

# Thermostable refractive index sensors based on whispering gallery modes in a microsphere coated with poly(methyl methacrylate)

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This study proposes a thermostable refractive index (RI) sensor consisting of a silica microsphere coated with a poly(methyl methacrylate) (PMMA) layer. The first-order and second-order whispering gallery modes (WGMs) of both TE and TM polarizations are considered theoretically. The layer thickness is carefully optimized to eliminate the thermal drift and enhance the RI sensitivity and detection limit. In various WGMs, at the thermostable thickness of the PMMA layer, the first-order TM mode corresponds to the highest sensitivity and the smallest detection limit. The theoretical predictions provide guidelines for the design and fabrication of thermostable RI sensors. © 2011 Optical Society of America

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## 1. Introduction

Optical microresonators of various shapes, such as microspheres, microdisks, and microrings, have been widely studied [1–12]. Resonators with whispering gallery modes (WGMs) can achieve high quality factors ( $Q$ -factor), i.e., long light-material interaction paths. For instance, a  $Q$ -factor on the order of  $10^9$  has been reported for WGMs in fused silica microspheres [1]. Because of the high  $Q$ -factor and small mode volume, microresonators with WGMs are widely used as optical refractive index (RI) sensors for biological material detection, chemical composition measurement, concentration changes, etc. [2–8]. Krioukov *et al.* propose a RI sensor based on inte-

grated optical microcavities that can measure a RI change of  $10^{-4}$  RIU (refractive index unit) of the surrounding medium [7]. Hanumegowda *et al.* demonstrate a fused silica microsphere-based refractometric sensor with a sensitivity of nearly 30 nm/RIU and a detection limit on the order of  $10^{-7}$  RIU [8]. Vollmer *et al.* use a perturbation theory to calculate the resonance shift induced by refractive index change for WGMs in a transparent microsphere [9,10]. They also investigate the properties of WGMs in a microsphere coated with a high-refractive-index layer, and find that the high-refractive-index layer can enhance the sensitivity of WGM wavelength shift sensors [11,12].

However, WGMs are susceptible to thermal fluctuations due to environmental temperature variations and probe-induced energy absorptions. Especially for RI sensors with high  $Q$ -factors, the

instability of WGMs due to temperature-induced fluctuations significantly impairs the sensor performance [13]. Han and Wang theoretically demonstrate that it is possible to employ a surface layer with a negative thermo-optic coefficient to compensate the thermal drift of the resonances [14]. He *et al.* experimentally and theoretically investigate the feasibility of using a thin layer of polydimethylsiloxane (PDMS) to compensate the thermal refraction effect in a silica microtoroid [15]. But, due to the large absorption loss of light in PDMS, the  $Q$ -factor decreases quickly with coating thickness increase, which eventually degrades the detection limit. This study proposes a thermostable RI sensor by applying a thin layer of poly(methyl methacrylate) (PMMA) to the surface of a silica microsphere. The thermal refraction coefficient of PMMA is  $-1.25 \times 10^{-4}/^\circ\text{C}$  at room temperatures [16], which makes it possible to reduce or even completely eliminate the resonance thermal drift of the silica microsphere. The transition temperature of PMMA is about  $90^\circ\text{C}$ , so the proposed RI sensor can work stably at room temperatures. Furthermore, the absorption loss of light in PMMA is very low at the wavelength of 780 nm (about 0.15 dB/m) [17], which corresponds to a very high absorption-limited  $Q$ -factor, on the order of  $10^8$ . Because of the noncrystal structure, PMMA material can form a smooth and uniform thin layer on the silica microsphere surface. Dong *et al.* experimentally show an ultrahigh  $Q$ -factor ( $>10^8$ ) of the PMMA-coated silica microsphere [18].

For the proposed sensor, this study investigates the thermal and sensing properties of the WGMs in a PMMA-coated silica microsphere. The numerical models for the thermal drift, RI sensitivity and detection limit are presented in Section 2. Based on the models, the effects of the PMMA coating are studied in detail in Section 3. The first-order and second-order WGMs of both TE and TM polarizations are considered. Major results are reported in Section 4.

