

• HEURISTICS:

pg. 8, 9, 10 : Uninformed search.

No information about the goal is taken into account.

Sometimes we Do have hints about the goal

**IDEA:** Use information about the goal to select paths with better "potential"  
"heuristic"

DEF heuristic function  $h(n)$  nodes  $\mapsto$  value

value: - an estimate of the cost of the least-cost-path from  $n$  to the goal.

We can extend  $h$  to paths:

$$h(\langle n_0, n_1, n_2, n_3, \dots, n_k \rangle) = h(n_k)$$

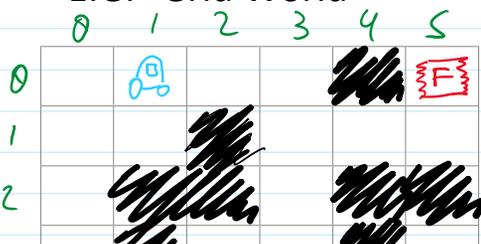
Note:  $h(n)$  should be quick to compute.

**Admissible Heuristic:** - Non-negative.  
- Underestimate of true cost

underestimate : - no path from  $n$  to the goal can have cost less than  $h(n)$

E.G.

E.G. "Grid World"



states:  $\langle \text{row}, \text{col} \rangle$

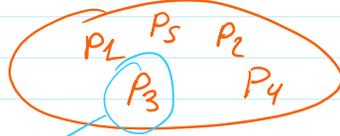
goal:  $\langle 0, 5 \rangle$

$h(n)$ : distance between  $n$  and goal



$h(n)$ : distance between  $n$  and goal (disregarding walls)

- Developing a heuristic is Domain Dependent
  - Try solving a "simpler" problem.
- SEARCH STRATEGY: Greedy Best-First Search
  - Treat Frontier as a priority queue where paths are ordered by their heuristic value.

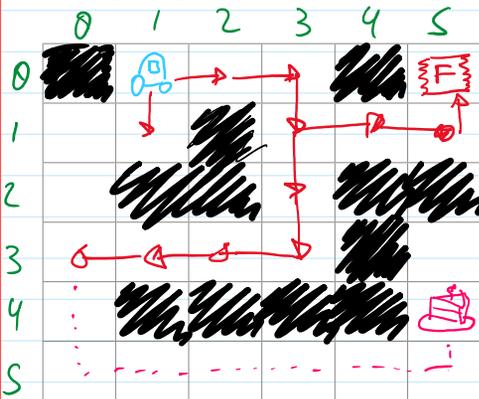


Lowest  $h()$  value.

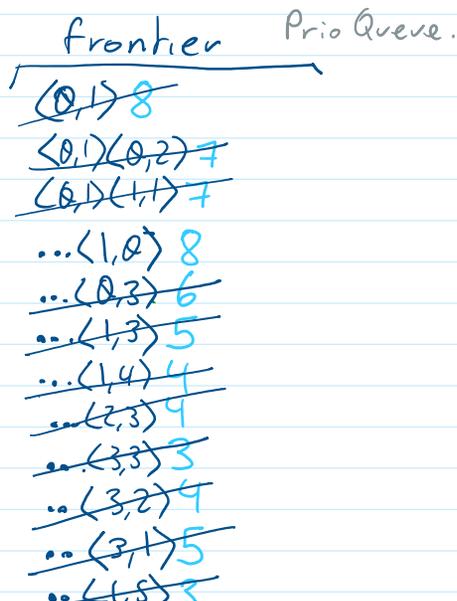
$P_3$  is the path with the lowest heuristic value  
 $P_3$  is tested  
 If  $P_3$  is not a solution extensions of  $P_3$  are inserted in the frontier.

E.G.

E.G. "Grid World"



- Action Order:  $\uparrow \rightarrow \downarrow \leftarrow$
- Tiebreaker: prefer last added.



• Tiebreaker: prefer last added.

- ~~∴ (3,1) 5~~
- ~~∴ (1,5) 3~~
- ~~∴ (0,5) 4~~
- ~~∴ (3,0) 6~~
- ∴ (2,0) 7
- ∴ (4,0) 5

• SEARCH STRATEGY: A-star Search

A\*

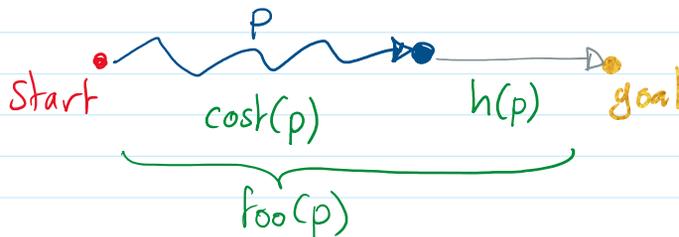
- A mix of Lowest-cost-first-search & Greedy-best-first search  
 $cost(p)$   $h(p)$

In A\*  $cost(p) + h(p)$

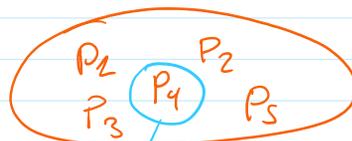
In A\* search strategy:

treat the frontier as a **priority queue** where the paths are ordered by  $foo(p)$  value where  $foo(p) = cost(p) + h(p)$

$foo(p)$  :- an estimate of the lowest cost of a path to the goal through p



frontier



$P_4$  has the lowest  $foo(p)$  value.  
 $P_4$  is tested

Extensions of  $P_4$  are inserted in the frontier.

