$$

Set up integrals and sub in limits from conditions.

$$\int_0^t dt = \int_8^v -\frac{1}{2}v^{-3}\,dv$$

Set up integrals and sub in limits from conditions.

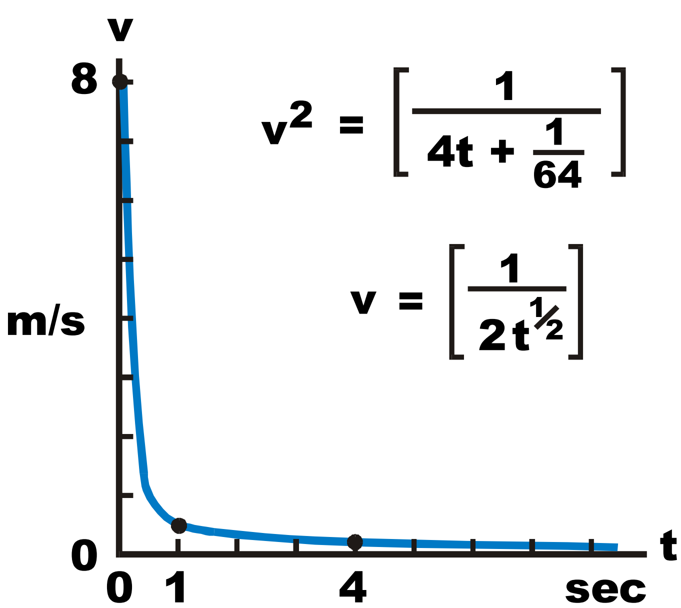
$$\int_0^t dt = \int_8^v -\frac{1}{2} v^{-3}\, dv$$

Integrate....

$$t = \frac{1}{4} v^{-2} \Big|_8^v = \frac{1}{4}\left[\frac{1}{v^2} - \frac{1}{64} \right]$$

Re-arrange to get v^2.....

$$v^2 = \left[\frac{1}{4t + \frac{1}{64}} \right]$$



$$v^2 = \left[\frac{1}{4t + \frac{1}{64}} \right]$$

$$v = \left[\frac{1}{2t^{1/2}} \right]$$

At t = 4 sec...

$$v^2 = \left[\frac{1}{16 + \frac{1}{64}} \right]$$

t	v
0	8
1	.5
4	.25

$$v \approx \frac{1}{4}$$

v = 0.25 m/s

For the second part of this problem, how would you find the *position, s,* of the particle at t = 4 sec ?

You now have *two* equations to use as starting points….*plus the defining eqns.*

The initial a(v) equation: $a = -2v^3 \ \text{m/s}^2.$

or the v(t) equation: $v^2 = \left[\dfrac{1}{4t + \frac{1}{64}} \right]$

Defining Eqns

① $a = \dfrac{dv}{dt}$

② $v = \dfrac{ds}{dt}$

③ $a\,ds = v\,dv$

So, which equation can be combined with a defining equation to get position?

Answer: You can use either one! The a(v) eqn with (3) (easier) *or* the v(t) eqn with (2)…(harder). We'll do both

Hard way: Use the v(t) equation plus defining equation (2)….

We found this v(t) equation….

$$v^2 = \left[\frac{1}{4t + \frac{1}{64}}\right]$$

Solve for v and set equal to ds/dt…..

$$v = \left[\frac{1}{4t + \frac{1}{64}}\right]^{\frac{1}{2}} = \frac{ds}{dt}$$

Separate variables and set up the integral….

$$\int_0^{10} ds = \int_0^t \left[4t + \frac{1}{64}\right]^{-.5} dt$$

The integration is difficult. You must use a table or MathCad or other solver to solve it. Fortunately, there is an easier way….

See the next page….

Easier approach: Use the a(v) eqn plus defining equation (3)….

Defining equation: $a\,ds = v\,dv$

Sub in our function: $(-2v^3)\,ds = v\,dv$

Separate variables: $$ds = \frac{v\,dv}{-2v^3} = -\frac{1}{2}v^{-2}\,dv$$

Set up integrals and sub in limits from conditions: $$\int_{10}^{s} ds = \int_{8}^{v} -\frac{1}{2}v^{-2}\,dv$$

Integrate: $$s - 10 = \frac{1}{2}v^{-1}\Big|_{8}^{v} = \frac{1}{2}\left[\frac{1}{v} - \frac{1}{8}\right]$$

Re-arrange: $$s = 10 + \frac{1}{2}\left[\frac{1}{v} - \frac{1}{8}\right]$$

Easier approach: Use the a(v) eqn plus defining equation (3)….

$$s = 10 + \frac{1}{2}\left[\frac{1}{v} - \frac{1}{8}\right]$$

At v = .25 m/s (found earlier)…

$$s = 10 + \frac{1}{2}\left[4 - \frac{1}{8}\right] = 10 + \frac{1}{2}\left[3.875\right]$$

s = 11.94 m

At v = .25 m/s…