## 2. Theory

### A. Thermal Drift

Over the years, WGM modal structures and resonance spectra of microspheres have been widely studied [19–21]. WGM is characterized by a set of integers:  $l$ ,  $m$ , and  $v$ , which represent angular, azimuthal, and radial mode numbers, respectively. For an ideal sphere, modes with the same  $l$  and  $v$  but arbitrary  $m$  have the same resonant wavelength. When  $m = l$  and  $v = 1$ , the modes frequently are called fundamental WGMs. Also, a WGM has either TE or TM polarization, and the two polarizations can be selectively excited by controlling the polarization of the coupling light. In general, the resonance spectrum of WGMs is determined by the size of the microsphere and spatial distribution of the refractive index inside/outside the microsphere. This study considers an ideal microsphere with radius of  $a_0$  and refractive index of  $n_s$ , coated by a PMMA layer with thickness of  $h_0$

and refractive index of  $n_p$ . The coated microsphere is surrounded by a medium with the refractive index of  $n_0$ . The characteristic equation to specify the resonant wavelengths  $\lambda_R$  of the WGMs can be expressed by [11,12]

$$N_0 \frac{\chi'_l(n_0ka_1)}{\chi_l(n_0ka_1)} = \frac{B_l \psi'_l(n_pka_1) + \chi'_l(n_pka_1)}{B_l \psi_l(n_pka_1) + \chi_l(n_pka_1)},$$

$$N_0 = \begin{cases} n_0/n_p, & \text{TE modes} \\ n_p/n_0, & \text{TM modes} \end{cases}, \quad (1)$$

where  $k = 2\pi/\lambda_R$  is the resonant wave vector;  $a_1 = a_0 + h_0$  is the total radius of the entire microsphere including the PMMA layer;  $\psi_l$  and  $\chi_l$  are spherical Ricatti–Bessel function and spherical Ricatti–Neumann function, respectively;  $B_l$  is a coefficient, and its expression is given by

$$B_l = \frac{N_s \psi'_l(n_ska_0) \chi_l(n_pka_0) - \psi_l(n_ska_0) \chi'_l(n_pka_0)}{\psi'_l(n_pka_0) \psi_l(n_ska_0) - N_s \psi_l(n_pka_0) \psi'_l(n_ska_0)},$$

$$N_s = \begin{cases} n_s/n_p, & \text{TE modes} \\ n_p/n_s, & \text{TM modes} \end{cases}. \quad (2)$$

For a given value of  $l$ , there are multiple values of  $\lambda_R$  that satisfy the characteristic equation. These resonant modes are called the first-order mode, the second-order mode..., and the  $v$ th order mode in decreasing value of  $\lambda_R$ .

Considering a temperature variation,  $\delta T$ , due to the thermal expansion effect,  $a_0$  and  $h_0$  change to  $a_0(1 + \alpha_s \delta T)$  and  $h_0(1 + \alpha_p \delta T)$ .  $\alpha_s$  and  $\alpha_p$  are the linear thermal expansion coefficients of the microsphere and the surface layer, respectively. Because of the thermal-optic effect,  $n_s$  and  $n_p$  change to  $n_s + (dn_s/dT)\delta T$  and  $n_p + (dn_p/dT)\delta T$ .  $dn_s/dT$  and  $dn_p/dT$  are the thermal refraction coefficients of the microsphere and the surface layer, respectively [14]. According to Eq. (1), a resonant wavelength shift  $\delta\lambda_R$  would take place correspondingly.  $\delta\lambda_R$  can be numerically calculated by substituting the size and refractive index variations induced by  $\delta T$  into Eq. (1). However, it is difficult to derive an explicit formula to describe the relation between  $\delta\lambda_R$  and  $\delta T$  from the characteristic equation. The resonant wavelength shift versus temperature can be given by [22]

$$\frac{d\lambda_R}{dT} = \lambda_R \left( \frac{1}{n_{\text{eff}}} \frac{dn_{\text{eff}}}{dT} + \frac{1}{D} \frac{dD}{dT} \right), \quad (3)$$

