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Thermal effects on measurements of dynamic processes in composite structures using piezoelectric sensors

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Abstract. Piezoelectrics have recently gained popularity as an integral part of smart materials. Piezoelectric materials exhibit both direct and converse piezoelectric effects. The former effect, i.e. generation of an electric field as a response to mechanical strain, is employed in piezoelectric sensors. The latter effect, i.e. mechanical strain produced as a result of an electric field, is used in actuators. The converse effect is usually relatively weak in present-day piezoelectrics, limiting their applications as actuators. However, piezoelectric materials have been found to be both efficient and reliable in sensory applications.

Temperature has a profound effect on the properties of piezoelectrics. Moreover, it affects the properties of the substrate (the structure whose response the sensor must monitor). The latter effect is particularly prominent if the substrate is manufactured from a polymeric composite material, because polymeric matrices are more affected by temperature than metallic or ceramic materials. These two effects, i.e. the effects of temperature on piezoelectric sensors and on a composite substrate, represent the subject of the present paper. It is shown that even moderate fluctuations of temperature within 200 °C can significantly change the voltage reading from a piezoelectric sensor. Therefore, a reliable interpretation of data from piezoelectric sensors requires an engineer to account for thermal effects on the sensor and the substrate material.

1. Introduction

The application and theory of piezoelectric sensors have been intensively studied [1–3]. However the effect of thermal changes on the accuracy of measurements from piezoelectric sensors. Temperature effects the voltage generated in the sensors due to the following three effects:

(i) the direct influence of temperature on the properties of the piezoelectric sensor material;

(ii) the effect of temperature on the properties of the material of the structure whose behavior the sensor must monitor (substrate);

(iii) thermally induced stresses in the sensor and in the substrate.

Information regarding the first effect is limited and, to the best knowledge of the author, reliable analytical relationships reflecting this effect are not currently available. Several manufacturers have published the properties of piezoelectric films that can be used in sensory applications as functions of temperature [4, 5, 6]. In the present paper, piezoceramics manufactured by Morgan Matroc, Inc., are considered.

The second effect may vary in its severity, dependent on the substrate material and the range of temperature. As follows from experimental studies [4–7], even variations of temperature within the range $-200\,^{\circ}\mathrm{C} < T < 200\,^{\circ}\mathrm{C}$ result in significant changes of piezoceramic properties. In this rather limited range of temperatures the properties of steels, aluminum and titanium alloys remain fairly close to those measured at room temperature. However, the properties of polymeric matrices exhibit noticeable fluctuations even in this temperature range. Thermally induced stresses may have a significant influence on the response of a structure. However, the method of incorporating these stresses into the analysis of composite structures is well understood [8]. Therefore, this paper does not elaborate on the effect of thermal stresses on the response, and, accordingly, on the measurements obtained from the sensors.

Nevertheless, one important phenomenon which should be a subject of a separate study has to be discussed here. This is a zone of three-dimensional thermal stress state at the edge of the piezoelectric sensor. Due to the mismatch between the coefficients of thermal expansion of the substrate and the sensor, transverse stresses acting

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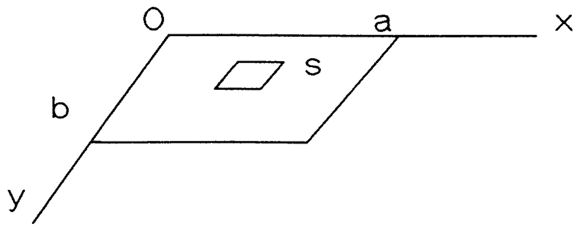

Figure 1. Rectangular plate with a piezoelectric sensor (s). Note that the sensor can be either bonded to the surface or embedded within the plate.

perpendicular to the plane of the sensor increase with temperature. The situation is similar to that at the edge of lap joints with the difference in loading, i.e. thermal loading in the case of the sensors versus mechanical loading in lap joints. This three-dimensional stress state may result in debonding of the sensor.

Moisture is another environmental effect that may affect the measurements. Although moisture does not notably affect the properties of piezoelectric sensors, it results in a degradation of the properties of polymeric composite materials that can be accounted for by using analytical expressions of Chamis [9]. In addition, elevated moisture induces hygral stresses whose effect is mathematically similar to that of their thermal counterparts [10].

