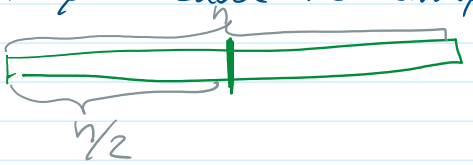
6.2 Applications of Decrease-by-Constant-Factor

Thursday, April 3, 2025 1:30 PM

- **Example Algorithm: Binary-Search**

Problem: Given a Sorted Array A[0...n-1]
answer whether a value x is in A

(i) How can you reduce the array you are searching x into?

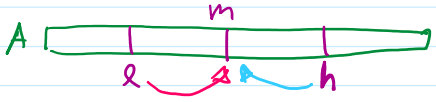


Algorithm
compare x to the element in the middle of the array.

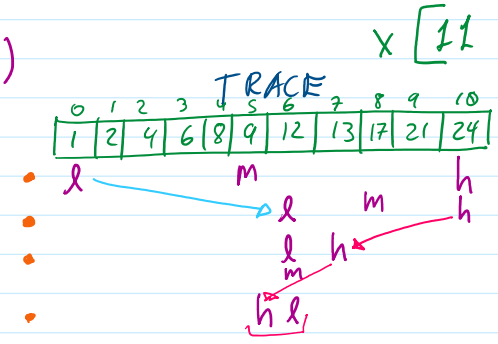
cases
- x = A[m] success!!
- x < A[m] : search on left subarray
- x > A[m] : search on right subarray

- Solution Statement is Recursive.
  but final implementation can be iterative.

- Iterative version.




```
FUNCTION BinarySearch(A[0...n-1], X)
    l ← 0
    h ← n-1
    WHILE l ≤ h DO
        m ← ⌊(l+h)/2⌋
        IF x = A[m] THEN
            RETURN m
        ELSIF x < A[m] THEN
            h ← m-1
        ELSE // x > A[m]
            l ← m+1
    RETURN -1
```



Analysis: basic operation =

C(n) = 1 C(m) = 1 + C(n/2)

Analysis: basic operation

$C_{best}(n) = 1$ $\quad C_{worst}(n) = 1 + C_{worst}(n/2)$

Argument: at every iteration we consider half of the array

$C_{worst}(n) = \log_2 n \in \Theta(\log n)$

- Finding elements is extremely useful
  It's good that we can do it fast.

n	Linear Search	Binary Search
30,000	30,000	15
3,000,000	3,000,000	22
300,000,000	300,000,000	29

- Example: Fake-Coin Problem
  - Given n identical coins
  - one coin is fake
  - the fake coin is lighter than the real coins
  - you only have balance to compare groups of coins.

How to find the fake coin?

Algorithm:

case: n is even: $\frac{n}{2}$ vs $\frac{n}{2}$

- continue search on pile that is lighter.

case: n is odd $\lfloor \frac{n}{2} \rfloor$ vs $\lfloor \frac{n}{2} \rfloor$ vs 1

case a) continue search on pile that is lighter.

case b) if pile balance, the left over coin is the fake one.

Analysis: basic operation Weighting.

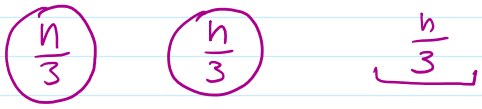
$W_{best}(n) = 1$ $\quad W_{worst}(n) = 1 + W\left(\frac{n}{2}\right)$

$\vdots$

$W(n) \in \Theta(\log n)$

Can we do better?

Split by 3.....



- n is divisible by 3 :- case a) plates balance
  continue search on left over group
  case b) plates don't balance.
  continue search on the lighter one.
- 1 left over coin :- put it in the outside stack.
- 2 left over coins :- put one each on the weighted piles.

- Example: Russian Peasant Multiplication

$$n \cdot m = \underbrace{m + m + m + m \ldots m}_{n\text{-times.}}$$

How can we reduce by a constant factor?

$n \cdot m$ to $\frac{n}{2}$?

- n is even - $n \cdot m = \frac{n}{2} \cdot 2m$

- n is odd - $n \cdot m = \frac{n-1}{2} \cdot 2m + m$

Hwy? - multiplying by 2 is easy
- dividing by 2 is easy

Eg: n m.

50 * 68

25 136

2 2

136

Eg: n m.

50	68
25	136
12	272
6	544
3	1088
1	2176

2 2

136
1088
× 2176
3400

Note: multiplying and dividing in binary is even easier!!

11010	1101
1101	10101
110	101000
11	1010000
1	10100000