where  $D = 2(a_0 + h_0)$  is the total diameter of the coated microsphere including the PMMA layer;  $n_{\text{eff}}$  is the effective refractive index of the WGMs, which can be approximately expressed as  $n_{\text{eff}} = \eta_s n_s + \eta_p n_p + \eta_0 n_0$ .  $\eta_s$ ,  $\eta_p$ , and  $\eta_0$  denote the fractions of light energy distributed in the microsphere, surface layer, and surrounding medium, respectively. Thus,  $d\lambda_R/dT$  can be further expressed as

$$\frac{d\lambda_R}{dT} \approx \lambda_R \left( \frac{\eta_s (dn_s/dT) + \eta_p (dn_p/dT)}{\eta_s n_s + \eta_p n_p + \eta_0 n_0} + \frac{\alpha_s a_0 + \alpha_p h_0}{a_0 + h_0} \right). \quad (4)$$

In Eq. (4), the refractive index change of the surrounding medium induced by temperature variation is not considered. The energy fractions can be calculated by [11,12]

$$\begin{aligned} \eta_s &= \frac{n_s^2 I_s}{n_s^2 I_s + n_p^2 I_p + n_0^2 I_0}, \\ \eta_p &= \frac{n_p^2 I_p}{n_s^2 I_s + n_p^2 I_p + n_0^2 I_0}, \\ \eta_0 &= \frac{n_0^2 I_0}{n_s^2 I_s + n_p^2 I_p + n_0^2 I_0} \quad (\text{TE modes}), \end{aligned} \quad (5a)$$

$$\begin{aligned} \eta_s &= \frac{I_s}{I_s + I_p + I_0}, & \eta_p &= \frac{I_p}{I_s + I_p + I_0}, \\ \eta_0 &= \frac{I_0}{I_s + I_p + I_0} \quad (\text{TM modes}), \end{aligned} \quad (5b)$$

where

$$\begin{aligned} I_s &= \int_0^{a_0} [A_l \psi_l(n_s k_0 r)]^2 dr, \\ I_0 &= \int_{a_1}^{\infty} [C_l \chi_l(n_0 k_0 r)]^2 dr, \\ I_p &= \int_{a_0}^{a_1} [B_l \psi_l(n_p k_0 r) + \chi_l(n_p k_0 r)]^2 dr. \end{aligned} \quad (5c)$$

$A_l$  and  $C_l$  are the coefficients determined by

$$\begin{aligned} A_l &= \frac{B_l \psi_l(n_p k a_0) + \chi_l(n_p k a_0)}{\psi_l(n_s k a_0)}, \\ C_l &= \frac{B_l \psi_l(n_p k a_1) + \chi_l(n_p k a_1)}{\chi_l(n_0 k a_1)}. \end{aligned} \quad (5d)$$

Substituting Eq. (5) into Eq. (4), the thermal drift for TE and TM modes in the coated microsphere can be respectively expressed by

$$\begin{aligned} \left( \frac{d\lambda_R}{dT} \right)_{\text{TE}} &\approx \lambda_R \left( \frac{n_s^2 (dn_s/dT) I_s + n_p^2 (dn_p/dT) I_p}{n_s^3 I_s + n_p^3 I_p + n_0^3 I_0} \right. \\ &\quad \left. + \frac{\alpha_s a_0 + \alpha_p h_0}{a_0 + h_0} \right), \end{aligned} \quad (6a)$$

$$\begin{aligned} \left( \frac{d\lambda_R}{dT} \right)_{\text{TM}} &\approx \lambda_R \left( \frac{(dn_s/dT) I_s + (dn_p/dT) I_p}{n_s I_s + n_p I_p + n_0 I_0} \right. \\ &\quad \left. + \frac{\alpha_s a_0 + \alpha_p h_0}{a_0 + h_0} \right). \end{aligned} \quad (6b)$$

As shown by Eq. (6), the thermal drift can be reduced or even eliminated by adjusting the thickness of the PMMA layer, due to its negative thermal refraction coefficient. But, the layer thickness may significantly influence other sensor performance factors, especially the RI sensitivity and detection limit.

