Lecture 4: Motion in two dimensions

- Position, velocity, and acceleration in 2-d
- Separation of motion in x-and y-direction
- Equations for 2-d kinematics at constant acceleration
- Projectile motion

Velocity

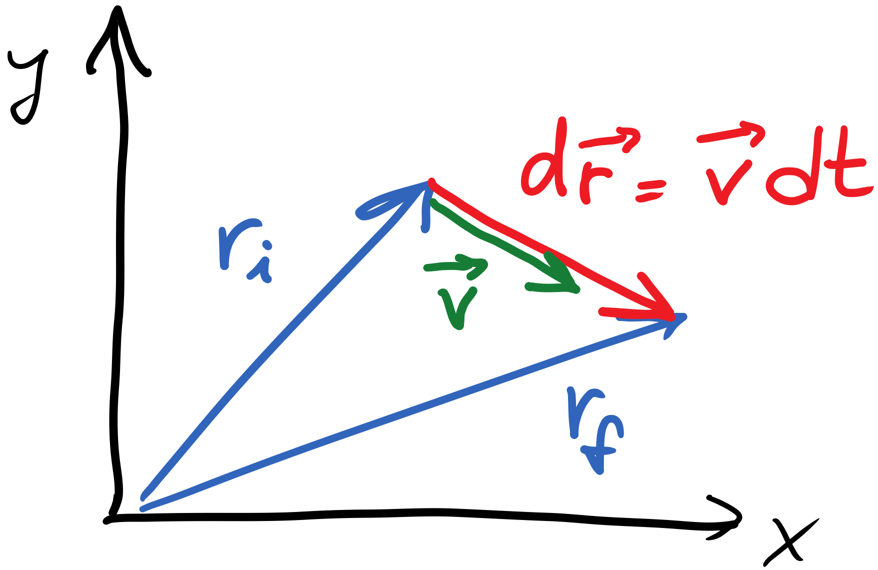
Position vector $\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath} = x\hat{\imath} + y\hat{\jmath}$

Average velocity $\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t}$

Instantaneous velocity: $\vec{v} = \frac{d\vec{r}}{dt}$

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$$

$$v_x = \frac{dx}{dt} \quad , \quad v_y = \frac{dy}{dt}$$



Small change of position vector in the direction of velocity vector

$$\vec{r}_f = \vec{r}_i + \vec{v}dt$$

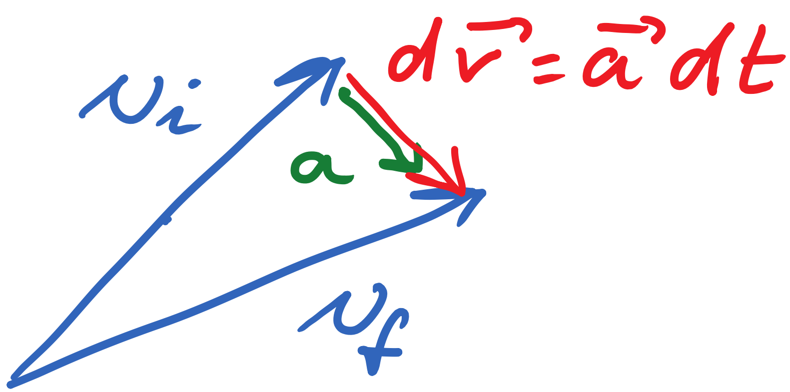
Acceleration

Particle has velocity vector $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$

Acceleration: $\vec{a} = \frac{d\vec{v}}{dt}$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath}$$

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \quad , \quad a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$



Small change in velocity vector occurs in the direction of the acceleration vector

$$\vec{v}_f = \vec{v}_i + \vec{a}dt$$

Acceleration changes velocity, i.e. speed and direction of motion.

