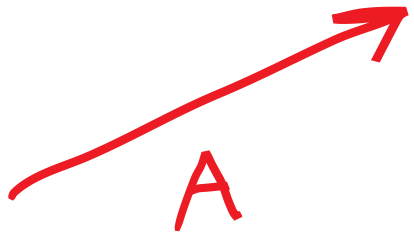
Lecture 3: Vectors

- Definition
- Graphical addition and subtraction of vectors
- Unit vector notation
- Vector components, magnitude and direction
- Addition and subtraction of vectors in unit vector notation

Vectors

A vector is a quantity that has size (magnitude) and direction. It can be symbolized by an arrow.



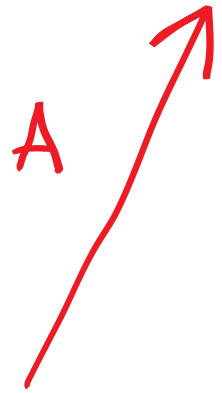
Length of the arrow represents magnitude

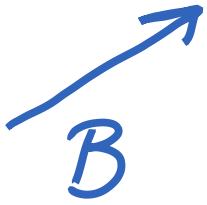
Notation convention:

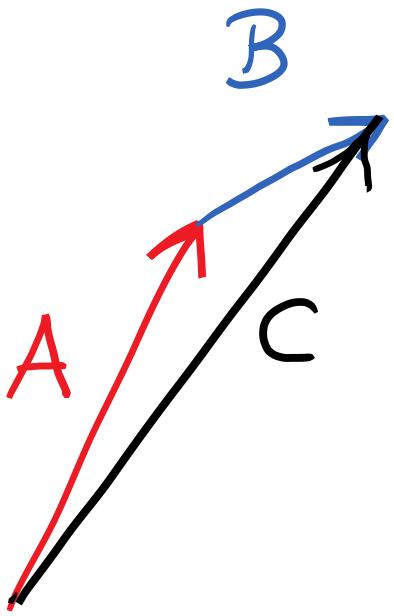
$\vec{A}$ denotes vector of magnitude $A = |\vec{A}|$