### B. Refractive Index Sensitivity

With regard to a refractive index change of the surrounding medium  $\delta n_0$ , the resonant wavelength shift  $\delta \lambda_R$  induced by  $\delta n_0$  can be calculated by  $\delta \lambda_R = \delta n_0 \times S$ .  $S$  is the sensitivity of the RI sensor, which is expressed by

$$S = \frac{\delta \lambda_R}{\delta n_0} = - \frac{\lambda_R}{\delta n_0} \cdot \frac{\delta k}{k_0}, \quad (7)$$

where  $k_0$  is the wave vector before refractive index change, and  $\delta k$  is the wave vector shift induced by  $\delta n_0$ . If the refractive index change is uniform and  $\delta(n_0^2) \ll 1$ , the fractional shift  $\delta k/k_0$  can be given as [11,12]

$$\left( \frac{\delta k}{k_0} \right)_{\text{TE}} = - \frac{\delta(n_0^2) I_0}{2(n_s^2 I_s + n_p^2 I_p + n_0^2 I_0)}, \quad \text{TE modes}, \quad (8a)$$

$$\left( \frac{\delta k}{k_0} \right)_{\text{TM}} = - \frac{\delta(n_0^2) [-T_0(a_1) T_0'(a_1^+) + n_0^2 k_0^2 I_0]}{2n_0^4 k_0^2 (I_s + I_p + I_0)}, \quad \text{TM modes}, \quad (8b)$$

where  $T(r)$  is the function to describe the electric field distribution of WGMs along the radial direction, and  $T_0$  denotes the field before the refractive index change;  $a_1^+$  is infinitesimally greater than  $a_1$ .  $T(r)$  is given by

$$T(r) = \begin{cases} A_l \psi_l(n_s k r) & r < a_0 \\ B_l \psi_l(n_p k r) + \chi_l(n_p k r) & a_0 < r < a_1 \\ C_l \chi_l(n_0 k r) & r > a_1 \end{cases} \quad (9)$$

Substituting Eqs. (8) and (9) into Eq. (7), and considering  $\delta(n_0^2) \approx 2n_0 \delta n_0$  if  $\delta(n_0^2) \ll 1$ , the RI sensitivity for TE and TM modes in the PMMA-coated microsphere can be respectively expressed as

$$S_{\text{TE}} = \frac{n_0 \lambda_R I_0}{n_s^2 I_s + n_p^2 I_p + n_0^2 I_0} = \frac{\lambda_R}{n_0} \eta_0, \quad (10a)$$

$$\begin{aligned} S_{\text{TM}} &= \frac{\lambda_R^2 [n_0 k_0 I_0 - C_l^2 \chi_l(n_0 k_0 a_1) \chi_l'(n_0 k_0 a_1)]}{2\pi n_0^2 (I_s + I_p + I_0)} \\ &= \frac{\lambda_R}{n_0} \eta_0 - \frac{C_l^2 \lambda_R^2 \chi_l(n_0 k_0 a_1) \chi_l'(n_0 k_0 a_1)}{2\pi n_0^2 (I_s + I_p + I_0)}. \end{aligned} \quad (10b)$$

As shown by Eq. (10), the sensitivity increases linearly with the energy fraction in the surrounding medium. The unit of sensitivity is nm/RIU.

### C. Detection Limit

The detection limit, defined as the smallest detectable change of refractive index, is proportional to  $1/(S \times Q)$  [23]. The  $Q$ -factor is defined as the ratio of the resonant wavelength to the full width at half maximum (FWHM) of the resonance. In practical applications of RI sensors, various noises can perturb the resonance spectrum. In these cases, the accurate detection of resonant wavelength shift is difficult for a broad resonance line width, which suggests that a narrow resonance (thus a high  $Q$  value) is preferred to achieve a good detection performance. The  $Q$ -factor of a well-fabricated silica microsphere with  $a_0 > 15 \mu\text{m}$  is limited by the optical absorption loss of light in silica [21]. In our case, the proposed RI sensor utilizes the PMMA coating as an intermediate layer to compensate the thermally-induced spectral fluctuations. But, unfortunately, the PMMA layer may reduce the  $Q$ -factor due to its larger absorption loss of light compared with silica. Considering a silica microsphere ( $a_0 > 15 \mu\text{m}$ ) coated with a defect-free and smooth-surface PMMA layer, the  $Q$ -factor is mainly determined by the absorption loss of light in silica and PMMA [15,18], which can be estimated by

