

6.1 Topological Sort

Thursday, April 3, 2025 1:03 PM

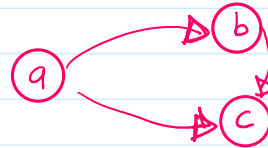
• Application of Decrease and Conquer

Sorting a digraph.

digraph :- directed graph
a graph where edges have direction

E.g

$G = \{V, E\}$
 $V = \{a, b, c\}$
 $E = \{(a, b), (a, c), (b, c)\}$



NOTE: Both DFS and BFS work on digraphs.

Problem:

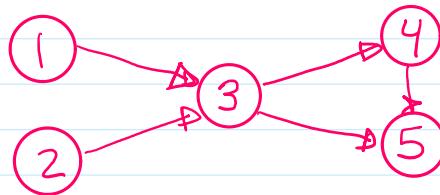
Suppose you have courses C_1, C_2, C_3, C_4, C_5

- C_1 and C_2 have no prerequisites
- C_3 requires C_1 and C_2
- C_4 req C_3
- C_5 req C_3 and C_4

¿ In which order to take the classes?

Abstraction:

use a directed graph



Problem (Re-stated)

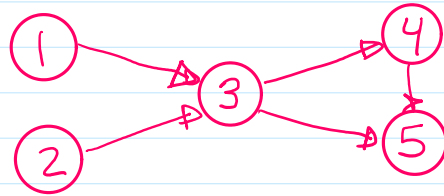
- order the vertices $\langle v_0, v_1, v_2 \dots v_k \rangle$

in such a way that for every edge $\langle v_a, v_b \rangle$

$a < b$ in the order

Note: not solvable if the digraph has a cycle.

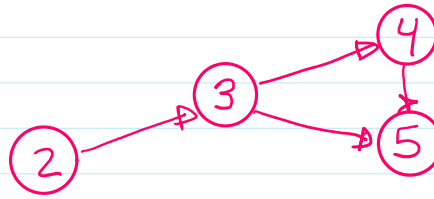
Solution: Decrease and Conquer.



Size=5, now can reduce this problem to size=4

(i) get rid of one node; one with no incoming edges push it in the vertex order.

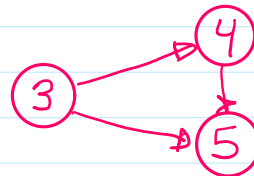
$\langle 1, \$



Algorithm

- Select a node with no incoming edges
- place it next in the order
- Simplify graph.

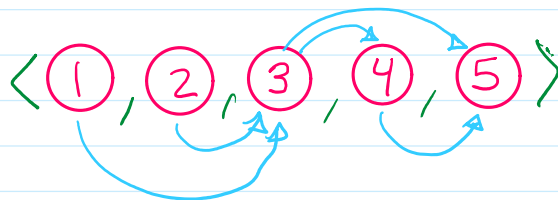
$\langle 1, 2, \$



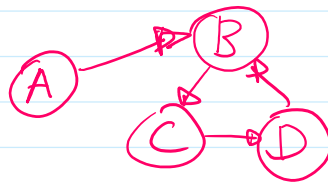
$\langle 1, 2, 3, \$



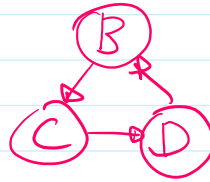
$\langle 1, 2, 3, 4, \$



• What if the digraph has cycles?



<A,



Fai|!

—0—0— EOF