Lecture 8: Circular motion

- Uniform and non-uniform circular motion
- Centripetal acceleration
- Problem solving with Newton's 2nd Law for circular motion

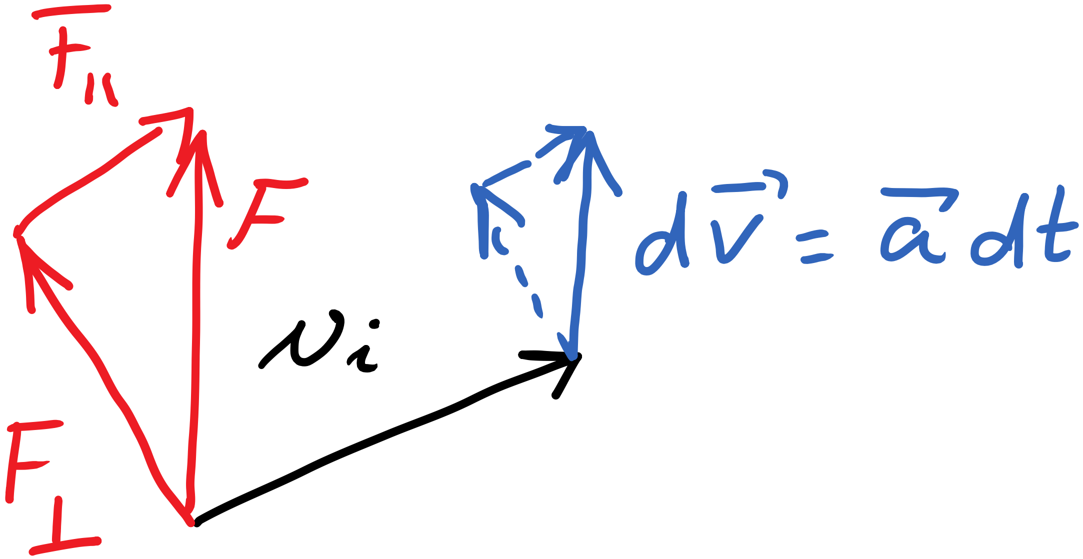
Effect of force components

Components of force parallel and perpendicular to velocity have different effects.

$$d\vec{v} = \vec{a}dt = \frac{\vec{F}}{m}dt$$

$F_{||}$ causes change in magnitude of velocity vector (speed)

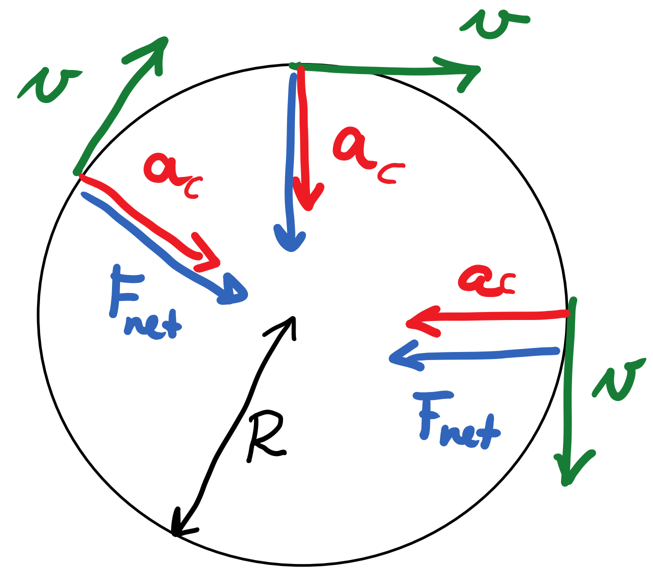
$F_{\perp}$ causes change in direction



Uniform circular motion

Motion in a circle with constant speed

Caution:
velocity is a vector and has magnitude and direction
⟹ constant *speed* does not mean constant *velocity*. There will be acceleration!