*Sometimes bold-face type also indicates a vector – hard to do in handwriting

Vector addition - graphically

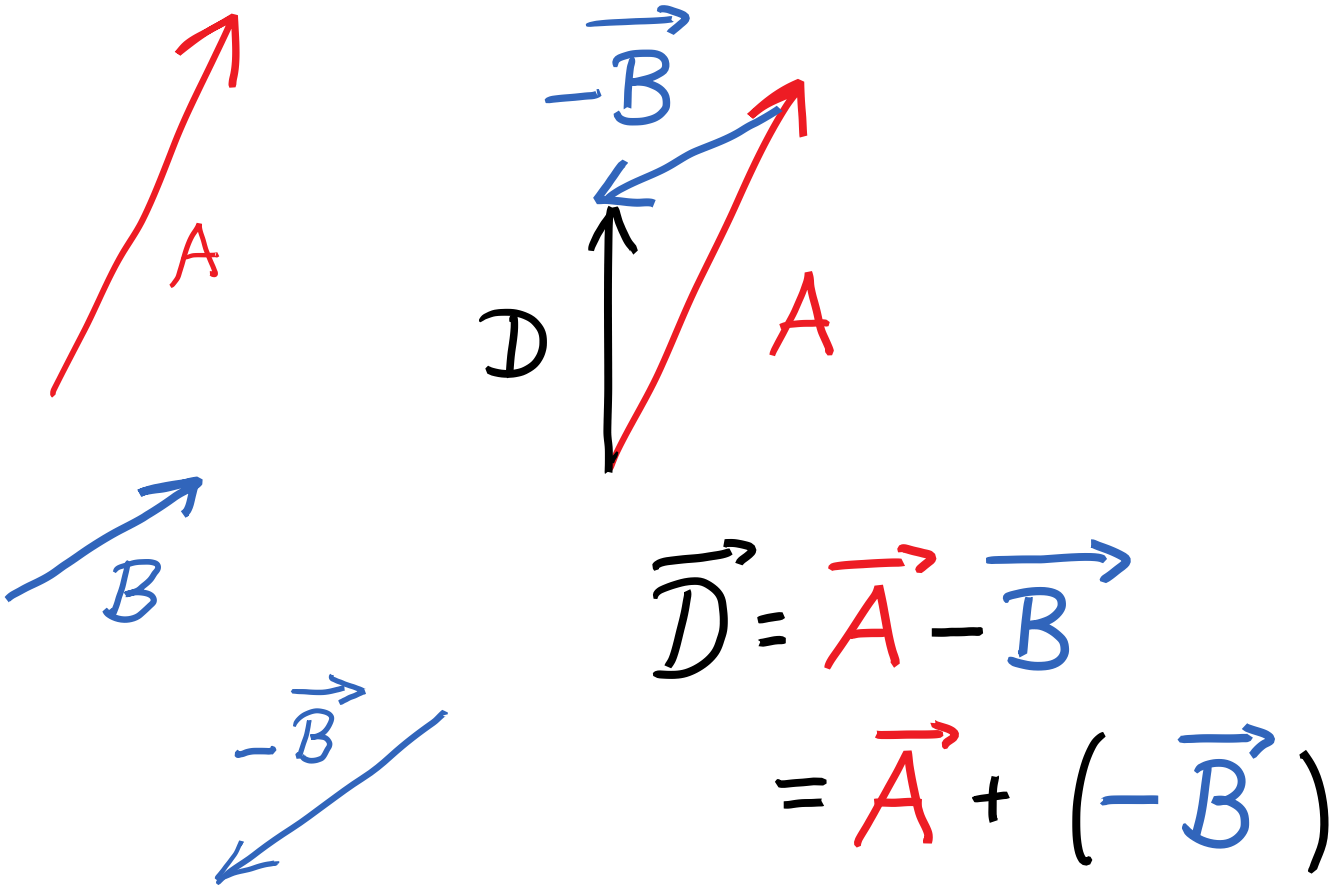






$$\vec{C} = \vec{A} + \vec{B}$$

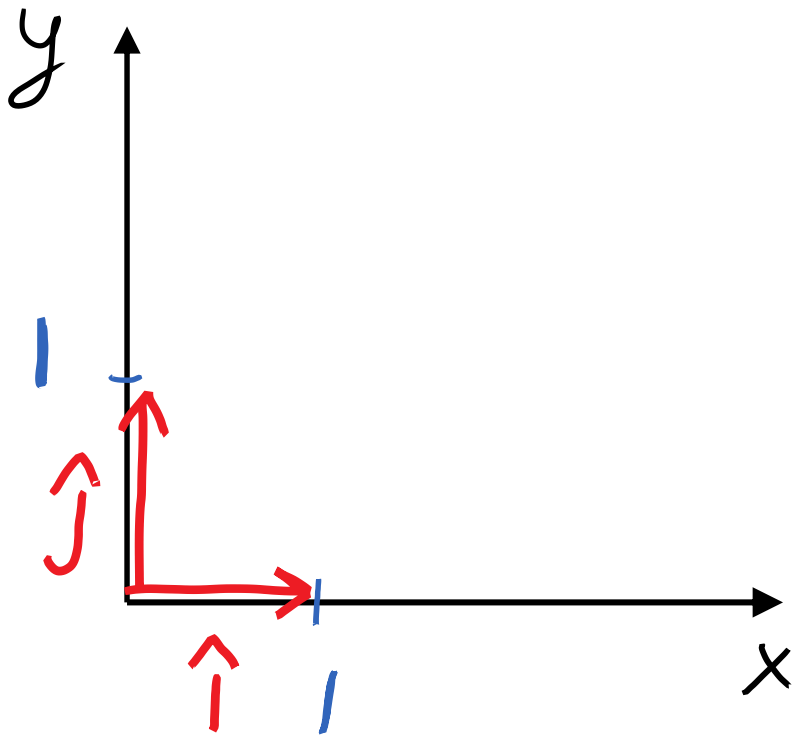
Vector subtraction - graphically



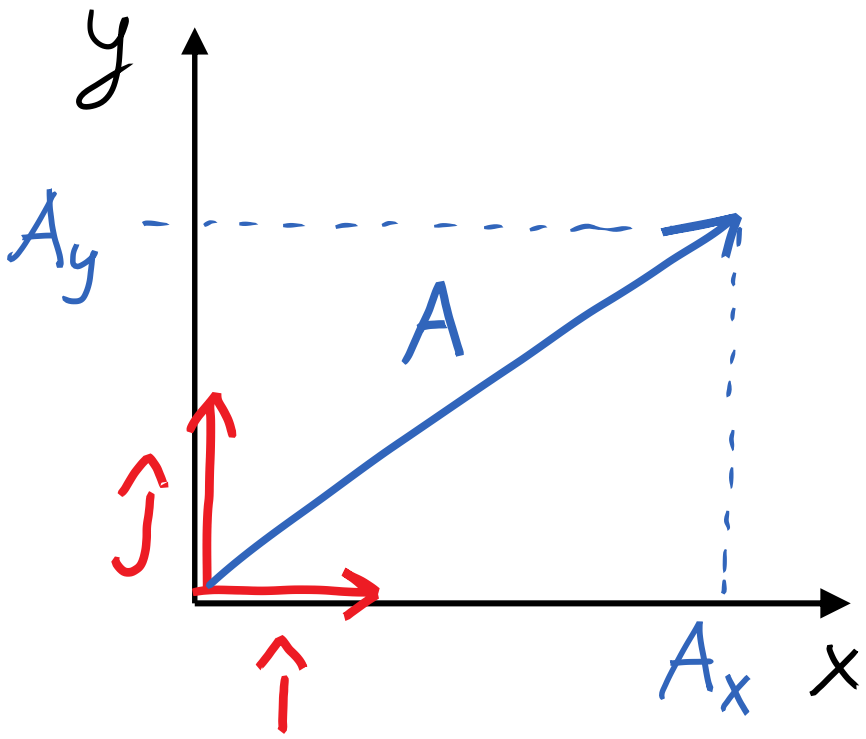
$$\vec{D} = \vec{A} - \vec{B}$$