$$\frac{1}{Q} \approx \eta_s \frac{\sigma_s \lambda_R}{2\pi n_s} + \eta_p \frac{\sigma_p \lambda_R}{2\pi n_p} = \eta_s \frac{1}{Q_0} + \eta_p \frac{\sigma_p \lambda_R}{2\pi n_p}, \quad (11)$$

where  $\sigma_s$  and  $\sigma_p$  are the material absorption loss of light in silica and that in PMMA, respectively, and  $Q_0 = 2\pi n_s / \sigma_s \lambda_R$  denotes the principle limit for the  $Q$ -factor of the silica microsphere before PMMA coating. This study mainly considers the PMMA layer influence on the  $Q$ -factor. The absorption loss of light in the surrounding medium is disregarded.

### 3. Results and Discussion

In the case study, a silica microsphere of  $n_s = 1.452$  coated with a PMMA layer is considered. The coated microsphere is assumed to be in aqueous environment of  $n_0 = 1.320$ , and coupled to a laser operating in the 780 nm band. The refractive index of PMMA is  $n_p = 1.49$  at room temperature [16]. The thermal refraction coefficients of silica and PMMA are  $dn_s/dT = 1.19 \times 10^{-5}/^\circ\text{C}$  [15] and  $dn_p/dT = -1.25 \times 10^{-4}/^\circ\text{C}$  [16], respectively. The linear thermal expansion coefficients of silica and PMMA are  $\alpha_s = 5.5 \times 10^{-7}/^\circ\text{C}$  [15] and  $\alpha_p = 2.02 \times 10^{-4}/^\circ\text{C}$  [24], respectively. Figure 1 shows  $d\lambda_R/dT$  for both TE and TM modes of  $v = 1$  and 2, as a function of the PMMA layer thickness. The radius of the silica microsphere is  $a_0 = 50 \mu\text{m}$ , and the angular mode numbers  $l$  are chosen to have a similar resonant wavelength of  $\lambda_R \approx 780 \text{ nm}$ . As shown in Fig. 1, the changes of  $d\lambda_R/dT$  with  $h_0$  are similar for the TE and TM modes. As the layer thickness increases,  $d\lambda_R/dT$  decreases quickly for the first-order WGMs. A near-zero thermal drift can be achieved when  $h_0 \approx 0.165$  and  $0.195 \mu\text{m}$ , for the TE and TM modes, respectively, while, for the second-order WGMs, a near-zero ther-

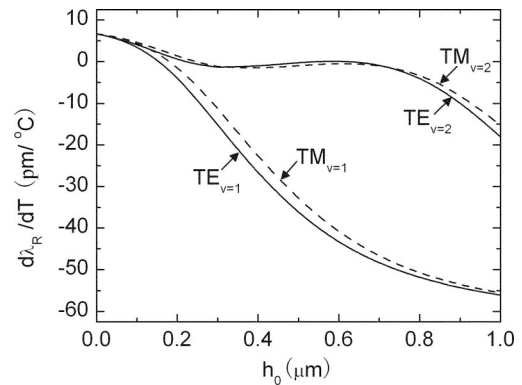


Fig. 1.  $d\lambda_R/dT$  for TE modes (solid curves) and TM modes (dashed curves) of two orders ( $v = 1, 2$ ) as a function of the PMMA layer thickness  $h_0$ .

mal drift can be achieved in a much wider range of PMMA layer thicknesses from 0.2 to  $0.7 \mu\text{m}$ .