$$a_c = \frac{v^2}{R}$$

Centripetal acceleration

Directed **towards center** of the circle

Non-uniform circular motion

Motion in a circle with non- constant speed

Centripetal acceleration
Towards the center
changes direction

$$a_c = \frac{v^2}{R}$$

v is speed at that instant, does not have to be constant

Tangential acceleration
tangential to circle,
changes speed

$$a_{tan} = \frac{dv}{dt}$$

Forces create centripetal acceleration

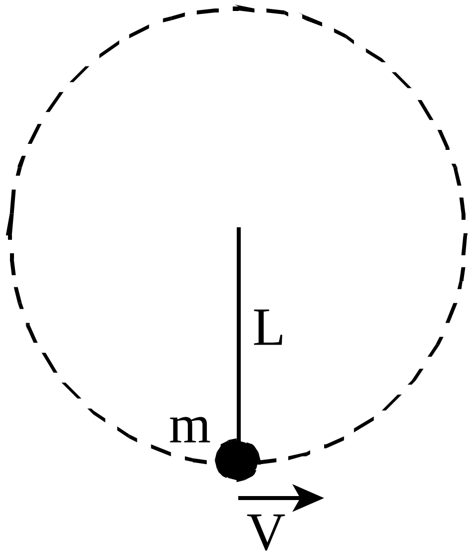
The acceleration towards the center must be created by a force that is acting towards the center.

$$\Sigma F_r = ma_c = m\frac{v^2}{R}$$

Example: http://www.walter-fendt.de/ph1i1e/carousel.htm

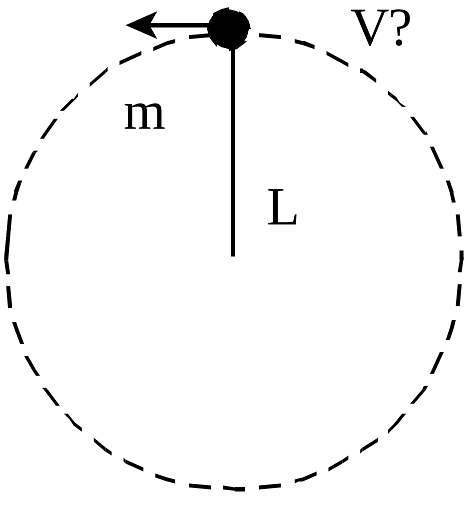
Example: ball in vertical circle

A Ball of mass *m* at the end of a string of length *L* is moving in a vertical circle. When it is at its lowest point, it has speed *V*. What is the tension in the string at that instant?



Example: ball in vertical circle- Minimum speed?

A Ball of mass *m* at the end of a string of length *L* is moving in a vertical circle. What must be its minimum speed at the highest point?



Demo: An instructor gets wet…

… or maybe not?

Twirling a bucket full of water in a vertical circle

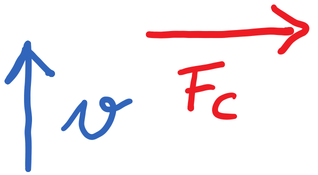
Pseudoforces

In non-inertial rotating reference frame: Pseudoforces

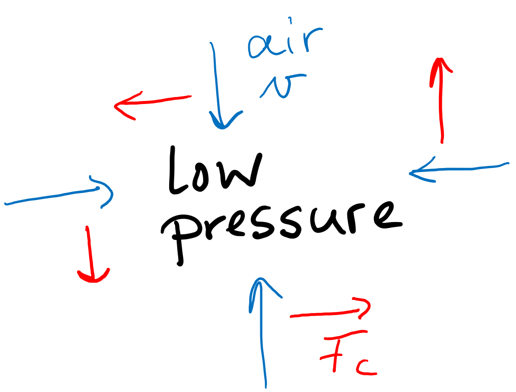
- Centrifugal force
- Coriolis force

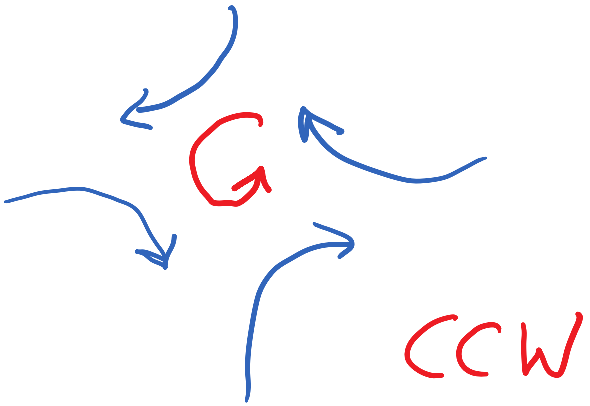
Coriolis force

- Due to Earth's rotation
- Relevant for very large masses (air masses, ocean currents) that are moving
- Responsible for formation of hurricanes



