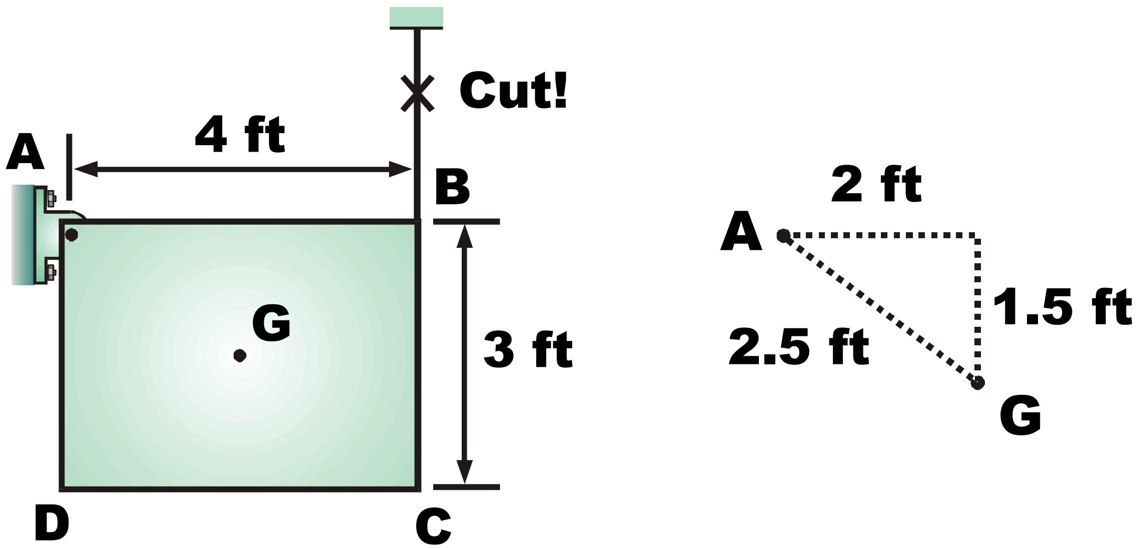
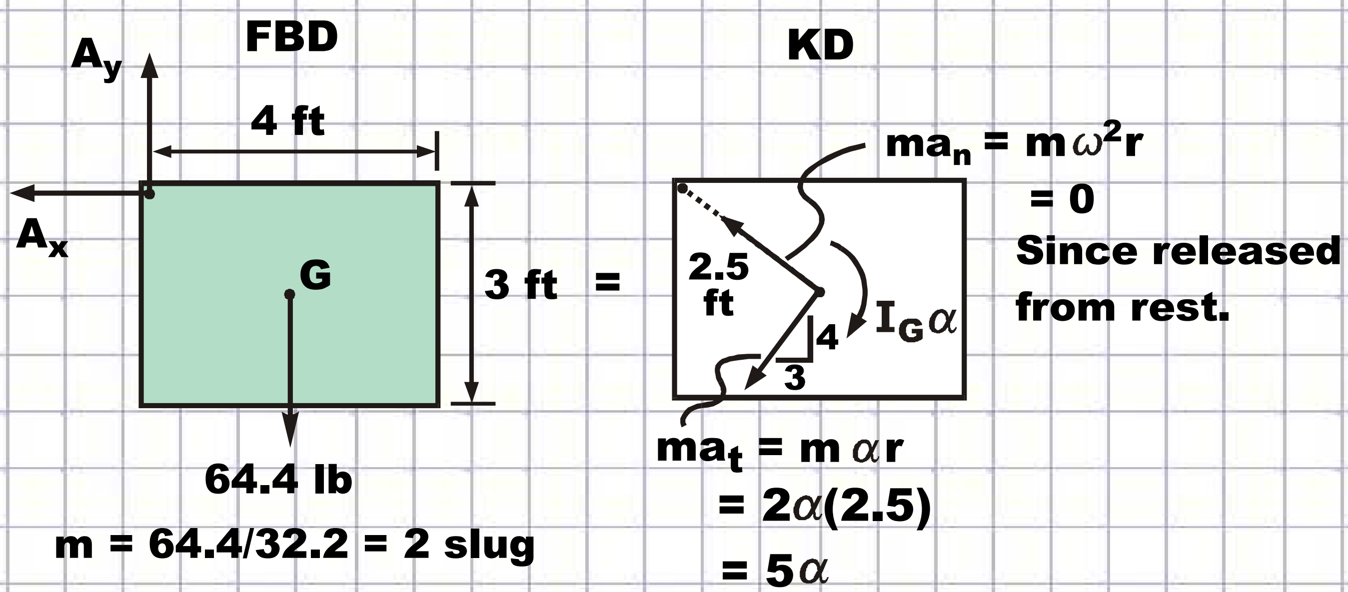
Rigid Body F=ma Fixed Axis Rotation: Example 3

A 64.4 lb rectangular plate is suspended by a pin and a cord. At the instant the cord is cut, please determine: (a) Pin reactions at A; (b) The plate's angular acceleration, α .