In this paper, the analysis is concerned with the effects of temperature on measurements. It is illustrated that thermal effects on both the piezoelectric sensor properties and on the properties of the polymeric composite substrate can be significant. Therefore, data obtained from a piezoelectric sensor should be interpreted considering these effects.

2. Direct piezothermoelectric effect in a thin piezoelectric sensor

Consider a thin patch of piezoelectric material (sensor) bonded to or embedded within the structure (figure 1). The analysis employs the following assumptions:

(i) The piezoelectric patch is in the state of plane stress.

(ii) The problem can be treated using linear geometric and physical models.

(iii) The material is polarized in the thickness (z) direction.

(iv) The electric field generated in the sensor does not result in strains sufficient to affect the response of the structure, i.e. the converse piezoelectric effect is negligible compared with the direct effect.

(v) The influence of the bonding layer between the sensor and the structure on the strains and stresses is negligible.

Constitutive equations for an elastic piezoelectric material in the presence of temperature read [11, 12]

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\epsilon_{kl} - e_{ijm}E_m - \beta_{ij}T \\ D_m &= \varepsilon_{mk}E_k + e_{mij}\epsilon_{ij} + p_mT \end{aligned} \tag{1}$$

where σ_{ij} and ϵ_{kl} are stresses and strains, E_m are the components of the electric field, D_m are electric displacements and T is the difference between the current and reference temperatures. The following coefficients appear in equations (1):

$C_{ijkl}(E,\ T)$ = elements of the matrix of elastic moduli;
$e_{ijm}(\epsilon,\ T)$ = piezoelectric constants;
$\beta_{ij}(\epsilon,\ T)$ = thermal expansion constants;
$\varepsilon_{mk}\epsilon,\ T)$ = dielectric permittivities;
$p_m(\epsilon,\ T)$ = pyroelectric constants.

Note that all coefficients are dependent on the electric field (E), temperature (T) or strains (ϵ). Therefore, constitutive equations (1) are nonlinear and the analysis has to be carried out using some simplifying assumptions or experimental data. The latter approach is utilized in the present paper.

In the case where a macroscopic axis is aligned with the z-direction and the material is isotropic in the planes z = constant, as is the case in piezoelectric sensors considered in the paper, equations (1) are simplified. These simplified equations are presented here in the semi-inverted form employing piezoelectric constants d_{ij}:

$$\begin{Bmatrix} \epsilon_x - \alpha_x T - d_{31}E_z \\ \epsilon_y - \alpha_y T - d_{31}E_z \\ \epsilon_z - \alpha_z T - d_{33}E_z \\ \epsilon_{yz} - \alpha_{yz} T - d_{15}E_y \\ \epsilon_{xz} - \alpha_{xz} T - d_{15}E_x \\ \epsilon_{xy} \end{Bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{12} & 0 & 0 & 0 \\ & s_{11} & s_{12} & 0 & 0 & 0 \\ & & s_{33} & 0 & 0 & 0 \\ & & & s_{44} & 0 & 0 \\ & & & & s_{44} & 0 \\ \text{sym} & & & & & s_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} \tag{2}$$

$$\begin{aligned} D_x &= d_{15}\sigma_{xz} + \varepsilon_{11}^T E_x + p_x T \\ D_y &= d_{15}\sigma_{yz} + \varepsilon_{11}^T E_y + p_y T \\ D_z &= d_{31}(\sigma_x + \sigma_y) + d_{33}\sigma_z + \varepsilon_{33}^T E_z + p_z T. \end{aligned} \tag{3}$$

In equations (2) and (3), s_{ij} are elastic compliances, α_i are the coefficients of thermal expansion ($\alpha_x = \alpha_y$, $\alpha_{xz} = \alpha_{yz}$, $\alpha_{xy} = 0$), and the superscript 'T' is added to dielectric permittivities to comply with the standard notation. It can be shown that if $T = 0$, equations (2) converge to those published by Smith [13].