Figure 2 shows the resonant wavelength shift as a function of the temperature change for the first-order TE modes in the silica microsphere of  $a_0 = 50 \mu\text{m}$  coated with PMMA layers of various thicknesses. The curves represent the approximate results evaluated by  $\delta\lambda_R = (d\lambda_R/dT) \times \delta T$ . The symbols denote the accurate results of  $\delta\lambda_R$  that are numerically calculated based on the resonance condition of Eq. (1). As shown in Fig. 2, the resonance of the microsphere before coating has a total redshift of about 33 pm with the  $5^\circ\text{C}$  temperature change. It is evident that a PMMA layer with the thickness around  $0.165 \mu\text{m}$  significantly reduces the thermal drift. However, as the layer thickness continuously increases, a blueshift of the resonance occurs and rapidly becomes much more significant. For example, when  $h_0 = 1.0 \mu\text{m}$ , the resonance has a total blueshift of about 280 pm with the  $5^\circ\text{C}$  temperature change, which can be used for thermal sensing. Figure 3 shows  $d\lambda_R/dT$  for the first-order and second-order TE modes in the silica microspheres of different radii as a function of the PMMA layer thickness. It is shown that the thermostable thickness shifts slightly to a larger value for the first-order modes in a larger

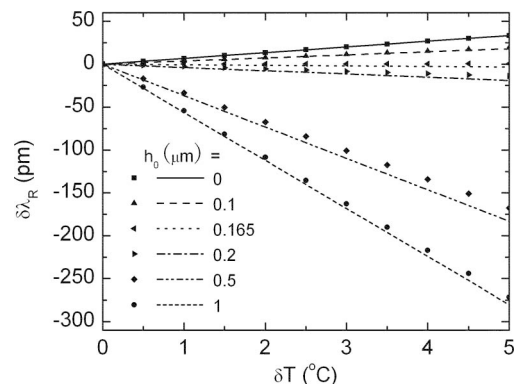


Fig. 2. Resonant wavelength shift  $\delta\lambda_R$  as a function of the temperature change  $\delta T$  for TE modes of  $v = 1$  at various PMMA layer thicknesses. The discrete spots represent  $\delta\lambda_R$  calculated by Eq. (1). The solid curves represent  $\delta\lambda_R = (d\lambda_R/dT) \times \delta T$ .

sphere. For example, a near-zero thermal drift is achieved when  $h_0 \approx 0.1$  and  $0.2 \mu\text{m}$ , for the silica microspheres of  $a_0 = 25$  and  $100 \mu\text{m}$ , respectively. For the second-order modes, the thermostable thickness range is wider for a larger sphere.

The primary performance factor of a RI sensor is the detection sensitivity. With an increase in layer thickness, the RI sensitivity is also affected due to the refractive index difference between silica and PMMA. Figure 4 shows the RI sensitivity as a function of the PMMA layer thickness, for both TE and TM modes of  $v = 1$  and 2 at  $a_0 = 50 \mu\text{m}$ . As shown in Fig. 4, the RI sensitivity is higher for the TM mode than the TE mode, and the peak of sensitivity is higher and at a larger layer thickness for the first-order mode than the second-order mode. At the thermostable thicknesses, the sensitivities are 25.6 and 21.6 nm/RIU, for the first-order TM and TE modes, respectively. Before PMMA coating, the sensitivities are 16.2 and 13.8 nm/RIU for the corresponding modes. Thus, the PMMA layer of the thermostable thickness not only significantly reduces the thermal drift, but also enhances the sensitivity. However, for the second-order WGMs, the sensitivity enhancement is less evident in the thermostable thickness range. Figure 5 shows the resonant wavelength shift as a function of the refractive index change of the surrounding medium for the first-order WGMs, at  $a_0 = 25 \mu\text{m}$  with PMMA layers of the thermostable thicknesses. The curves represent the approximate results evaluated by  $\delta\lambda_R = \delta n_0 \times S$ . The symbols denote the accurate results of  $\delta\lambda_R$  that are numerically calculated by substituting  $\delta n_0$  into the resonance condition, Eq. (1). Figure 5 also shows the resonance shifts in the microsphere before the PMMA coating. The silica microsphere still exhibits an excellent linear sensing characteristic in the range of refractive index changes after the PMMA coating, and the curves are in agreement with the accurate values.