$$= \vec{A} + (-\vec{B})$$

Unit vectors

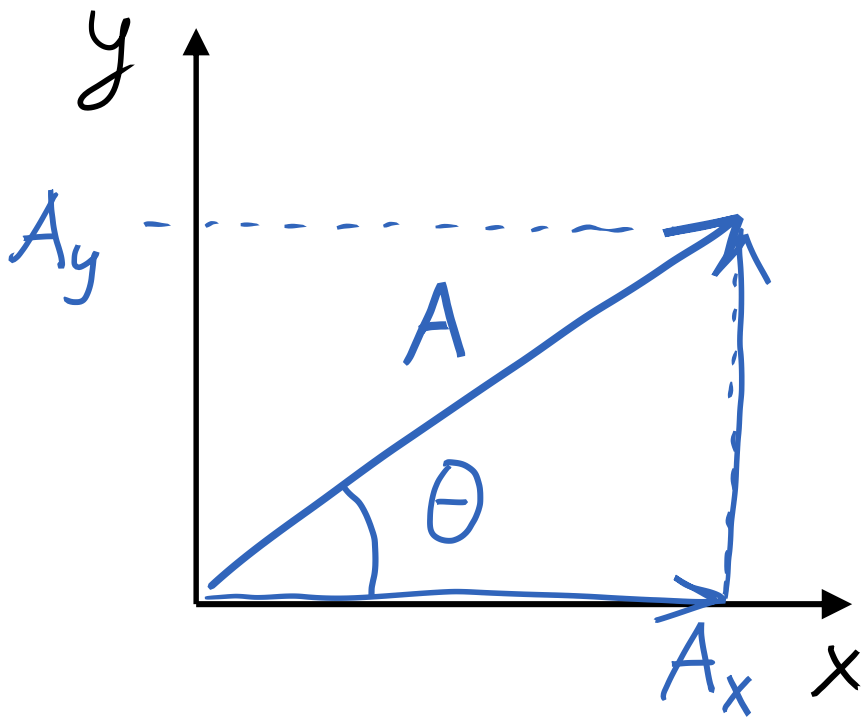


Unit vector notation



$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

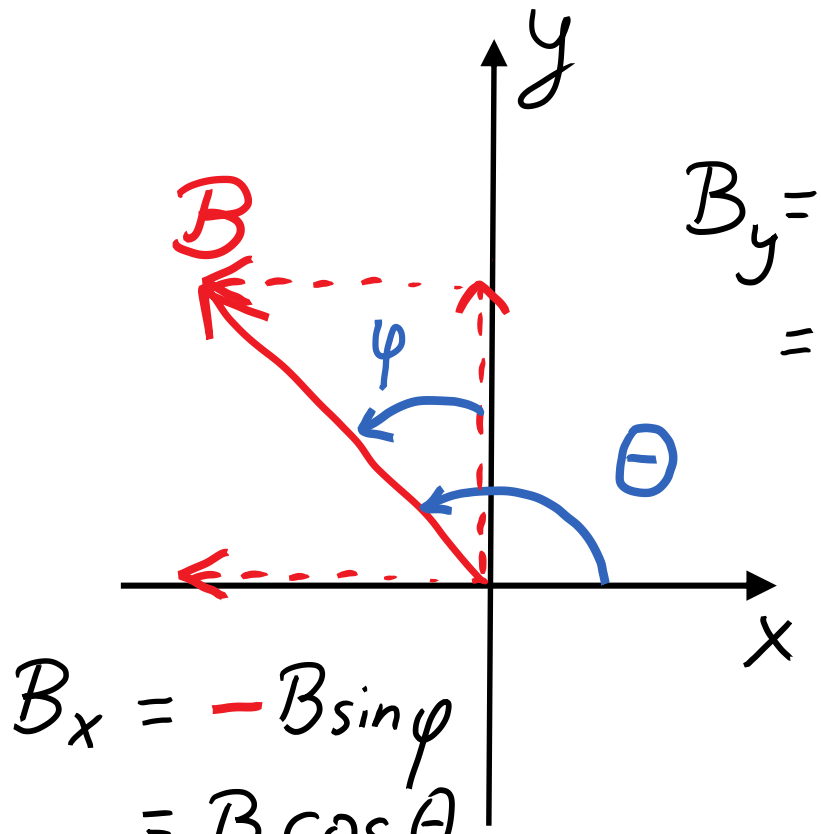
Vector components



$$A_x = +A\cos\theta$$
$$A_y = +A\sin\theta$$

$$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$$