If the sensor is in the state of plane stress, $\sigma_z = \sigma_{xz} = \sigma_{yz} = 0$ and equations (2) are simplified accordingly. Note that equations (2) are decoupled, i.e. the first three equations are independent of the fourth and fifth equations and the sixth equation is independent of the other equations. In the case of plane stress the first two equations (2) and the third equation (3) yield

$$\begin{aligned} \begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix} &= \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{11} \end{bmatrix} \begin{Bmatrix} \epsilon_x - \alpha_x T - d_{31}E_z \\ \epsilon_y - \alpha_x T - d_{31}E_z \end{Bmatrix} \\ D_z &= d_{31}\big[(c_{11} + c_{12})(\epsilon_x + \epsilon_y) \\ &\quad - 2(c_{11} + c_{12})(\alpha_x T + d_{31}E_z)\big] + \varepsilon_{33}^{\mathrm{T}} E_z + p_z T. \end{aligned} \tag{4}$$

The third equation (2) can be used to specify the strain ϵ_z as a function of in-plane stresses, temperature and the component of the electric field E_z. As follows from equations (4)

$$E_z = \{d_{31}(c_{11}+c_{12})(\epsilon_x+\epsilon_y) - [2d_{31}(c_{11}+c_{12})\alpha_x - p_z]T - D_z\}[2d_{31}^2(c_{11}+c_{12}) - \varepsilon_{33}^T]^{-1}. \quad (5)$$

If the variations of temperature are quasistatic, the terms dependent on temperature in the numerator of equation (5) do not affect the readings of dynamic voltage. In sensory applications the electric circuit remains open, so that the total charge over the area of the electrode can be assumed zero. The voltage generated in the sensor can be obtained as

$$\phi = -\frac{1}{A_s}\int_{A_s}\int_{h_s} E_z \, \mathrm{d}z \, \mathrm{d}A_s \quad (6)$$

where h_s is the thickness of the sensor and A_s is its surface area. The result of integration for a dynamic voltage generated as a result of a dynamic process in the presence of a static thermal field is

$$\phi = -f_1 f_2 \quad (7)$$

where

$$f_1 = \frac{d_{31}(c_{11}+c_{12})}{2d_{31}^2(c_{11}+c_{12}) - \varepsilon_{33}^T} \quad (8)$$

$$f_2 = \frac{1}{A_s}\int_{A_s}\int_{h_s} (\epsilon_x+\epsilon_y) \, \mathrm{d}z \, \mathrm{d}A_s. \quad (9)$$

It is convenient to introduce a ratio

$$R(\phi) = k_1 k_2 \quad (10)$$

where

$$k_1 = f_1(T)/f_1(T_\mathrm{R}) \qquad k_2 = f_2(T)/f_2(T_\mathrm{R}). \quad (11)$$

In equations (11), k_1 and k_2 represent the ratios of the corresponding coefficients measured at a current temperature T to their counterparts at the reference temperature T_R. Note that the coefficient k_1 reflects the effects of temperature on the properties of a piezoelectric sensor, while coefficient k_2 depends on the strains in the sensor. These strains are, in theory, affected by the properties of the sensor material and the stiffness of the sensor. However, in practical applications, the sensors are usually very thin compared with the substrate whose response they monitor. Therefore, the stiffnesses of the substrate have a dominant influence on the strains, while the contribution of the sensor can be neglected.

The previous discussion results in an important conclusion, i.e. the effect of temperature on the measurements from piezoelectric sensors can be represented by a product of two factors. One of these factors reflects the influence of temperature on the properties of a piezoelectric sensor, while the second factor represents the effects of temperature on the properties of the structure (substrate). Thermally induced stresses will also be included in the second factor.

Although the first factor, k_1, can be evaluated using temperature-dependent properties of a piezoelectric material, the second factor, k_2, depends on a particular structure and mechanical and thermal loading. In the following section the latter factor is evaluated for multilayered symmetrically laminated angle-ply plates, and specially orthotropic plates in a uniform thermal field experiencing periodic forced vibrations.

3. Forced vibrations of composite plates in the presence of a constant uniform temperature

Consider a rectangular composite plate subjected to forced vibrations (figure 1). A thin piezoelectric sensor has a negligible influence on the plate stiffnesses and mass. Therefore, the equation of forced motion of the plate can be written in the form

$$D_{11}w_{,xxxx} + 2(D_{12}+2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} + N_x^\mathrm{T}w_{,xx} + N_y^\mathrm{T}w_{,yy} = q - \rho w_{,tt} \quad (12)$$

where D_{ij} are plate bending stiffnesses, ρ is the mass of the plate per unit area, N_x^T and N_y^T are thermal stress resultants and w are transverse deflections. Equation (12) describes forced vibrations of specially orthotropic plates.

