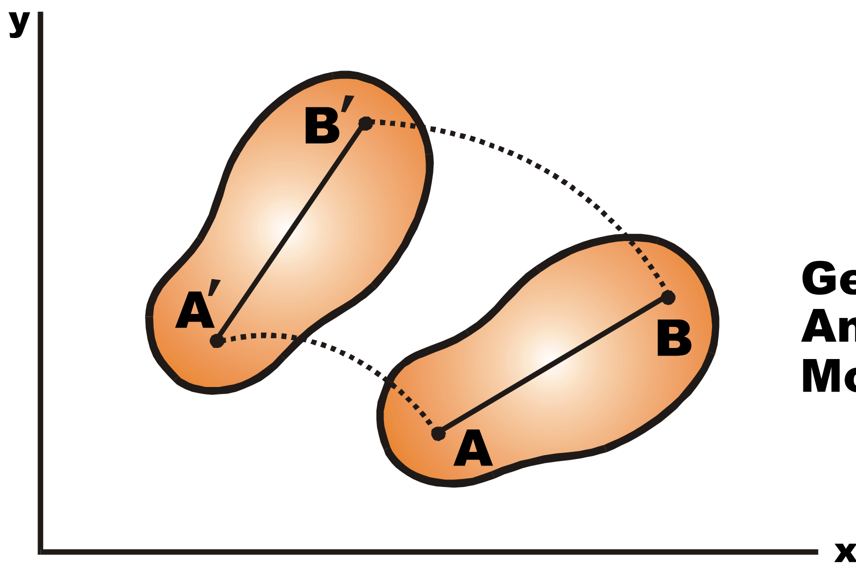
Rigid Body Planar Motion

- General Angular Motion

General Angular Motion: Definitions
θ = angular displacement, radians
$\omega = \frac{d\theta}{dt} = \dot{\theta}$ = angular velocity, rad/sec
$\alpha = \frac{d\omega}{dt} = \dot{\omega}$ = angular acceleration, rad/sec^2



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Defining Kinematic Equations	
Along a line:	**Angular:**
① $a = \frac{dv}{dt}$	① $\alpha = \frac{d\omega}{dt}$
② $v = \frac{ds}{dt}$	② $\omega = \frac{d\theta}{dt}$
③ $a\,ds = v\,dv$	③ $\alpha\,d\theta = \omega\,d\omega$

Constant Accel Kinematic Equations	
Motion along a path: a = constant eqns	**Angular Motion:** α = constant eqns
① $v = v_0 + at$	① $\omega = \omega_0 + \alpha t$
② $s = s_0 + v_0 t + \frac{1}{2}at^2$	② $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
③ $v^2 = v_0^2 + 2a(s - s_0)$	③ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

All of these eqns apply to ANY angular motion, not just to fixed axis rotation.

But, we usually only use these for bodies undergoing fixed axis rotation.

General Angular Motion: Types of Problems

Cases:

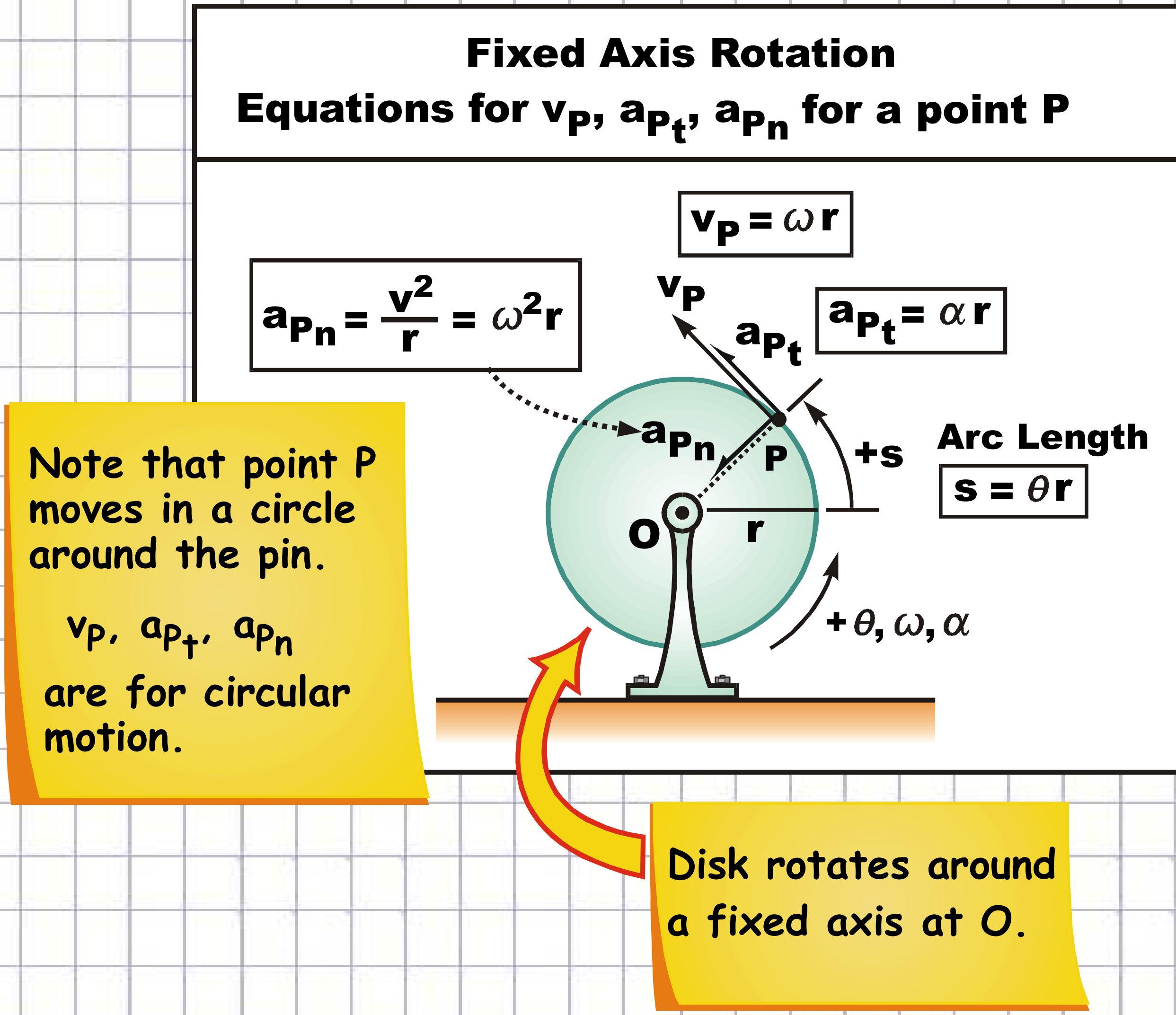
1. α = constant
2. $\alpha = f(t)$
3. $\alpha = f(\theta)$
4. $\alpha = f(\omega)$
5. $\omega = f(\theta)$
6. etc.

Angular Motion: α **= constant eqns**
① $\omega = \omega_0 + \alpha t$
② $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
③ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Angular Motion Defining Kinematic Equations
① $\alpha = \frac{d\omega}{dt}$
② $\omega = \frac{d\theta}{dt}$
③ $\alpha\, d\theta = \omega\, d\omega$

Fixed Axis Rotation

- Velocity and accelerations of a POINT on a rotating rigid body.



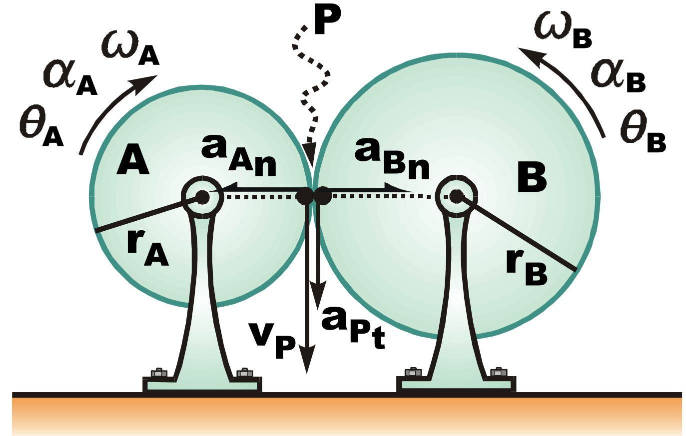
Fixed Axis Rotation (Gears Touching)

- Use the ratio of the radii of adjacent gears to transfer ω's, α's, and θ's from gear to gear….

- Use your intuition about whether the next gear is turning faster or slower.

Two gears touching at P, with no slip.

At this contact point, the two disks share the same: v_P, a_{P_t}, s



Key idea here:

Use the ratio of the gear radii $\frac{r_A}{r_B}$ to transfer ω's, α's from gear to gear.

$$\omega_A r_A = v_P = \omega_B r_B \quad\Rightarrow\quad \omega_B = \frac{r_A}{r_B}\omega_A$$

$$\alpha_A r_A = a_{P_t} = \alpha_B r_B \quad\Rightarrow\quad \alpha_B = \frac{r_A}{r_B}\alpha_A$$

$$\theta_A r_A = s = \theta_B r_B \quad\Rightarrow\quad \theta_B = \frac{r_A}{r_B}\theta_A$$

The a_{P_n} terms are NOT equal.

$$a_{A_n} \neq a_{B_n}$$