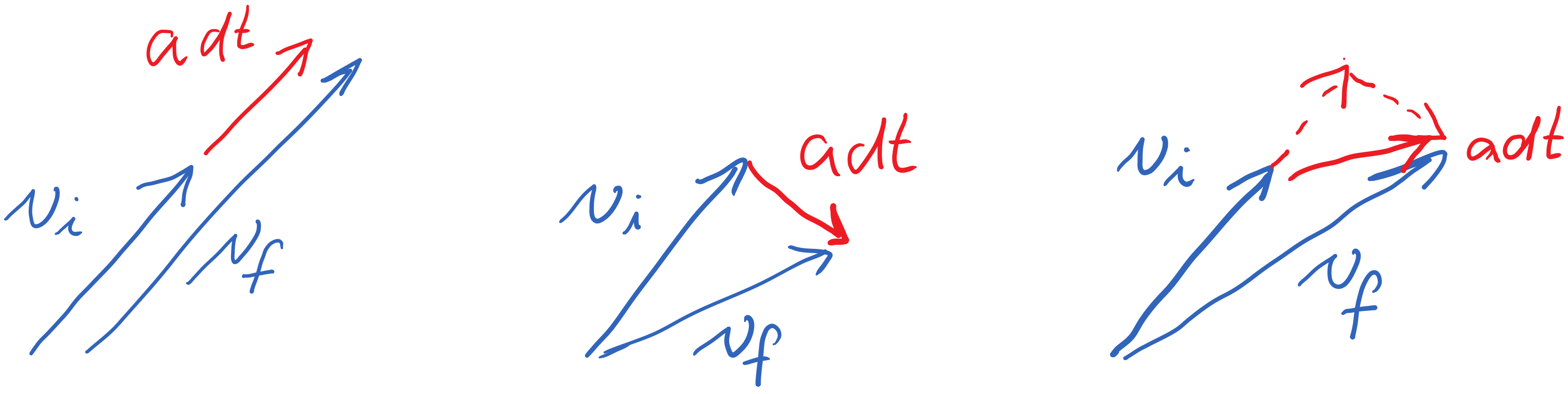
Effect of acceleration components

Components of acceleration parallel and perpendicular to velocity have different effects.

$$d\vec{v} = \vec{a}dt$$

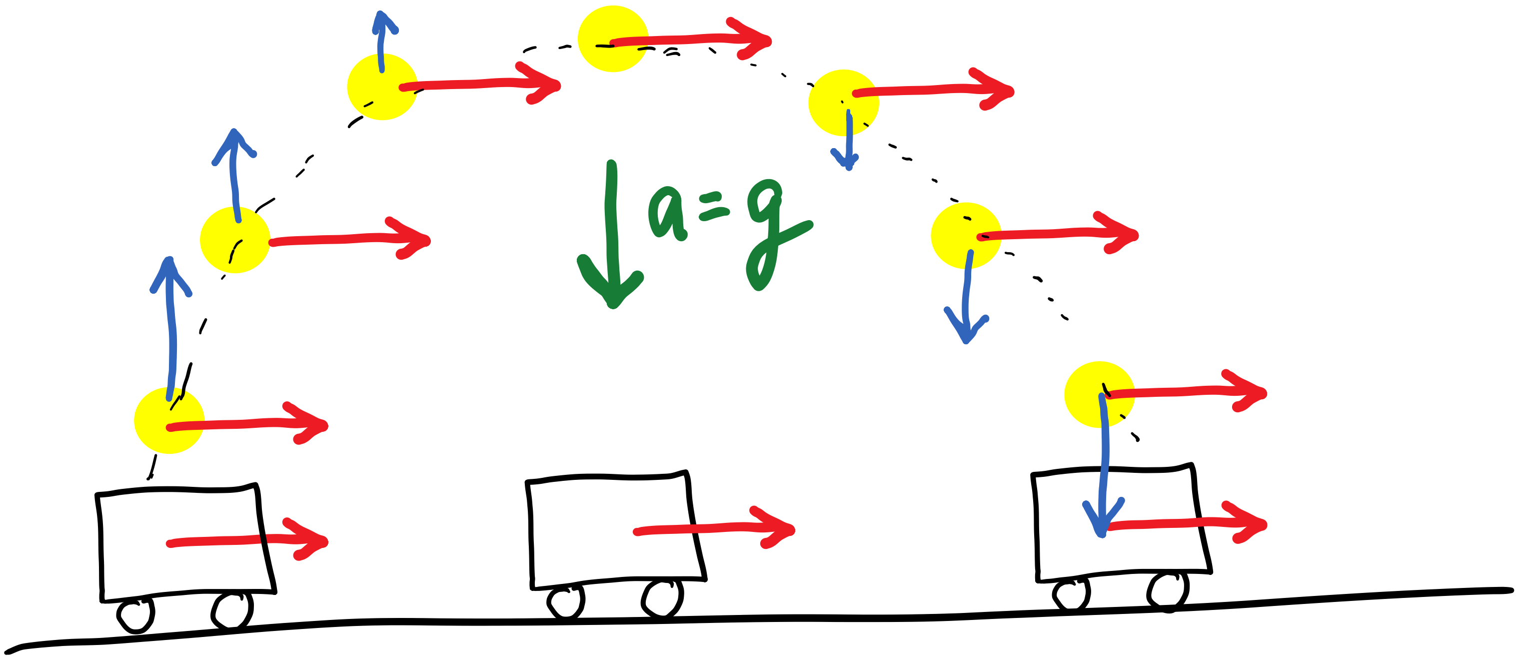
$a_{||}$ causes change in magnitude of velocity vector (speed)