Northern hemisphere: Deflection to the right as seen in direction of motion





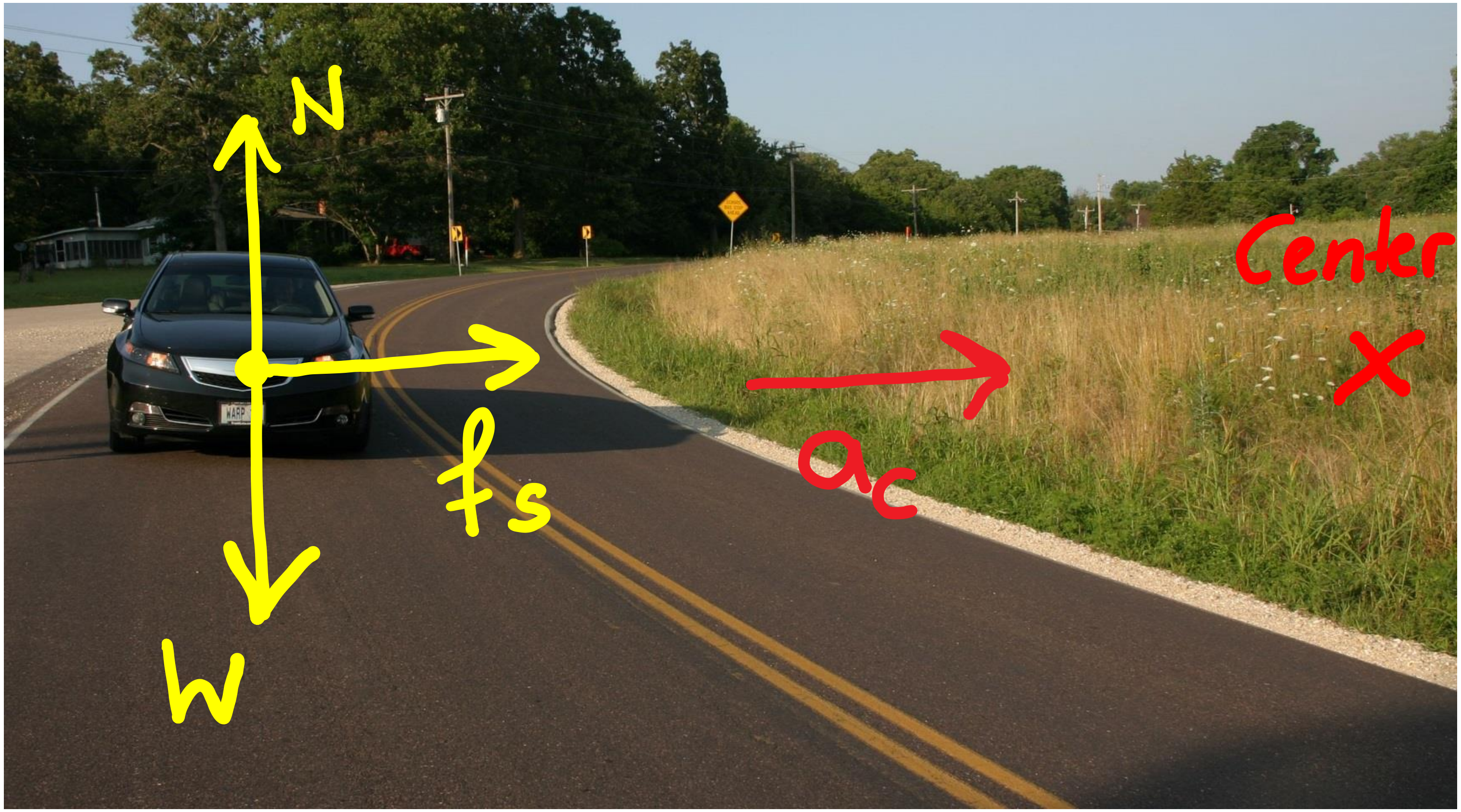
Hurricane Florence seen from the International Space Station
September 12, 2018
(photo: Alexander Gerst @ISS)

In this course, we will never describe circular motion in a rotating coordinate system.
Attach coordinate system to Earth,
treat Earth as inertial reference frame

