Litany for Force Problems [©2008 RJ Bieniek]

Learning proper methods for solving homework problems will lead to improved performance on exams. To encourage the development of the appropriate skills, the following elements must appear in a solution (whether presented on the board or in collected homework) to earn full credit. A fully worked example is shown on the back of this sheet, with the relevant "suggestion" numbers below circled at the steps. *You will find that following this procedure will rather assuredly lead you to a correct answer*.

Suggested Steps ("suggested" means "must" in *Bieniek's Newspeak Dictionary*)

1. Draw a basic representative sketch of the physical situation.

2. Draw a free-body diagram with all forces shown as vectors originating from the *labeled* point mass you are considering. Show an appropriate acceleration vector next to the point mass (or write "a = 0" if the acceleration is zero). This can be superimposed on the sketch if it can be done clearly, with the vectors drawn ***darker*** than the lines of the sketch. **Each of these vectors must be distinctly and legibly drawn**.
   Note: Label each vector as you go along with a unique appropriate symbol, e.g., *T* for tension, *N* for normal force, *a* for acceleration. You need not put in the vector sign above the symbol in free body diagram because the large arrow indicates the vector property. If there are two or more of any type of quantity (e.g., two tensions), then each must have a different symbol, e.g., T_1 *and* T_2 for tensions.

3. *Lightly* draw an appropriate coordinate axis-system near to or superimposed on the free-body diagram, with an arrow at one end of each line indicating the positive direction of that axis. Choose the orientation of the axes to fit the problem; e.g., for an inclined plane problem, tilt the axes to be parallel and perpendicular to the incline. If the direction of acceleration is constant and known, it is almost always best to choose one of the axes to be in the direction of (or at least parallel to) the acceleration vector. Otherwise, choose axis orientation to handle force components conveniently.

4. *Lightly* draw in component projections of all vectors that are not parallel to a coordinate axis using "*x*-avenue and *y*-street" method. Make sure there are arrows at the end of these vectors to indicate their direction. It will probably be best for you to show them in a right triangle, as if you were adding two perpendicular vectors to get a resultant. This will give you a quick way of properly determining the sign of the component in a force equation.

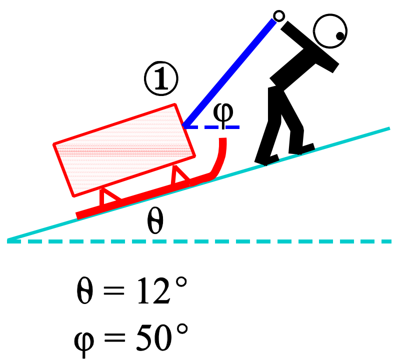
5. As an initial mathematical step, you **MUST** begin with an appropriate *Official Starting Equation*. *All subsequent steps must mathematically follow from this beginning point and reference to your diagram*.

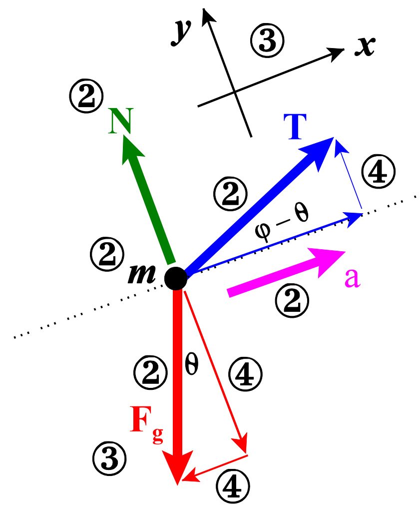
6. Write out the sum of force components explicitly, with the number of terms matching the number of force vectors in your free body diagram. If a component in that direction is zero, list the term algebraically and indicate it has a value of zero. In this way, you can make sure that you have considered all the forces in your diagram by counting the number of terms in your equation and verifying that the sum agrees with the number of forces you see in your free-body diagram. Sometimes you may want to leave the component as a symbol, but make sure it has the proper axis-subscript, e.g., x in T_{1x}. **It is important that you write a sum as a sum, i.e., with addition signs**. If you know a component is negative, the negative sign can be either retained unseen within the symbol for the component, e.g. $+T_{1x}$, or displayed within parentheses in front of a magnitude, e.g. $+(-T)$. This demands that you make individual decisions about signs. Remember that a scalar symbol without a component subscript is generally a magnitude, i.e., positive.

7. Solve for the desired quantity algebraically. If you use a symbol in an equation, it must appear in your diagram. Make sure you do this **before** you substitute any values for the symbols, to decrease chance of error or inconsistency. Let the mathematics flow from the *Official Starting Equation* you have chosen and the symbol-labels in your diagram. Hold off on the substitution of numerical values for the symbols until the end (or toward the end) of the solution. Draw a box around your final algebraic answer.

Example of a Force Problem

A boy pulls a high-tech frictionless sled up a snow-covered slope using a rope. The slope makes an angle of 12° with the horizontal, while the rope makes an angle of 50° with the horizontal, as shown in the diagram below. The mass of the sled is 26 kg. If the sled accelerates up the slope at 4 m/s^2, what is the tension in the rope?





$$(5) \qquad \sum F_x = m\,a_x$$

Note: There are three forces acting that produce the resultant net force

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{T}} + \vec{\mathbf{F}}_g + \vec{\mathbf{N}}\,.$$

(6) (preferred) $\qquad T_x + F_{gx} + \cancel{N}_x = m\,a_x$

then necessary: $\qquad T\cos(\varphi - \theta) + [-F_g \sin(\theta)] + 0 = m(+a)$

Note: three terms on left means we have accounted for all three forces.

Remember: symbols associated with vectors but without subscripts are magnitudes.

$$(7) \quad T\cos(\varphi-\theta) = ma + mg\sin\theta = m[a + g\sin\theta]$$

$$\boxed{T = \frac{m[a+g\sin\theta]}{\cos(\varphi-\theta)}} = \frac{26[4.0+9.8\sin(12°)]}{\cos(50°-12°)} = 199\text{ N}$$