Forced vibrations of symmetrically laminated multilayered angle-ply plates with stiffnesses D_{16} and D_{26} that are negligible compared to other bending stiffnesses are also characterized by equations similar to equation (12). This is because if the stiffnesses D_{16} and D_{26} are negligible, so are thermal shear stress resultants N_{xy}^T. This observation is particularly important because symmetrically laminated multilayered angle-ply plates are the most popular composite configuration.

The driving dynamic pressure is represented by double Fourier series

$$q = \sum_m \sum_n q_{mn}\,\mathrm{e}^{\mathrm{i}\omega t} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \quad (13)$$

ω being the frequency of excitation.

Thermal stress resultants in equation (12) are given by

$$\begin{Bmatrix} N_x^T \\ N_y^T \end{Bmatrix} = \int_h \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \end{bmatrix}_k \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_k \mathrm{d}z \quad (14)$$

where h is the plate thickness, Q_{ij} are transformed reduced stiffnesses and α_i ($i = x, y, xy$) are the coefficients of thermal expansion of the corresponding kth layer. Obviously, in specially orthotropic plates, $Q_{16} = Q_{26} = 0$ and $\alpha_{xy} = 0$.

Forced vibrations of simply supported plates can be represented by

$$w = \sum_m \sum_n W_{mn}\,\mathrm{e}^{\mathrm{i}\omega t} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}. \quad (15)$$

The substitution of equation (15) into equation (12) and straightforward transformations yield the values of W_{mn}. The dynamic bending strains in the sensor can be calculated as

$$\{\epsilon_x, \epsilon_y\} = -z_s\{w_{,xx}, w_{,yy}\} \quad (16)$$

z_s being the coordinate of the midplane of the sensor. Note that, in typical applications, the sensor is so thin that the equation (16) adequately predicts strains throughout its entire thickness.

The sum of in-plane strains in the sensor can be easily evaluated, being

$$\epsilon_x(T) + \epsilon_y(T) = \sum_m \sum_n z_s \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] \times \left\{\left(\frac{m\pi}{a}\right)^4 D_{11}(T) + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2\left[D_{12}(T) + 2D_{66}(T)\right] + \left(\frac{n\pi}{b}\right)^4 D_{22}(T) - \left(\frac{m\pi}{a}\right)^2 N_x^{\mathrm{T}} - \left(\frac{n\pi}{b}\right)^2 N_y^{\mathrm{T}} - \rho\omega^2\right\}^{-1} q_{mn}\,\mathrm{e}^{\mathrm{i}\omega t} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}. \tag{17}$$

Note that although the factors f_2 are periodic functions of time, the coefficients k_2 are constant. For example, in the case where a dynamic pressure is represented by the first term of the series (13), i.e. $m = n = 1$,

$$k_2 = F_{11}(T_{\mathrm{R}}) / F_{11}(T) \tag{18}$$

where

$$F_{11}(T) = \left(\frac{\pi}{a}\right)^4 D_{11}(T) + 2\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2\left[D_{12}(T) + 2D_{66}(T)\right] + \left(\frac{\pi}{b}\right)^4 D_{22}(T) - \left(\frac{\pi}{a}\right)^2 N_x^T - \left(\frac{\pi}{b}\right)^2 N_y^T - \rho\omega^2. \tag{19}$$

The thermal stress resultants depend on temperature, both explicitly, as shown by equations (14), and implicitly since the transformed reduced stiffnesses and the coefficients of thermal expansion are affected by temperature.

In this paper, we consider effects of temperature on the properties of polymeric composite plates. While the fibers are usually relatively insensitive to moderate variations of temperature (say, within the range $-200\,^\circ\mathrm{C} < T < 200\,^\circ\mathrm{C}$), polymeric matrices are significantly affected. Chamis [9] proposed the following formula to account for environmental degradation of resin matrices (all temperatures are in °F):

$$F_m = \left(\frac{T_{\mathrm{gw}} - T}{T_{\mathrm{g0}} - T_{\mathrm{R}}}\right)^{1/2} \tag{20}$$

where T_{g0} and T_{gw} are glass transition temperatures at the reference temperature corresponding to dry and wet conditions respectively. T_{gw} is found from

$$T_{\mathrm{gw}} = \left(0.005M_{\mathrm{r}}^2 - 0.1M_{\mathrm{r}} + 1.0\right)T_{\mathrm{g0}} \tag{21}$$

where M_{r} is a weight per cent of moisture in the matrix resin.

