## *Litany for Work-Kinetic Energy Problems* [©2008 RJ Bieniek]

1. Draw a basic representative sketch of the process.

2. Draw a free body diagram with all forces shown as vectors originating from the point mass. Show the displacement vector next to the point mass in the diagram This can be superimposed on the sketch, as long as the vectors are darker than the lines of the sketch.

3. If friction is involved, you will generally have determine the magnitude of the normal force. Make sure you consider all forces in this calculation.

4. Write down the appropriate fundamental equation. Normally, this will most likely be:

$$W_{net} = \int \vec{\mathbf{F}}_{net} \bullet d\vec{\ell} \qquad \text{or} \qquad K_f - K_i = (W_{net})_{i \to f}$$

   For a constant force $\vec{\mathbf{F}}_C$, you must explicitly state $W_F = \vec{\mathbf{F}}_C \bullet \vec{\mathbf{D}} = F_C\, D \cos\xi$, where $\xi$ is whatever angle you have used in your diagram between the vectors $\vec{\mathbf{F}}_C$ and $\vec{\mathbf{D}}$ (unless you have a special dispensation situation where $\xi = 0°$, $90°$, or $180°$). This will help keep you from incorrectly applying $W_F = \vec{\mathbf{F}} \bullet \vec{\mathbf{D}}$ to forces that are not constant over the path.

5. Decompose the net work into separate terms for each force in the free-body diagram, with an unique identifying subscript for each term. You may then zero out any work term whose force is perpendicular to $d\vec{\ell}$. For example if tension, gravity, normal, spring and frictional forces act on a box pulled up an inclined plane:

$$K_f - K_i = (W_{net})_{i \to f}$$

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = W_t + W_g + \cancel{W}_N + W_{spring} + W_f\,.$$

   Note that the decomposition of the net work is a sum, but some of the terms may be negative in value; e.g., the work of friction ($W_f$ here) is often less than zero.

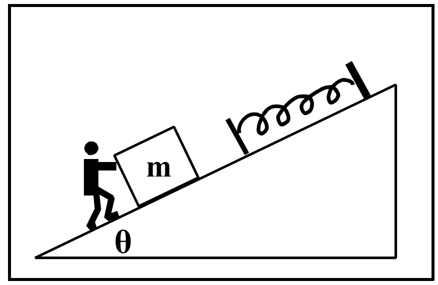
6. Write out explicit formulae for each work term, using the information in your diagram; e.g., substitute $\vec{\mathbf{F}}_g \bullet \vec{\mathbf{D}}$ for $W_g$, and $-\tfrac{1}{2}k\,(s_f^2 - s_i^2)$ for $(W_{spring})_{i \to f}$.

   When you use such derived position-dependent formulae, you *must* clearly include the initial and final position symbols in your diagram, along with the location of the reference zero value; e.g., the location of $s_i$ and $s_f$. For any force vector that is not parallel or perpendicular to the displacement vector, show the angle between them using a different symbol for each angle; e.g., $\theta$ and $\varphi$.

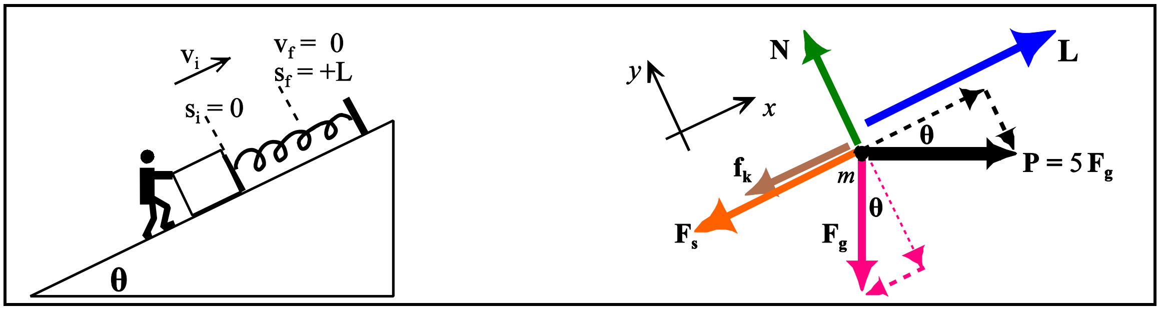
7. This is most of the physics. You are now on your own!

## Example of a Work-KE Problem Fully Done

You are pushing a crate of mass *m* up a ramp that has an incline of θ. You push *horizontally* with a force with a magnitude that is 5 times the weight magnitude of the crate. The coefficient of kinetic friction is $\mu$. At the top of the ramp, the crate encounters a large spring of force constant *k* that is aligned down the ramp. You find that you compress the spring a distance L before the crate comes to a halt. Use Work-KE methods to determine an algebraic expression for the speed of the crate just before it started to compress the spring.




Note the force of the spring is NOT constant $\Rightarrow$ you CANNOT use constant acceleration formula




$$K_f - K_i = [W_{net}]_{i\to f} = W_{fric} + W_{spring} + \cancel{W_N} + W_P + W_{grav}$$

$$\tfrac{1}{2}m\cancel{v_f}^2 - \tfrac{1}{2}mv_i^2 = -f_k L + [-\tfrac{1}{2}k(s_f^2 - \cancel{s_i}^2)] + \vec{\mathbf{N}}\cdot\vec{\mathbf{L}} + \vec{\mathbf{P}}\cdot\vec{\mathbf{L}} + \vec{\mathbf{F}}_g\cdot\vec{\mathbf{L}}$$

$$-\tfrac{1}{2}mv_i^2 = -\mu N L - \tfrac{1}{2}k(L)^2 + 0 + PL\cos\theta + F_{g\parallel}L$$

insightful note: $\vec{\mathbf{F}}_g\cdot\vec{\mathbf{L}} = F_g L\cos(90° + \theta) = F_g L(-\sin\theta) = (-F_g\sin\theta)L = F_{g\parallel}L$

$$f_k = \mu N \quad\Rightarrow\quad -\tfrac{1}{2}mv_i^2 = -\mu NL - \tfrac{1}{2}k(L)^2 + PL\cos\theta + (-F_g\sin\theta)L$$

multiply through by (–1) $\Rightarrow$ $\tfrac{1}{2}mv_i^2 = \mu NL + \tfrac{1}{2}kL^2 - (5F_g)L\cos\theta + mgL\sin\theta$

To get normal force N: $\sum F_y = \cancel{f}_y + (\cancel{F}_s)_y + N_y + P_y + F_{gy} = m\cancel{a}_y$

$$N + (-P\sin\theta) + (-F_g\cos\theta) = 0$$

$$N = P\sin\theta + mg\cos\theta = (5F_g)\sin\theta + mg\cos\theta$$

$$\therefore \quad \tfrac{1}{2}mv_i^2 = \mu(5mg\sin\theta + mg\cos\theta)L + \tfrac{1}{2}kL^2 - (5mg)L\cos\theta + mgL\sin\theta$$

$$\boxed{v_i^2 = 2g[\mu(5\sin\theta + \cos\theta) - 5\cos\theta + \sin\theta]L + kL^2/m}$$