$a_{\perp}$ causes change in direction



Demonstrations

- Vertical launch of ball from traveling car
- Simultaneously dropped and horizontally launched balls


a=g

Kinematics equations

For constant acceleration:

$$x = x_0 + v_{0x}t + ½a_x t^2$$
$$y = y_0 + v_{0y}t + ½a_y t^2$$

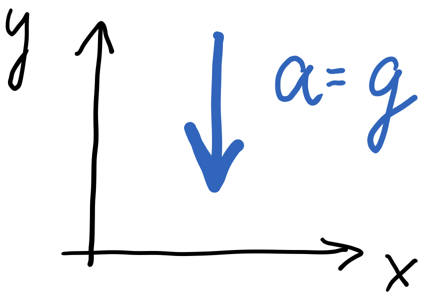
$$v_x = v_{0x} + a_x t$$
$$v_y = v_{0y} + a_y t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$
$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$$

Official Starting Equations

Projectile Motion

If only gravity acts on an object (free fall), then acceleration is a constant vector of magnitude g, directed down.



$a_x = 0$

$a_y = -g$

Effect on velocity:

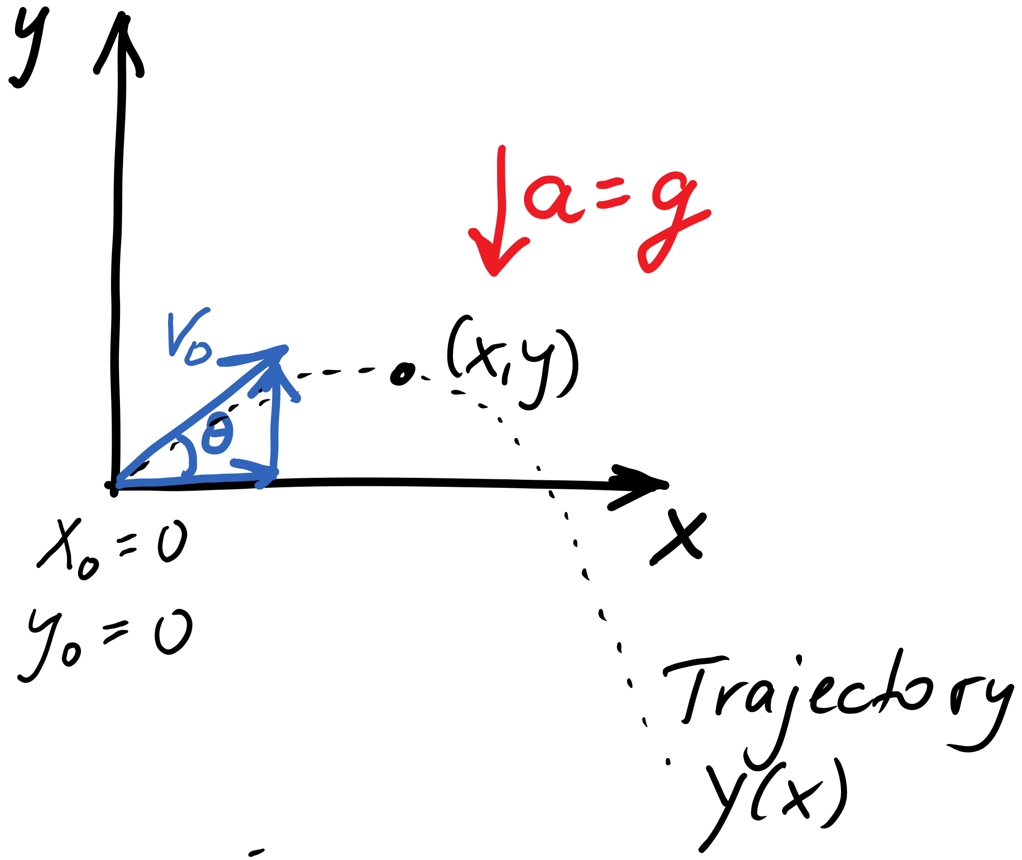
$$v_x = v_{0x} + a_x t = v_{0x}$$
$$v_y = v_{0y} + a_y t = v_{0y} - gt$$