64.4 lb Rectangular plate, released from rest.

Draw a Free Body Diagram and a Kinetic Diagram



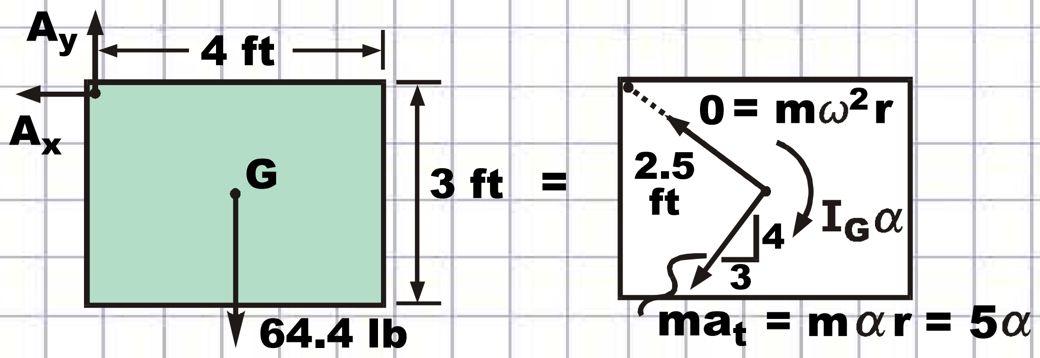
$m = 64.4/32.2 = 2$ slug

Calculate I_G for the plate: $[I_G = (1/12)m(a^2 + b^2)]$

$$I_G = (1/12)(2)(4^2+3^2) = 4.17 \text{ slug-ft}^2$$

Calculate I_{pin} from the parallel axis theorem:

$$I_{pin} = I_G + md^2 = 4.17 + (2)(2.5^2) = 16.67 \text{ slug-ft}^2$$



Write equations of motion: Sum moments about the pin:

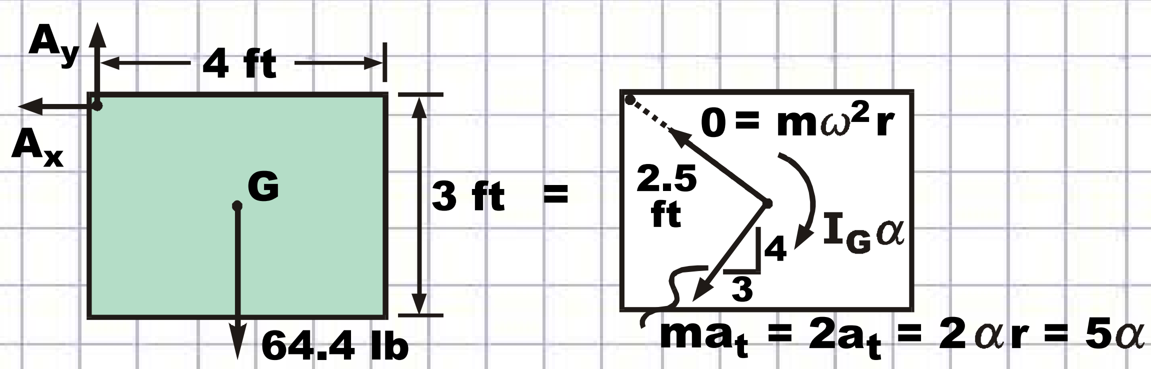
$$\Sigma M_{pin} = I_{pin}\alpha \;;\;\; (64.4)(2\ ft) = I_{pin}\alpha = 16.67\alpha$$

$$\boxed{\alpha = 7.73\ rad/sec^2}$$

from kinematics: $a_t = r\alpha = 2.5(7.73)$, $\boxed{a_t = 19.3\ \ fps^2}$

Note: Don't be alarmed about the fact that we drew $I_G\alpha$ on the KD and then used I_{pin} in the moment equation.

Remember the $I_{pin}\alpha$ term on the RHS is the equivalent of a kinetic moment of $I_G\alpha$ plus $ma_t(r)$ about the pin. Using I_{pin} simplifies the equation slightly.



