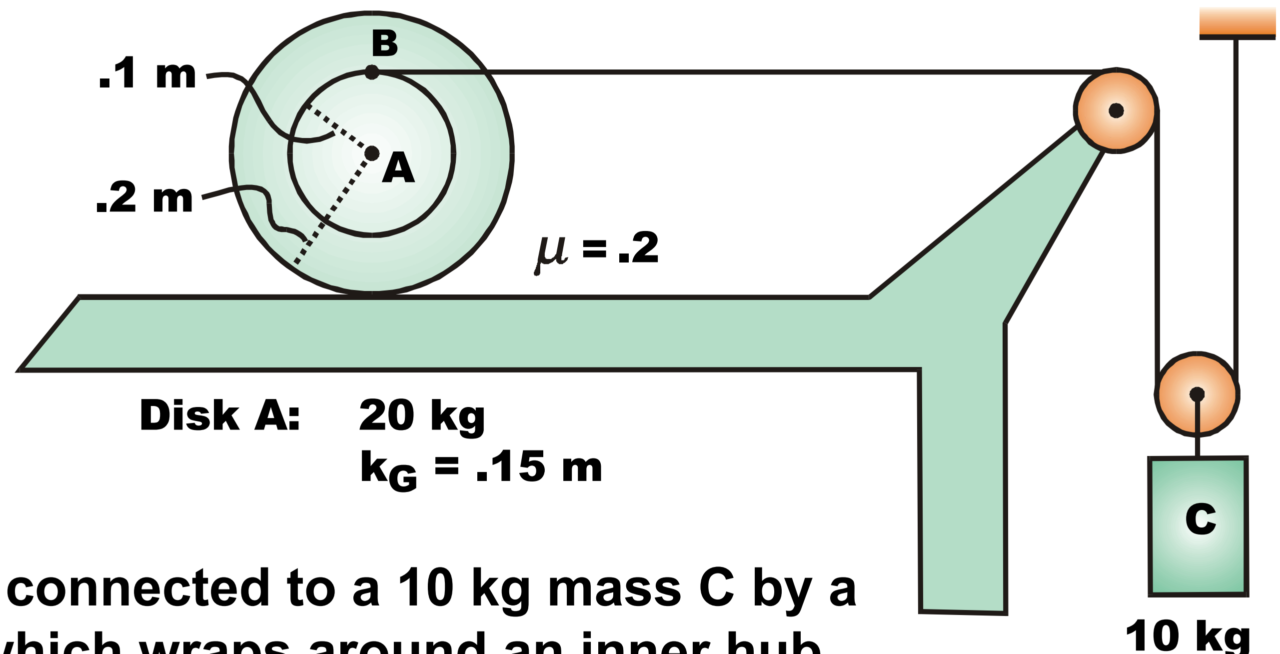
Rigid Body F=ma Intro (Gen Plane B): Example 1

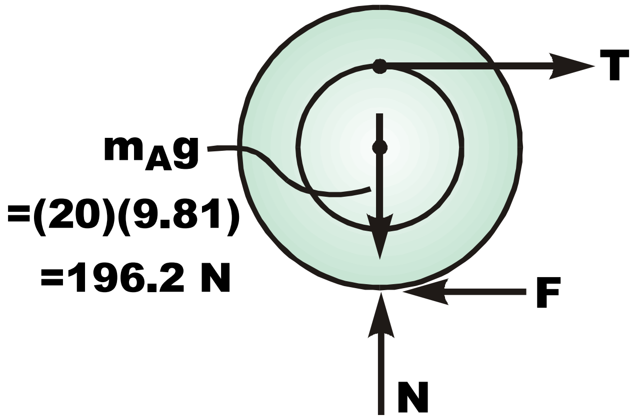
(Last class we worked a drum and weight connected by a cable problem, except the cable attached to the *center* of the drum. In this problem the cable is *not* at the center and we must use a relative acceleration equation for kinematics.



A 20 kg disk A is connected to a 10 kg mass C by a massless cable which wraps around an inner hub on A and around massless, frictionless pulleys. If the system is released from rest in the position shown, please determine the tension in the cable and the component accelerations.

