0 Math Review

Tuesday, January 21, 2025 12:31 PM

- Counting and sums:

Problem:

$0 + 1 + 2 + 3 + 4 + 5 \ldots + 97 + 98 + 99 + 100$

$\ldots\ldots + 48 + 49 + 50 + 51 + 52 + \ldots$

$= 50 \cdot 100 + 50 = 5050$

Generalizing:

$$
\begin{aligned}
1 + 2 + 3 + 4 + \ldots (n-2) + (n-1) + n &= S \\
n + (n-1) + (n-2) + (n-3) + \ldots 3 + 2 + 1 &= S \\
\hline
(n+1) + (n+1) + (n+1) + \ldots + (n+1) + (n+1) + (n+1) &= 2 \cdot S
\end{aligned}
$$

$$n \cdot (n+1) = 2 \cdot S$$

$$S = \frac{n \cdot (n+1)}{2}$$

$$1 + 2 + 3 + 4 + \ldots = \sum_{i=1}^{n} i = \frac{n \cdot (n+1)}{2} \approx \frac{1}{2} n^2$$

Other important sums:

- $\displaystyle\sum_{i=\ell}^{u} 1 = \underbrace{1 + 1 + 1 + \ldots + 1}_{u - \ell + 1 \text{ times}} = u - \ell + 1$

- $\sum_{i=1}^{n} 1 = n$

- $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{1}{3} n^3$

- $\sum_{i=1}^{n} i^k = 1^k + 2^k + 3^k + \dots = n^k = \frac{1}{k+1} n^{k+1}$

- $\sum_{i=0}^{n} a^i = a^0 + a^1 + a^2 + \dots + a^n = \frac{a^{n+1} - 1}{a-1}$
    - $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

Sum Manipulation:

- $\sum_{i=l}^{u} c \cdot a_i = c \cdot \sum_{i=l}^{u} a_i$

- $\sum_{i=l}^{u} (a_i \pm b_i) = \sum_{i=l}^{u} a_i \pm \sum_{i=l}^{u} b_i$

- $\sum_{i=l}^{u} a_i = \sum_{i=l}^{m} a_i + \sum_{i=m+1}^{u}$

- $\sum_{i}^{u} (a_i - a_{i-1}) = a_u - a_{l-1}$

- $\sum_{i=l}^{u} (a_i - a_{i-1}) = a_u - a_{l-1}$

logarithms.

- $x = a^k \qquad \log_a x = k$
- $\log_a xy = \log_a x + \log_a y$
- $\log_a x^y = y \cdot \log_a x$
- $a^{\log_b x} = x^{\log_b a}$
- $\underbrace{\log_a x} = \dfrac{\log_b x}{\log_b a} = \log_a b \cdot \underbrace{\log_b x}$