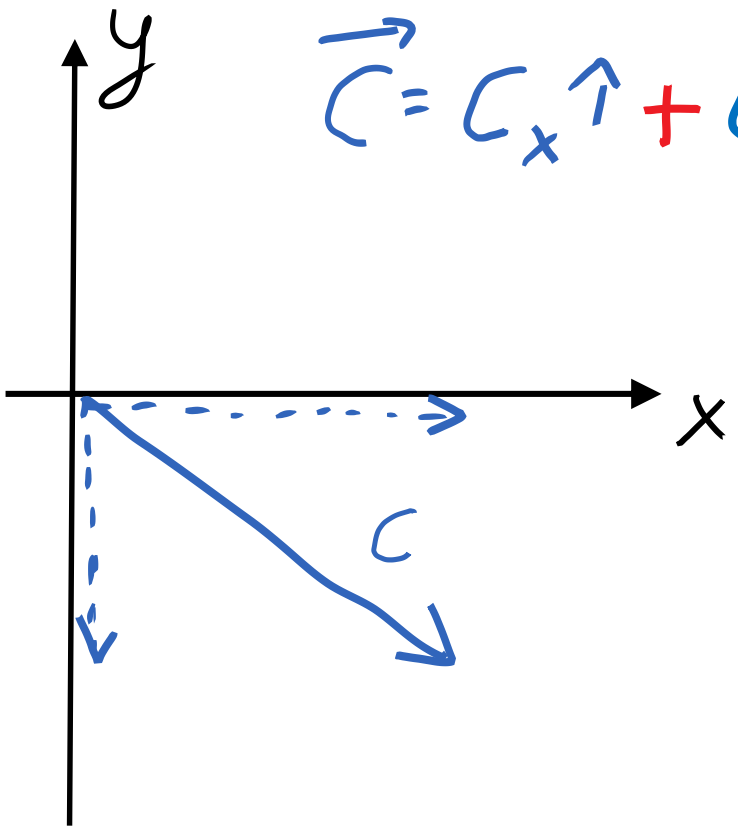
Vector components



$$B_y = +B\cos\varphi$$
$$= B\sin\theta$$

$$B_x = -B\sin\varphi$$
$$= B\cos\theta$$

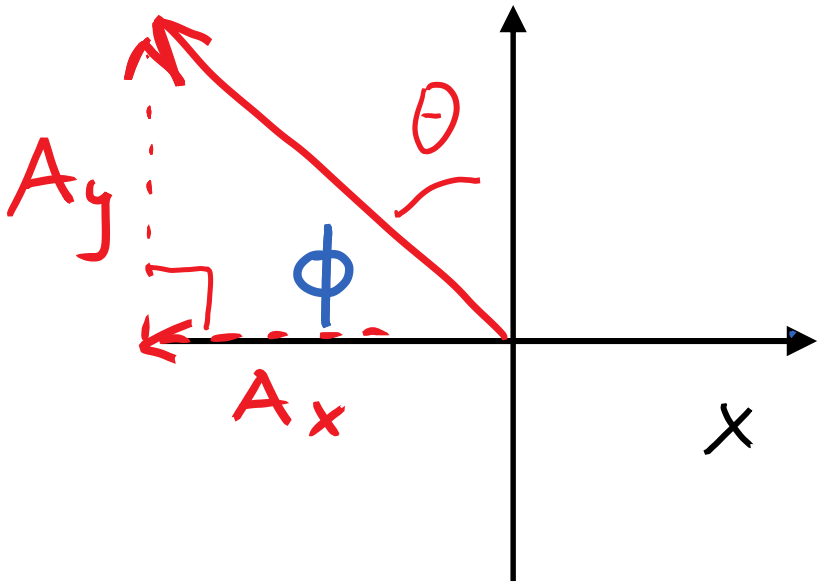
Unit vector notation



$\vec{C} = C_x \hat{\imath} + C_y \hat{\jmath}$

C_y is negative

Magnitude and direction