The aforementioned analysis demonstrates that the sensitivity enhancement is more evident for the first-order WGM compared with the second-order WGM, at the thermostable thickness. Furthermore, the first-order WGM has the highest  $Q$ -factor, which

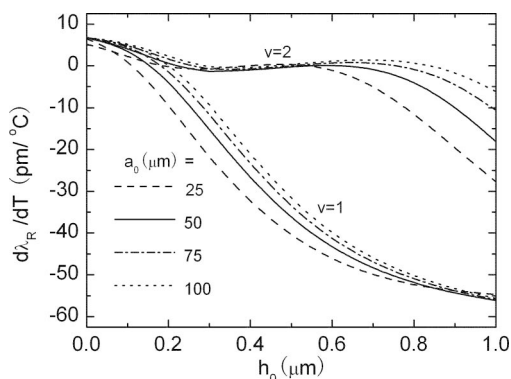


Fig. 3.  $d\lambda_R/dT$  for TE modes of two orders ( $v = 1, 2$ ) in microspheres of different radii as a function of the PMMA layer thickness  $h_0$ .

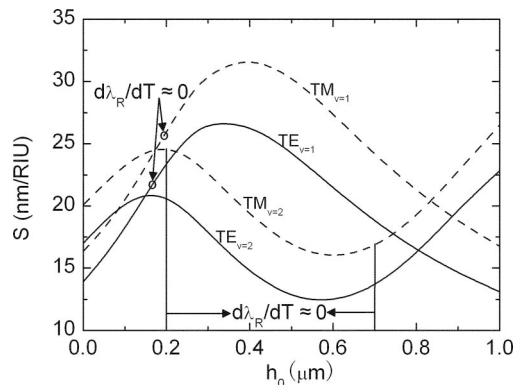


Fig. 4. RI Sensitivity  $S$  for TE modes (solid curves) and TM modes (dashed curves) of two orders ( $v = 1, 2$ ) as a function of the PMMA layer thickness  $h_0$ .

is optimal. So, in practical applications of the proposed sensor, the first-order WGM should be selectively excited by controlling the coupling condition. Figure 6 shows the  $Q$ -factor and the energy fraction in PMMA as a function of the PMMA layer thickness, for the first-order WGMs at  $a_0 = 50 \mu\text{m}$ . The quality factor of the silica microsphere before PMMA coating is taken to be  $Q_0 \approx 10^9$  [18]. As shown in Fig. 6, as the coating thickness increases, more light energy distributes in the PMMA layer, and the  $Q$ -factor decreases due to the greater material absorption loss of PMMA compared to silica. Fortunately, the decrease in  $Q$ -factor is less than 1 order of magnitude in the range of coating thickness changes. At the thermostable thicknesses, the principle limit for the  $Q$ -factor is up to about  $8.3 \times 10^8$  for both TE and TM modes. Figure 7 shows  $S \times Q$  as a function of the PMMA layer thickness for the first-order WGMs at  $a_0 = 50 \mu\text{m}$ . It is shown that the TM mode has a larger  $S \times Q$ , i.e., a smaller detection limit. In practical applications of the proposed sensor, the first-order TM mode would be exclusively excited to achieve the best detection performance.

In addition to the thermal fluctuation, there are other two factors that might also contribute to errors in the spectral location of the resonance and thus

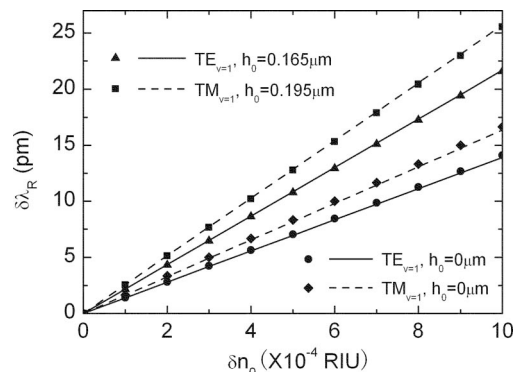


Fig. 5. Resonant wavelength shift  $\delta\lambda_R$  as a function of the refractive index change  $\delta n_0$  of the surrounding medium for WGMs of  $v = 1$  at  $a_0 = 50 \mu\text{m}$ . The curves represent  $\delta\lambda_R = \delta n_0 \times S$ . The symbols represent  $\delta\lambda_R$  calculated by Eq. (1).