Draw the FBD and KD for the drum and the mass:

FBD for A



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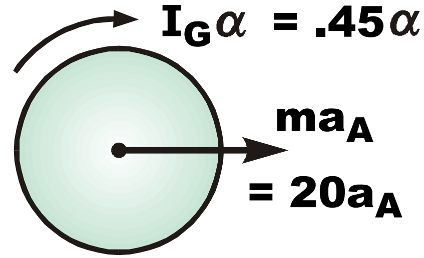
KD for A

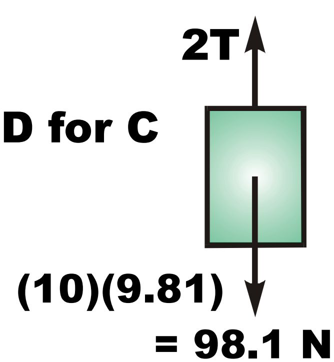


I_G from k_G : $\boxed{I_G = mk_G^2}$

$I_G = (20)(.15^2) = 0.45$ kg-m^2

Important! **Do not set $F = \mu N$ unless the problem says it slips. Treat F as unknown.**

FBD for C



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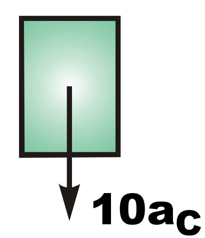
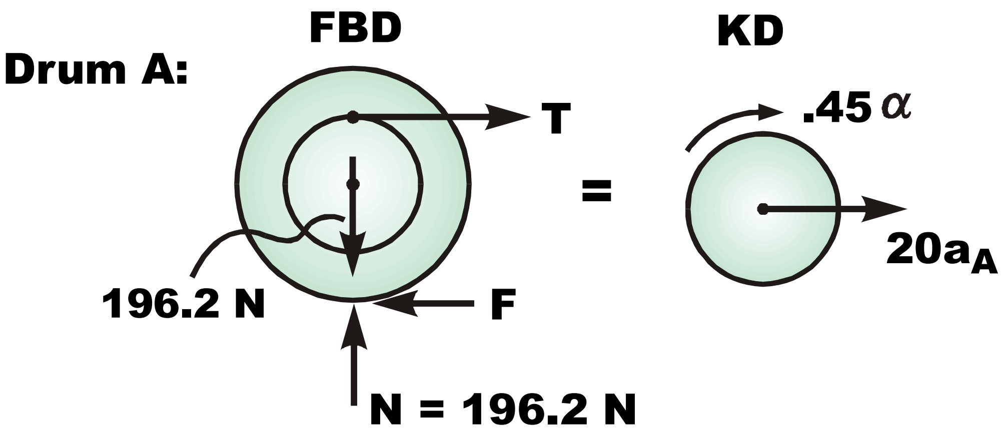
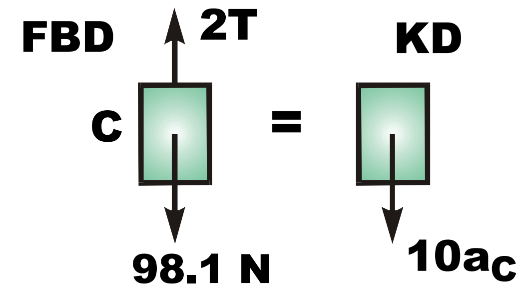
KD for C



Write the Equations of Motion:





Equations of Motion (Drum A):

$+\uparrow \Sigma F_y = ma_{Gy}$; $N = 196.2$ N

$\xrightarrow{+} \Sigma F_x = ma_{Gx}$; $T - F = 20a_A$ ①

$\overset{+}{\curvearrowright} \Sigma M_G = I_G \alpha$; $T(.1) + F(.2) = .45\alpha$ ②

Equation of Motion (Mass C):

$+\downarrow \Sigma F_y = ma_{Gy}$; $98.1 - 2T = 10a_C$ ③

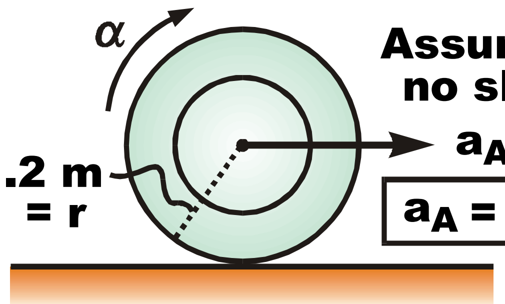
Count the unknowns in these THREE equations. How many do you get?