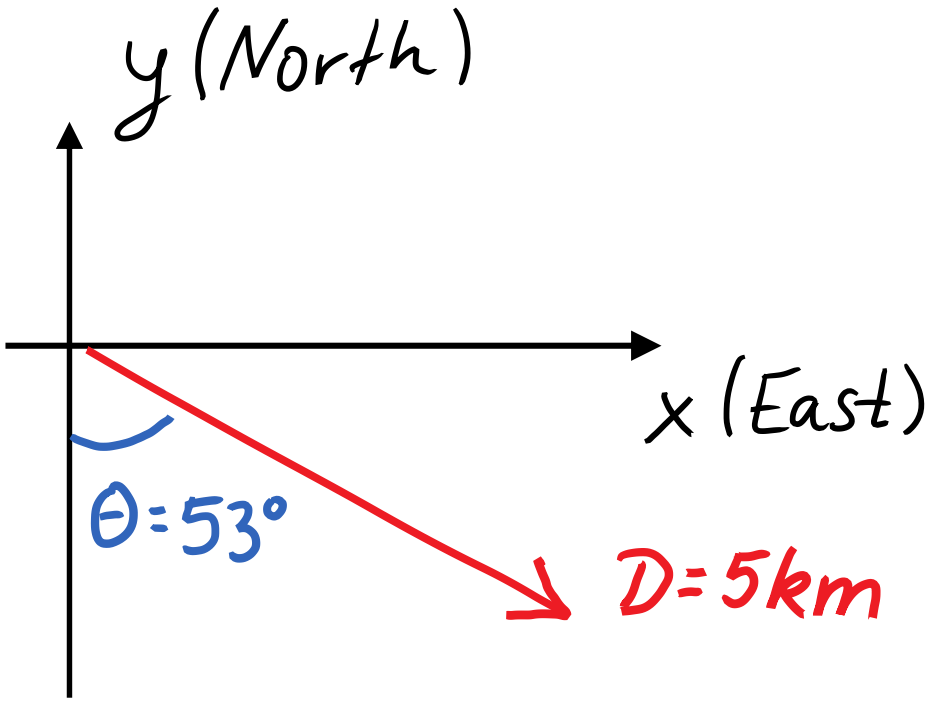
$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \left|\frac{A_x}{A_y}\right|$$

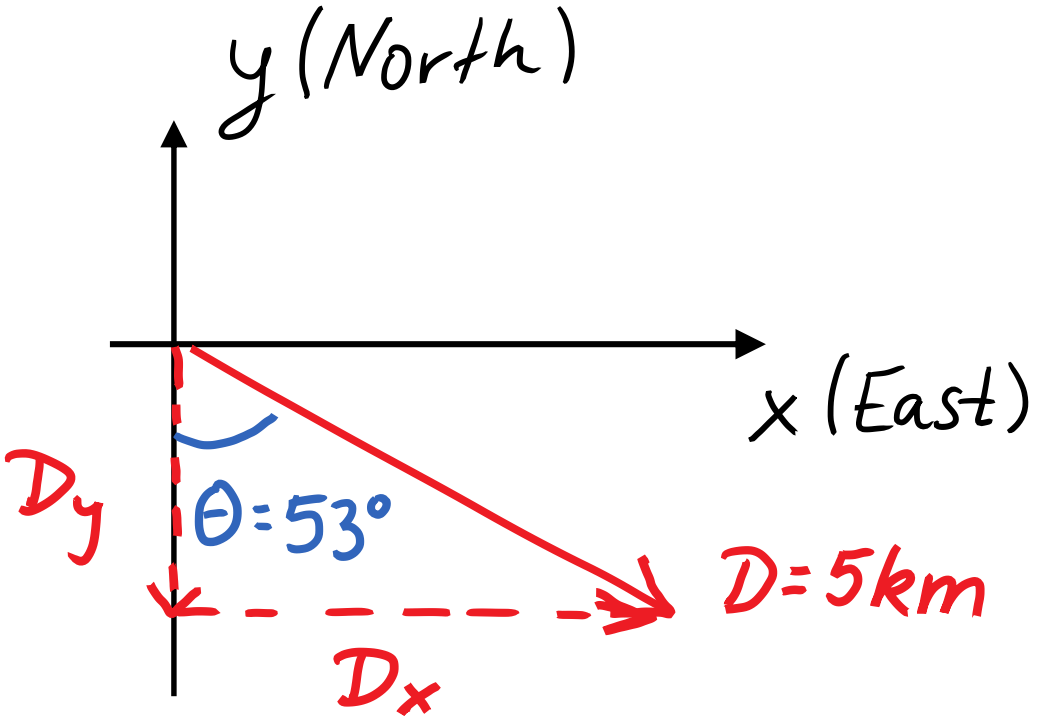
$$\tan\phi = \left|\frac{A_y}{A_x}\right|$$

Example

A displacement of 5 km is directed θ = 53° East of South. What is the displacement vector in unit-vector notation?