Note A\* always finds the optimal solution when:

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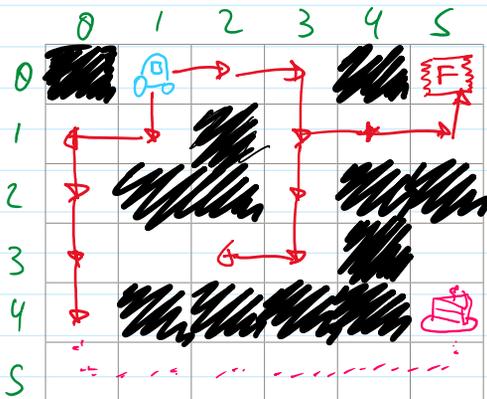
- The branching factor is finite
- The arc costs are "bounded"

$\exists \alpha \geq 0$  all costs are greater than  $\alpha$

- $h(p)$  is non-negative and an underestimate.

E.G.

E.G. "Grid World"

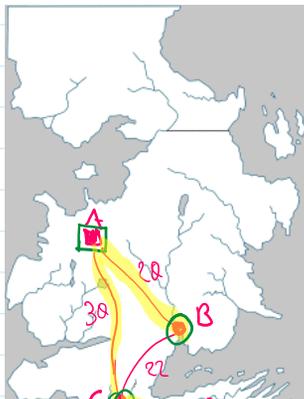


- Action Order:  $\uparrow \rightarrow \downarrow \leftarrow$
- Tiebreaker: prefer last added.

Frontier Priority queue

- ~~(0,1) 0+8 = 8~~
- ~~(0,1) (0,2) 1+7 = 8~~
- ~~(0,1) (1,1) 1+7 = 8~~
- ~~... (1,0) 2+8 = 10~~
- ~~... (0,3) 2+6 = 8~~
- ~~... (1,3) 3+5 = 8~~
- ~~... (1,4) 4+4 = 8~~
- ~~... (2,3) 4+4 = 8~~
- ~~... (3,3) 5+3 = 8~~
- ~~... (3,2) 6+4 = 10~~
- ~~... (1,5) 5+3 = 8~~
- ~~... (0,5) 6+4 = 10~~
- ~~... (3,1) 7+5 = 12~~
- ~~... (2,0) 3+7 = 10~~
- ~~... (3,0) 4+6 = 10~~
- ~~... (3,1) 5+5 = 10~~
- ~~... (4,0) 5+5 = 10~~

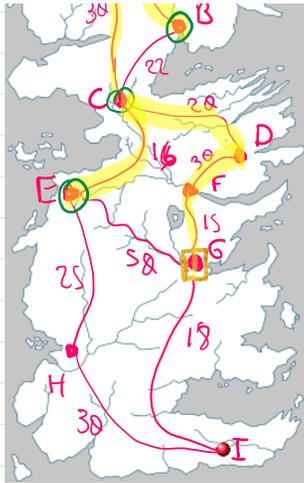
EG #2



Euclidean Distance is an Acceptable Heuristic.

- $h(n)$
- A 55
  - B 43
  - C 31
  - D 26
  - E 26
  - F 12
  - G 0
  - H 30

- Frontier
- ~~[A] 0+55 = 55~~
  - ~~[A,B] 20+43 = 63~~
  - ~~[A,C] 30+31 = 61~~
  - ~~[A,C,B] 52+43 = 95~~
  - ~~[A,C,D] 50+26 = 76~~
  - ~~[A,C,E] 46+26 = 72~~
  - ~~[A,C,E,G] 96+0 = 96~~
  - ~~[A,C,E,H] 71+30 = 101~~



G 0  
H 30  
I 15

~~[A,C,E,G]~~  $96 + 0 = 96$  ●  
~~[A,C,E,H]~~  $71 + 30 = 101$   
~~[A,C,D,F]~~  $80 + 12 = 92$   
~~[A,C,D,F,G]~~  $95 + 0 = 95$  ●

[A,C,D,F,G] ★

• VIZUALISATIONS :

- Dijkstra vs A\*  
 Lowest-cost-first cost + h

[https://upload.wikimedia.org/wikipedia/commons/2/23/Dijkstras\\_progress\\_animation.gif](https://upload.wikimedia.org/wikipedia/commons/2/23/Dijkstras_progress_animation.gif)

[https://upload.wikimedia.org/wikipedia/commons/5/5d/Astar\\_progress\\_animation.gif](https://upload.wikimedia.org/wikipedia/commons/5/5d/Astar_progress_animation.gif)

Pathfinding algorithm comparison: Dijkstra's



A\* (A-Star) Pathfinding Algorithm Visualization on a Real Map



- VISUALIZING THE DIFFERENT SEARCH ALGORITHMS:

<https://qiao.github.io/PathFinding.js/visual/>

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