NOT starting equations

Projectile motion: Simulation

http://www.walter-fendt.de/ph14e/projectile.htm

Free-fall trajectory

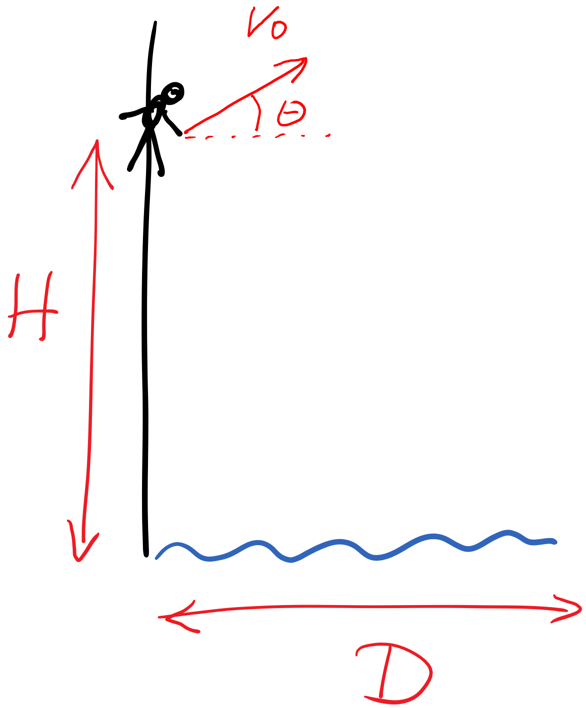


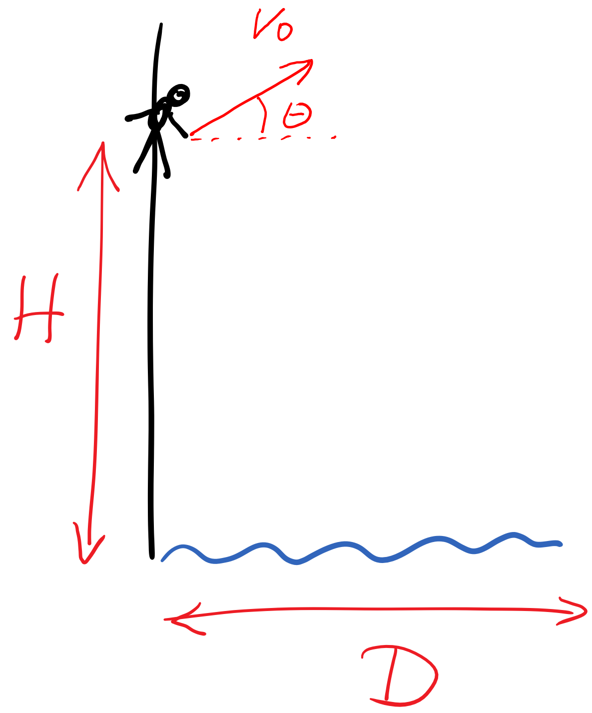
$x = x_0 + v_{0x}t + ½a_x t^2$ $\qquad$ $y = y_0 + v_{0y}t + ½a_y t^2$

Worked out on the board…

Example

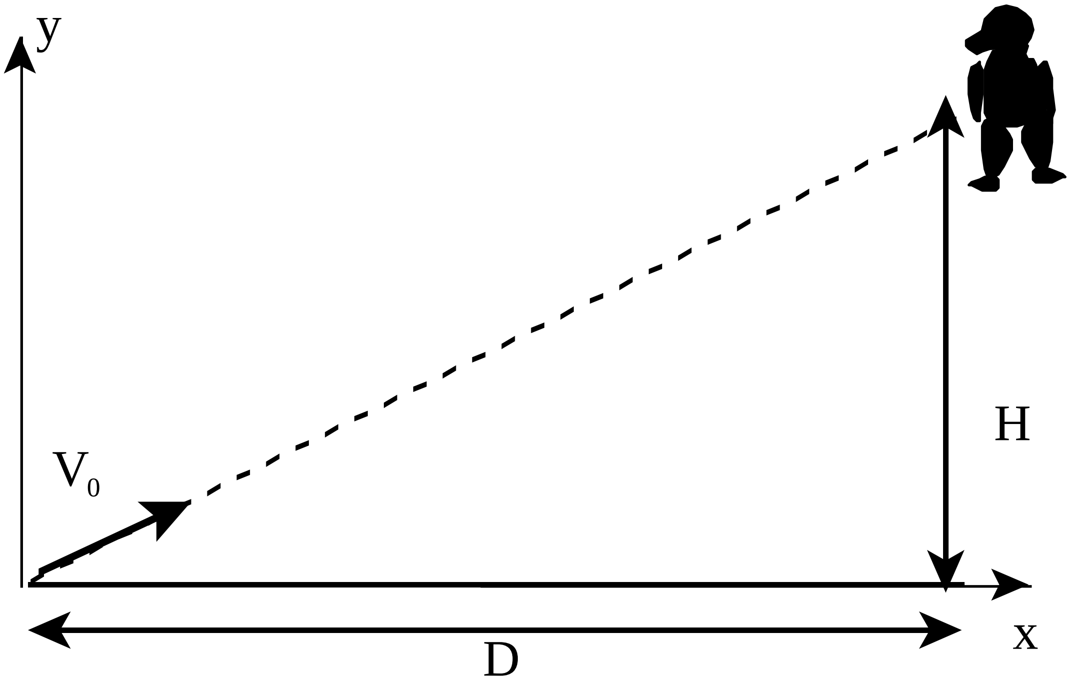
A person is stranded between a river and a high vertical cliff. To get help, they want to throw a bottle containing a message over the river. If they throw the bottle with an initial velocity V_0 and at a positive angle θ with respect to the horizontal, what is the minimum height *H* they need to climb up the cliff to ensure that the bottle just barely reaches the opposite river bank, a distance *D* away?





v_0
θ
H
D

Demo: The hunter and the monkey



*You will work this out in the Special Homework.

Hint: the angle θ between initial velocity and horizontal is not given, but knowing D and H will enable you to find sin θ and cos θ.