Use force summations to find the pin reactions at A:

$\Sigma F_y = ma_y$; $A_y - 64.4 = -\,2(19.3)(4/5)$

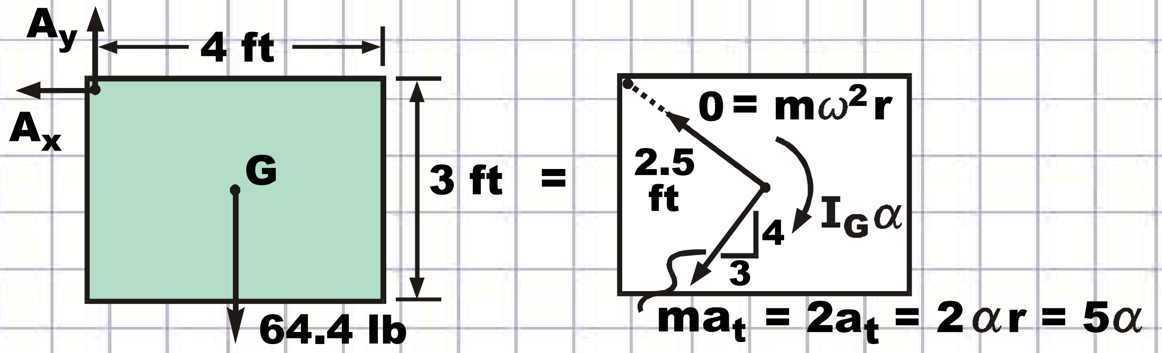
$A_y = 64.4 - 30.9$

$A_y = 33.5$ lb

$\Sigma F_x = ma_x$; $A_x = 2(19.3)(3/5)$

$A_x = 23.2$ lb

As noted earlier, you may choose to express the pin reactions at A in normal and tangential components, too. Then sum forces in n and t to compute A_n and A_t. (The mg term would need to be resolved into n and t coordinates, though.)



If you choose to sum moments about the pin using a **kinetic moment**, (instead of using $I_{pin}\alpha$) you get:

$$\Sigma M_{pin} = I_G\alpha + ma_t(r); \;\; (64.4)(2\text{ ft}) = 4.17\alpha + 2a_t(2.5)$$

$$128.8 = 4.17\alpha + 5a_t \;\; ; \text{ but here sub in } \;\; a_t = 2.5\alpha$$

$$128.8 = 4.17\alpha + 5(2.5\alpha) = 16.67\alpha$$

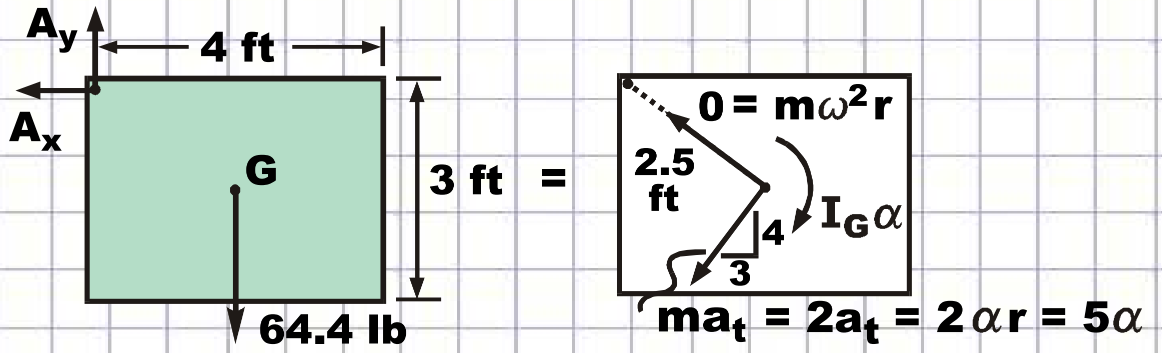
$$\boxed{\alpha = 7.73\text{ rad/sec}^2}$$

etc.

If you choose to sum moments about G, you introduce the pin reactions into the moment equation:

$$\Sigma M_G = I_G\alpha \; ; \;\; A_y(2) - A_x(1.5) = 4.17\alpha \qquad (1)$$

See the next page for the remainder of this approach....



Write the F = ma equations:

$\Sigma F_y = ma_y$; $A_y - 64.4 = -2(a_t)(4/5)$ (2)

$\Sigma F_x = ma_x$; $A_x = 2(a_t)(3/5)$ (3)

You need a fourth equation: $a_t = 2.5\alpha$. (4)

Solve these four equations for the four unknowns.

$$\begin{bmatrix} 2 & -1.5 & 0 & -4.17 \\ 1 & 0 & 1.6 & 0 \\ 0 & 1 & -1.2 & 0 \\ 0 & 0 & 1 & -2.5 \end{bmatrix} \begin{bmatrix} A_y \\ A_x \\ a_t \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ 64.4 \\ 0 \\ 0 \end{bmatrix}$$

$A_y = 33.5$ lb
$A_x = 23.2$ lb
$a_t = 19.3$ fps^2
$\alpha = 7.73$ r/s^2

These agree with our earlier results, but you can see that summing moments about G involves more solution effort. That's why we recommend summing moments about the pin.