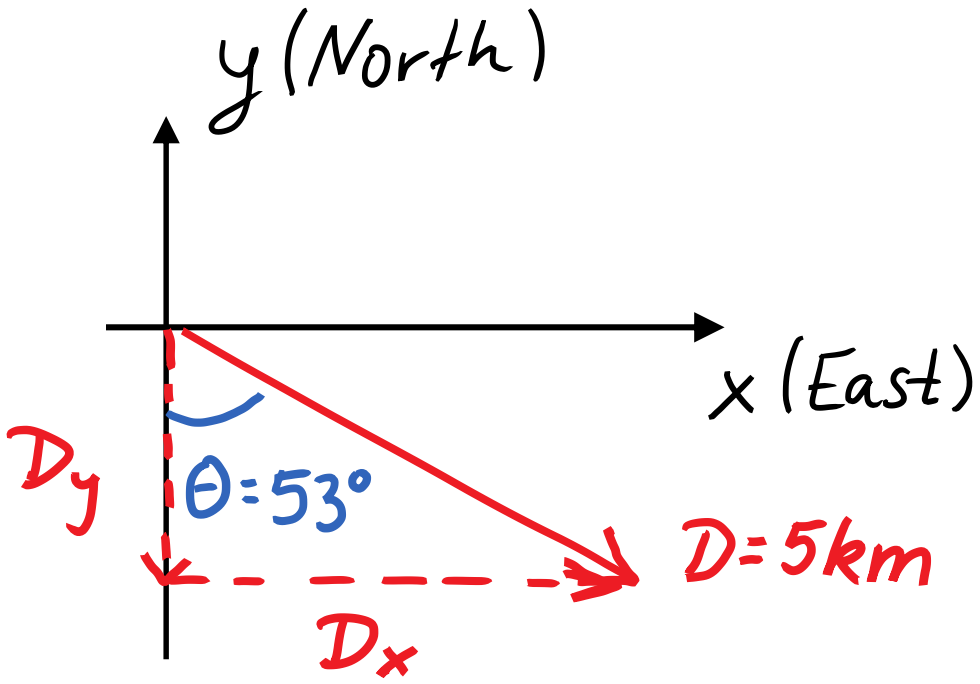


Example



$$\vec{D} = D_x \hat{i} + D_y \hat{j}$$

Example



$$D_x = +D\sin\theta = +5km(0.8) = +4km$$

$$D_y = -D\cos\theta = -5km(0.6) = -3km$$

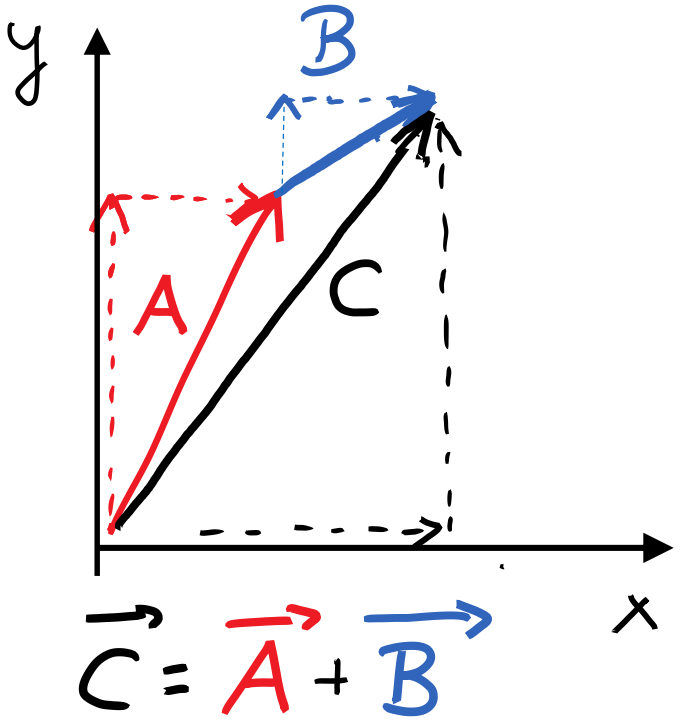
$$\vec{D} = +4km\,\hat{i} + (-3km)\,\hat{j}$$

Vector addition in components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

$$\vec{C} = \left(A_x\hat{\imath} + A_y\hat{\jmath}\right) + \left(B_x\hat{\imath} + B_y\hat{\jmath}\right)$$

$$\vec{C} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$
$$C_x = A_x + B_x$$
$$C_y = A_y + B_y$$