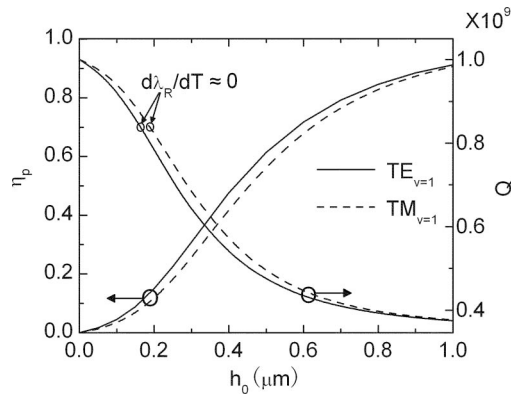


Fig. 6. Energy fraction  $\eta_p$  in PMMA and  $Q$ -factor for TE modes (solid curves) and TM modes (dashed curves) of  $\nu = 1$  as a function of the PMMA layer thickness  $h_0$ .

affect the detection limit, including the amplitude noise and spectral resolution of the detection system [13]. Consider a detection system with a spectral resolution of 1 fm, a SNR (signal-to-noise ratio) of 60 dB, and a thermal fluctuation of  $\pm 1$  °C. For the first-order TM mode in the silica microsphere before coating, the standard deviation of the amplitude noise and spectral resolution are 0.01 and 0.29 fm, respectively, while the standard deviation of the temperature stabilization is about 3.87 pm. The detection limit is determined by the thermal fluctuation, and calculated to be about  $7.2 \times 10^{-4}$  RIU. If the silica microsphere is coated with a PMMA layer of the thermostable thickness, the thermal fluctuation is eliminated and the detection limit is determined by the amplitude noise and spectral resolution. Then, a detection limit as low as  $3.4 \times 10^{-8}$  RIU can be obtained. If the spectral resolution of the detection system can be further improved, a detection limit on the order of  $10^{-9}$  RIU can be achieved. The theoretical minimum detection limit is determined by the amplitude noise and is about  $1.6 \times 10^{-9}$  RIU. In practice, however, it is difficult to control the thickness of the PMMA layer precisely to eliminate the thermal drift completely. Furthermore, some nonuniformity in the PMMA layer thickness would impair the SNR, and thus

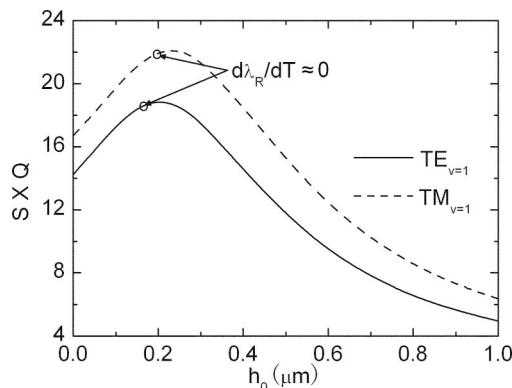


Fig. 7.  $S \times Q$  for the WGMs of  $\nu = 1$  as a function of the PMMA layer thickness  $h_0$ . The solid and dashed curves represent the values for the TE and TM modes respectively.

increase the amplitude noise. The practical detection limit for the proposed sensor thus should be larger than  $1.6 \times 10^{-9}$  RIU.

#### 4. Conclusions

This study proposes a thermostable RI sensor consisting of a silica microsphere coated with a PMMA layer. The thermal drift, RI sensitivity, and detection limit are theoretically studied. The simulation results show that a near-zero thermal drift can be achieved at a specific layer thickness, for the first-order WGM. The larger microsphere requires a slightly thicker layer. For the second-order WGM, a near-zero thermal drift is achievable in a wide range of the layer thicknesses. The PMMA layer of the thermostable thickness also enhances the RI sensitivity and detection limit. Especially for the first-order TM mode, the principle limit for the smallest detectable refractive change can be obtained on the order of  $10^{-9}$  RIU. The theoretical analysis provides physical insights for the design and optimization of microspheres for RI sensing applications.

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