I count FIVE!

We need TWO more equations.

We need two additional equations…Use kinematics:

Assume no slip: $a = \alpha r$



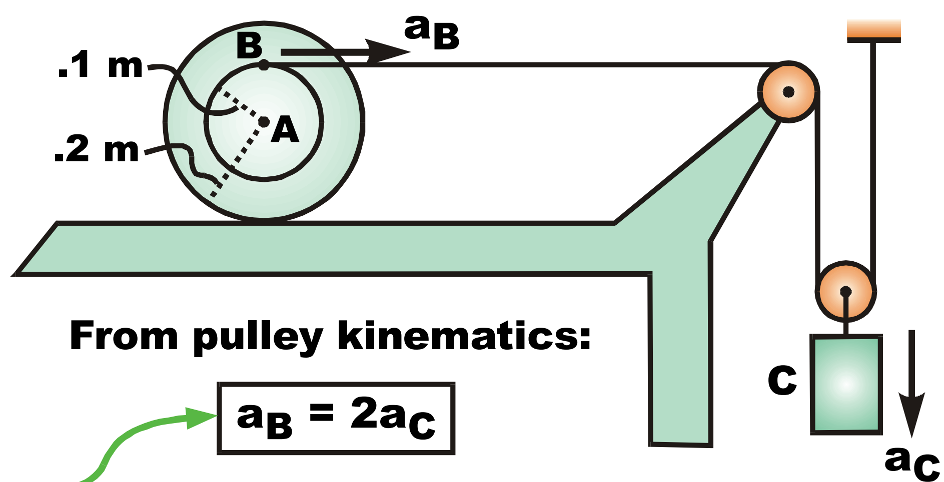
Assume no slip:

$a_A = \alpha r$

$\boxed{a_A = .2\alpha}$ ④

Note: This "no slip" assumption must be validated with your answers.

Relative Accel Equation for Drum A:



From pulley kinematics:

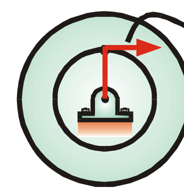
$\boxed{a_B = 2a_C}$

But a_B is not one of our current unknowns. How can we relate a_C to a_A? Use the relative accel equation.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$\vec{a}_B$: $\longrightarrow$

$\vec{a}_A$: $\longrightarrow$ $\alpha r = .2\alpha$ (no slip)



$\vec{a}_{B/A}$: $\alpha r = .1\alpha$

$$a_B = .2\alpha + .1\alpha = .3\alpha \longrightarrow$$

$$2a_C = a_B = .3\alpha$$

Finally: $\boxed{a_C = .15\alpha}$ ⑤

Set up the matrix and solve the system of eqns:

① $T - F = 20a_A$

② $T(.1) + F(.2) = .45\alpha$

③ $98.1 - 2T = 10a_C$

④ $a_A = .2\alpha$

⑤ $a_C = .15\alpha$

$$\begin{array}{c} \\ ① \\ ② \\ ③ \\ ④ \\ ⑤ \end{array} \begin{array}{c} \begin{array}{ccccc} T & F & a_A & a_C & \alpha \end{array} \\ \begin{bmatrix} 1 & -1 & -20 & 0 & 0 \\ .1 & .2 & 0 & 0 & -.45 \\ 2 & 0 & 0 & 10 & 0 \\ 0 & 0 & 1 & 0 & -.2 \\ 0 & 0 & 0 & 1 & -.15 \end{bmatrix} \end{array} \begin{bmatrix} T \\ F \\ a_A \\ a_C \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 98.1 \\ 0 \\ 0 \end{bmatrix}$$

Solve with calculator:

$T = 41.6$ N

$F = 1.66$ N ←

$a_A = 2.0$ m/s^2 →

$a_C = 1.50$ m/s^2 ↓

$\alpha = 9.98$ rad/s^2 ↻

One last step: **Check the no slip assumption…**

$(F/N)_{CALC} < \mu = .2$? Clearly true because F is small.