Vector subtraction in components

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$$

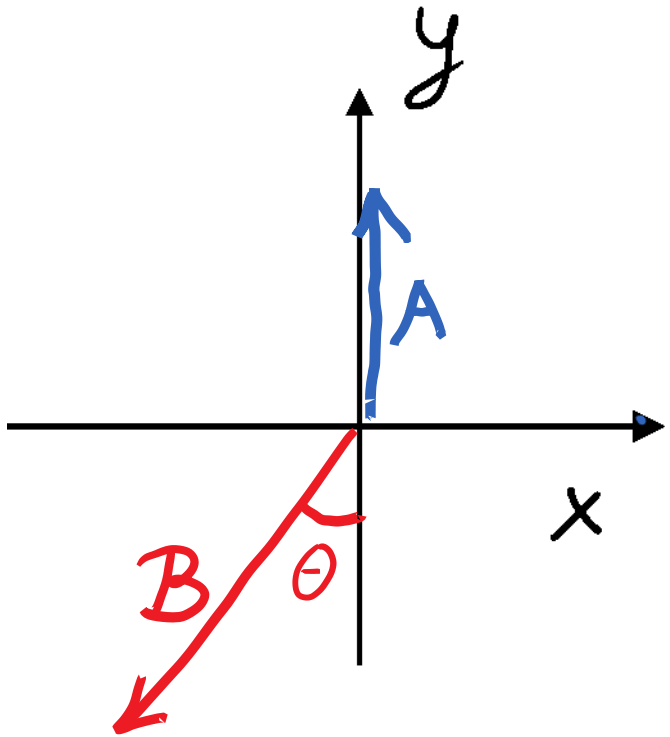
$$\vec{D} = \vec{A} - \vec{B}$$

$$\vec{D} = \left(A_x\hat{\imath} + A_y\hat{\jmath}\right) - \left(B_x\hat{\imath} + B_y\hat{\jmath}\right)$$

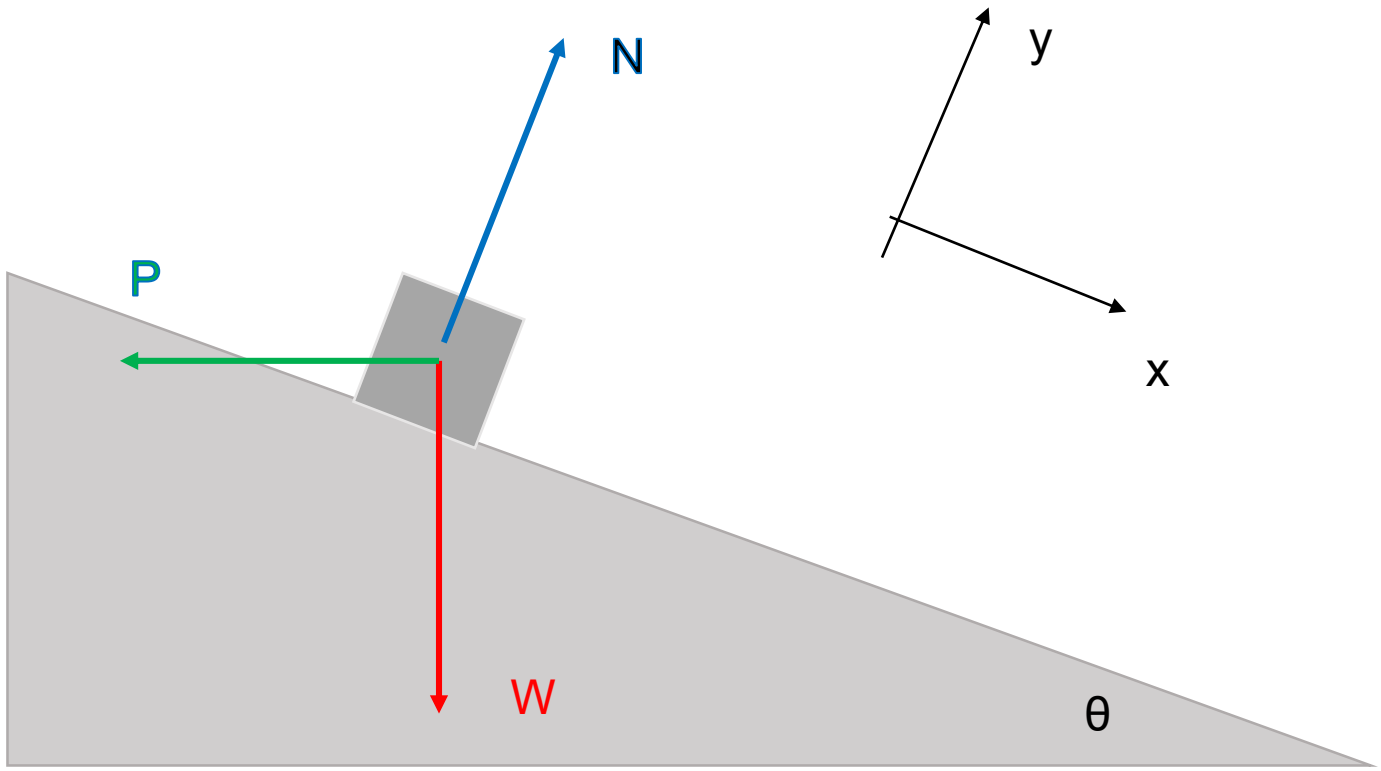
$$\vec{D} = (A_x - B_x)\hat{\imath} + (A_y - B_y)\hat{\jmath}$$
$$D_x = A_x - B_x$$
$$D_y = A_y - B_y$$