Equation (20) is applicable to the strength and stiffness of the matrix and its coefficient of thermal expansion. This means that the corresponding matrix properties have to be multiplied by the factor F_{m}. Poisson's ratios of the resin matrices are not noticeably affected by temperature or moisture.

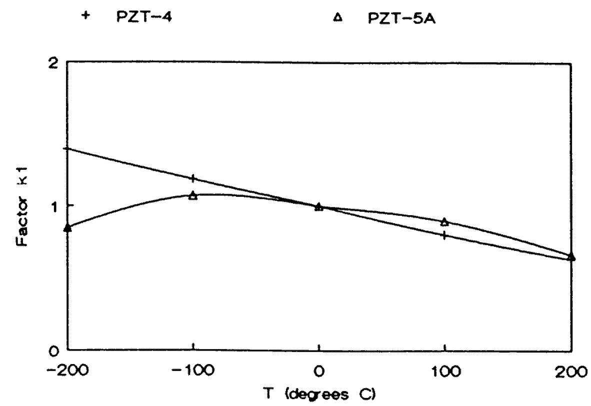

Figure 2. Ratio k_1 as a function of temperature (effect of temperature on the piezoelectric sensor).

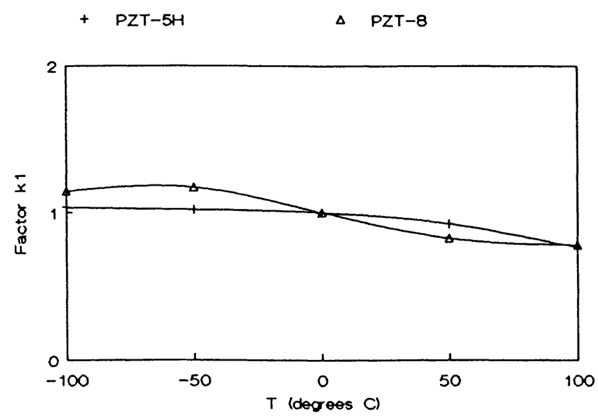

Figure 3. Ratio k_1 as a function of temperature (effect of temperature on the piezoelectric sensor).

4. Effect of temperature on the measurements associated with the changes of piezoelectric properties (k_1)

The effects of temperature on the factor k_1 were illustrated for four piezoceramic materials, i.e. PZT-4, PZT-5A, PZT-5H and PZT-8 [4]. The factor was calculated by assumption that elastic constants are not affected by variations of temperature within the range considered in that paper. The results presented in figures 2 and 3 represent factors calculated for a dynamic process in the presence of static temperatures. The analysis of these figures illustrates that, in general, dynamic voltage decreases due to the effect of an elevated temperature on the properties of piezoelectric sensors. Therefore, the magnitude of dynamic deformations measured at elevated temperatures can be underestimated if this effect is disregarded. This can yield a non-conservative conclusion regarding the amplitude of motion of the structure. The reverse applies to the case of low temperatures.

It is remarkable that the general tendency indicated above is expressed differently in different materials. For example, the tendency is obvious in PZT-4 and PZT-8.

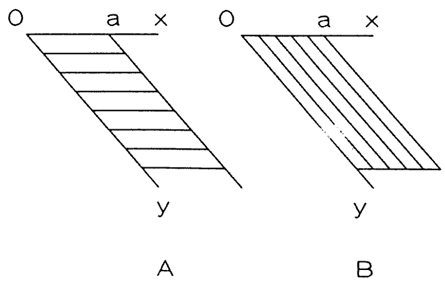

Figure 4. Large-aspect-ratio specially orthotropic composite plates. Case A: fibers are oriented along short edges. Case B: fibers oriented along long edges.

However, it is less pronounced in PZT-5A which is used in vibration pick-ups and other sensory applications. No clear effect was observed in PZT-5H at low temperatures, but the factor k_1 decreases when temperature exceeds 0 °C.

