

Thursday, April 3, 2025 1:30 PM

Problem: Given a Sorted Array $A[0 \dots n-1]$
answer whether a value x is in A

Diagram illustrating the first harmonic of a string fixed at both ends. The string is represented by a horizontal line with a green line above it. A vertical green line marks the center. A bracket below the left half is labeled $n/2$.

Compare x to the element in the middle of the array.



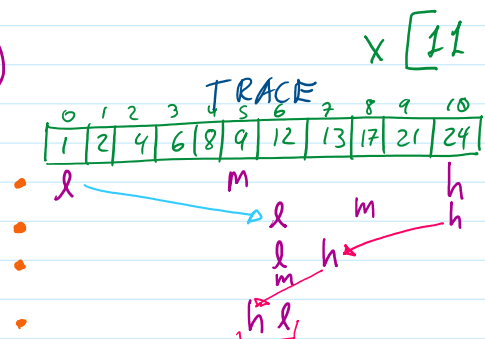
- Solution Statement is Recursive.
but final implementation can be iterative.

A diagram of a tube with nodes (N) and antinodes (A). The tube is divided into three sections by two vertical lines. The first section is labeled 'N' at the left end and 'A' at the first vertical line. The second section is labeled 'A' at the first vertical line and 'N' at the second vertical line. The third section is labeled 'N' at the second vertical line and 'A' at the right end. The labels 'N' and 'A' are written in blue. The vertical lines are drawn in blue. The tube itself is drawn in black.

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l ← 0
h ← n-1
WHILE l ≤ h DO
    m ← ⌊(l+h)/2⌋
    IF x = A[m] THEN
        RETURN m
    ELIF x < A[m] THEN
        h ← m-1
    ELSE // x > A[m]
        l ← m+1
RETURN -1

```


$$C(n) = 1 \quad C(n) = 1 + C\left(\frac{n}{2}\right)$$

analysis: basic operation

$$C(n)_{\text{best}} = 1$$

$$C(n)_{\text{worst}} = 1 + C\left(\frac{n}{2}\right)_{\text{worst}}$$

Argument: at every iteration we consider half of the array

$$C(n)_{\text{worst}} = \log_2 n \in \Theta(\log n)$$

- Finding elements is extremely useful
It's good that we can do it fast.

n	Linear Search	Binary Search
30,000	30,000	15
3,000,000	3,000,000	22
300,000,000	300,000,000	29

Example: Fake-Coin Problem

- Given n identical coins
- one coin is fake
- the fake coin is lighter than the real coins
- you only have balance to compare groups of coins.



How to find the fake coin?

Algorithm:

case: n is even:

$$\left(\frac{n}{2}\right) \text{ vs } \left(\frac{n}{2}\right)$$

- continue search on pile that is lighter.

case: n is odd

$$\left(\left\lfloor \frac{n}{2} \right\rfloor\right) \text{ vs } \left(\left\lfloor \frac{n}{2} \right\rfloor\right) \text{ vs } 1$$

case a) continue search on pile that is lighter.

case b) if pile balance, the left over coin is the fake one.

Analysis: basic operation Weighting.

$$W_{\text{best}}(n) = 1$$

$$W_{\text{worst}}(n) = 1 + W\left(\frac{n}{2}\right)$$

$\vdots$

$$W(n) \in \Theta(\log n)$$

Can we do better?

Split by 3.....

$$\left(\frac{n}{3}\right)$$

$$\left(\frac{n}{3}\right)$$

$$\left\lfloor \frac{n}{3} \right\rfloor$$

- n is divisible by 3 :- case a) plates balance
continue search on left over group
case b) plates don't balance.
continue search on the lighter one.
- 1 left over coin :- put it in the outside stack.
- 2 left over coins :- put one each on the weighted piles.

• Example: Russian Peasant Multiplication

$$n \cdot m = \underbrace{m + m + m + m \dots m}_{n\text{-times}}$$

How can we reduce by a constant factor?

$$n \cdot m \text{ to } \frac{n}{2} ?$$

- n is even - $n \cdot m = \frac{n}{2} \cdot 2m$

- n is odd - $n \cdot m = \frac{n-1}{2} \cdot 2m + m$

How? - multiplying by 2 is easy
- dividing by 2 is easy

Eg:

$$\begin{array}{r} n \\ 50 * 68 \\ \hline 25 \quad 136 \end{array}$$

$$\begin{array}{r} m \\ 22 \\ 136 \end{array}$$

Eg: n m.

$$\begin{array}{r}
 50 * 68 \\
 \begin{array}{r}
 25 \quad 136 \\
 12 \quad 272 \\
 6 \quad 544 \\
 3 \quad 1088 \\
 1 \quad 2176
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 22 \\
 136 \\
 1088 \\
 \times 2176 \\
 \hline
 3400
 \end{array}$$

Note: multiplying and dividing in binary is even easier!!

$$\begin{array}{r}
 110110 * 10101 \\
 \begin{array}{r}
 11011 \quad 101010 \\
 1101 \quad 1010100 \\
 110 \quad 10101000 \\
 11 \quad 101010000 \\
 1 \quad 1010100000
 \end{array}
 \end{array}$$

—o—o—