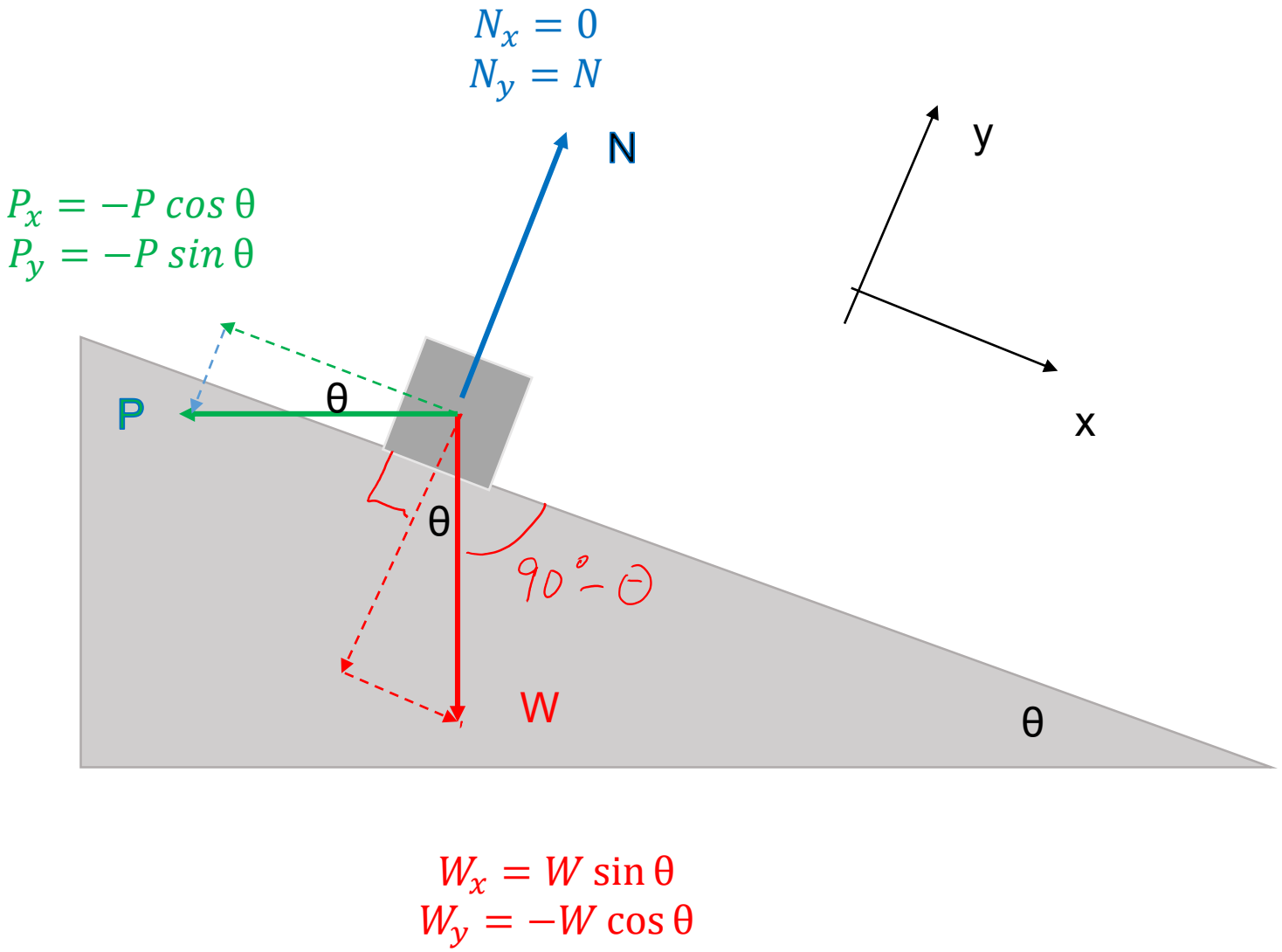
Example

Express the vectors $\vec{C} = \vec{A} + \vec{B}$ and $\vec{D} = \vec{B} - \vec{A}$ in unit vector notation in terms of A, B, and θ.



Example with tilted coordinate system





$$W_x = W \sin\theta$$
$$W_y = -W \cos\theta$$