5. Effect of temperature on measurements associated with the changes of the composite substrate properties

The variety of composite plates whose motion is described by equation (12) is infinite. However, we can consider two extreme cases. These are the large-aspect-ratio specially orthotropic plates shown in figure 4. Both plates exhibit a cylindrical mode shape of vibrations when driven by a dynamic pressure independent of the y-coordinate. However, in the first case the properties of the composite in the fiber (longitudinal) direction affect the motion, while in the second case the transverse properties in the direction perpendicular to the fibers are important. Longitudinal properties of a composite layer are mostly affected by the fiber properties. In contrast, the matrix properties make an important contribution to the transverse properties of the material. Accordingly, the first case in figure 4 corresponds to the situation where we expect a minimum impact of temperature on the measurements, while in the second case this impact will be maximum. Realistic composite plates usually include a combination of symmetric angle-ply layers. Accordingly, the effect of temperature on the material properties and on the strains generated in the sensors mounted on such plates will be somewhere between the effects estimated for the extreme cases introduced above.

To concentrate on the effects of temperature on the plate properties, and to eliminate well understood influences of in-plane thermal stress resultants, we assume that the edges $x = 0$ and $x = a$ are not restrained against in-plane displacements in the x-direction. Then $N_x^T = 0$ and the functions F_{11} in equation (19) reduce to

$$F_{10} = \left(\frac{\pi}{a}\right)^4 D_{11} - \rho\omega^2 \tag{22}$$

and

$$F_{10} = \left(\frac{\pi}{a}\right)^4 D_{22} - \rho\omega^2 \tag{23}$$

Table 1. Properties of fibers and matrix of composite plates considered in numerical examples [9].

Property	Fibers: Boron	Fibers: Kevlar	Intermediate-modulus high-strength matrix
E_l (Msi)	58.0	22.0	0.5
E_t (Msi)	58.0	0.6	0.5
ν_{lt}	0.20	0.35	0.35
ρ (lb in^{-3})	0.095	0.053	0.044

ρ is a mass density of the material.

for the cases A and B respectively.

In the following examples, the results were obtained for boron–epoxy and Kevlar–epoxy plates with the volume fraction of fibers equal to 0.5. The properties of the fibers and those of an intermediate-modulus high-strength matrix are presented in table 1. The overall properties of the composite materials were calculated using micromechanical equations based on Chamis' multicell model [9]:

$$\begin{aligned} E_l &= k_f E_{fl} + k_m F_m E_m \\ E_t &= \frac{F_m E_m}{1 - \sqrt{k_f}[1 - (F_m E_m / E_{ft})]} \\ G_{lt} &= \frac{F_m G_m}{1 - \sqrt{k_f}[1 - (F_m G_m / G_{flt})]]} \\ \nu_{lt} &= k_v \nu_{flt} + k_m \nu_m \end{aligned} \tag{24}$$

where E and G denote the moduli of elasticity and shear respectively, k_f and k_m are the volume fractions of the fibers and the matrix and ν is the Poisson ratio. The subscripts 'l' and 't' indicate longitudinal and transverse directions, and the subscripts 'f' and 'm' refer to fiber or matrix properties.

As was shown in the paper of Noor and Shah [14], the properties calculated in equations (24) using the multicell model are in excellent agreement with both experimental and finite element results. The width of the plates considered in the examples was 10″, and their thickness was equal to 0.3″. The effects of moisture on the matrix properties were excluded from consideration (dry conditions).

It was noted that in the case A, i.e. if the fibers are oriented in the x-direction, the effect of elevated temperatures on the properties was minimal. Within the range $20\,^\circ\mathrm{C} < T < 200\,^\circ\mathrm{C}$, the variations of the factor k_2 for both materials considered in the paper were limited to 3%.

Predictably, the effect of temperature on the composite properties was much stronger when the fibers were oriented along the y-axis (case B). The results shown in figures 5 and 6 illustrate that this effect depends on the relationship between the driving and natural frequencies. In both figures, the factor k_2 increased with temperature when the frequency of excitation was equal or close to zero (curve 1 corresponding to the static or quasistatic case). This reflects the fact that the properties of the matrix were degraded by temperature. When the driving frequency was equal to 25 rad s^{-1} the above tendency was preserved,

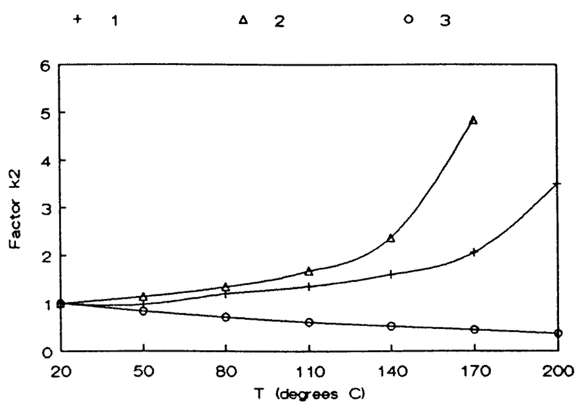

Figure 5. Effect of temperature on the factor k_2 for large-aspect-ratio specially orthotropic boron–epoxy plates. Fibers are oriented along long edges. Curves 1, 2 and 3 correspond to the excitation frequencies equal to zero (static case), 25 and 50 rad s^{-1} respectively.

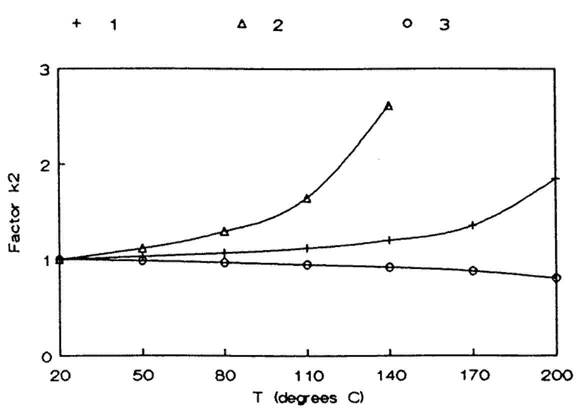

Figure 6. Effect of temperature on the factor k_2 for large-aspect-ratio specially orthotropic Kevlar–epoxy plates. Fibers are oriented along long edges. Curves 1, 2 and 3 correspond to the excitation frequencies equal to zero (static case), 25 and 50 rad s^{-1} respectively.

and as temperature increased the factor became very large (curve 2). The reason was a resonance between the fundamental frequency of the plates with degraded properties and the frequency of excitation. The situation was different when the driving frequency exceeded the fundamental frequency of the plates (curve 3). In this case, a higher temperature resulted in an increase of the gap between the fundamental and driving frequencies and the corresponding increase of the absolute value of $F_{10}(T)$. Accordingly, both the factor k_2 and the voltage in the sensor decrease. Due to the mode of excitation that includes only one half-wave in the x-direction ($m = 1$), we are not concerned here with possible resonances with the modes of vibrations corresponding to $m = 2$, 3, etc. If these modes were to be accounted for, curve 3 could be affected by resonances with these higher modes.

6. Conclusions

Temperature can affect measurements obtained using piezoelectric sensors. The effects of temperature on the measurements can be predicted based on analytical and/or experimental data. However, if this effect is disregarded, data obtained from piezoelectric sensors do not accurately represent the actual behavior of the structure.

Temperature affects voltage in piezoelectric sensors via three mechanisms, i.e. its influence on the properties of the piezoelectric and composite substrate materials, and thermal forces and moments in the structure. In this paper the effect of temperature on the material properties is considered. Analytically, this effect on the voltage generated in the sensor is represented by a product of two factors. One of these factors represents thermal effects on the piezoelectric properties, while the second factor reflects the influence of temperature on the substrate.

Note that although in the present paper the sensor is used to monitor the bending strains, the analysis would not be significantly altered in the case where the substrate experiences axial deformations. However, torsional deformations of the substrate cannot be detected, as follows from equation (5). This is due to the fact that the piezoelectric constant d_{36} is equal to zero in the case where the piezoelectric axes of the sensor coincide with the coordinate axes of the substrate. Measurements of torsional deformations become possible by introducing a non-zero skew angle between these two coordinate systems (for more details see [3]).

A general conclusion obtained in this paper is that it is impossible to predict whether an elevated temperature will result in an increase or decrease of the voltage in the sensor. The tendency, i.e. increase or decrease of the voltage, depends on a particular piezoelectric material, and on the properties, lay-up and geometry of a substrate. Moreover, in some cases the effect on the piezoelectric properties is dominant, while in other situations thermal effects on the substrate can become more important.

As follows from the solution presented in this paper, it is relatively easy to incorporate thermal effects in the analysis. It is desirable to develop analytical relationships reflecting variations of piezoelectric properties with temperature. In the absence of such relationships, experimental data can be used to generate analytical results, as was done in this paper.

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