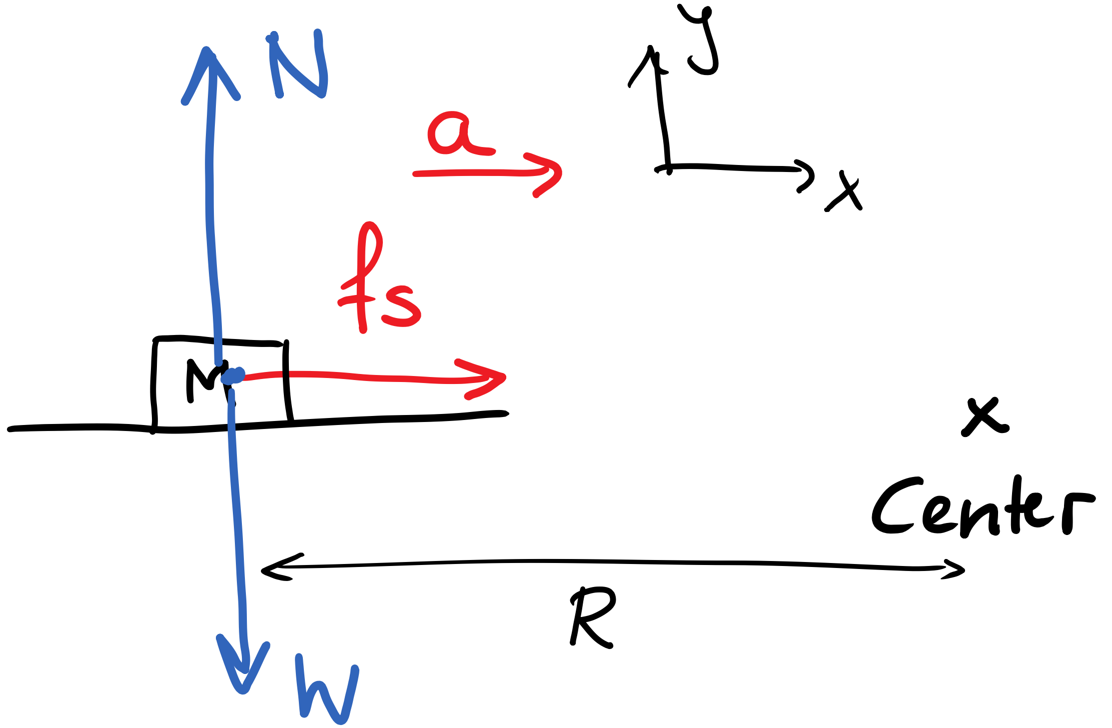
No centrifugal force

In inertial reference frame: **Inertia**
Object continues motion in straight line at constant speed unless force acts

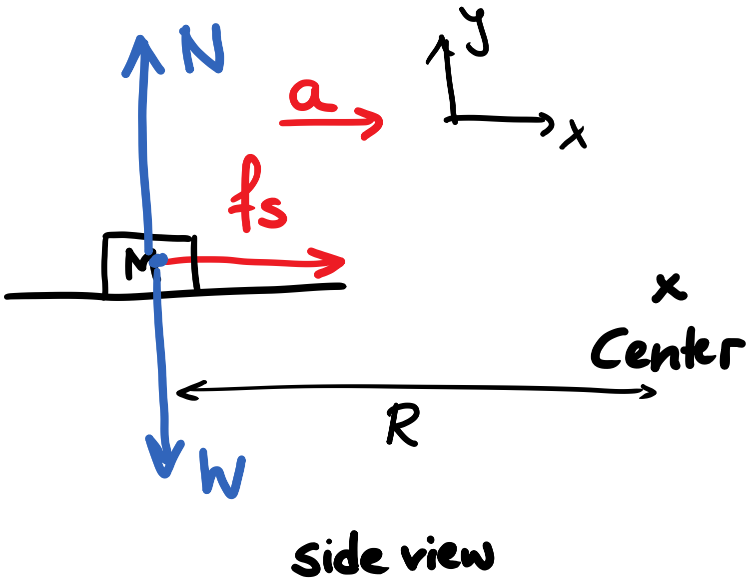
Car in flat curve



Car in flat curve



Car in flat curve worked out



$$\Sigma F_x = ma_x$$

$$f_s = m\frac{v^2}{R}$$

$$\Sigma F_y = ma_y$$

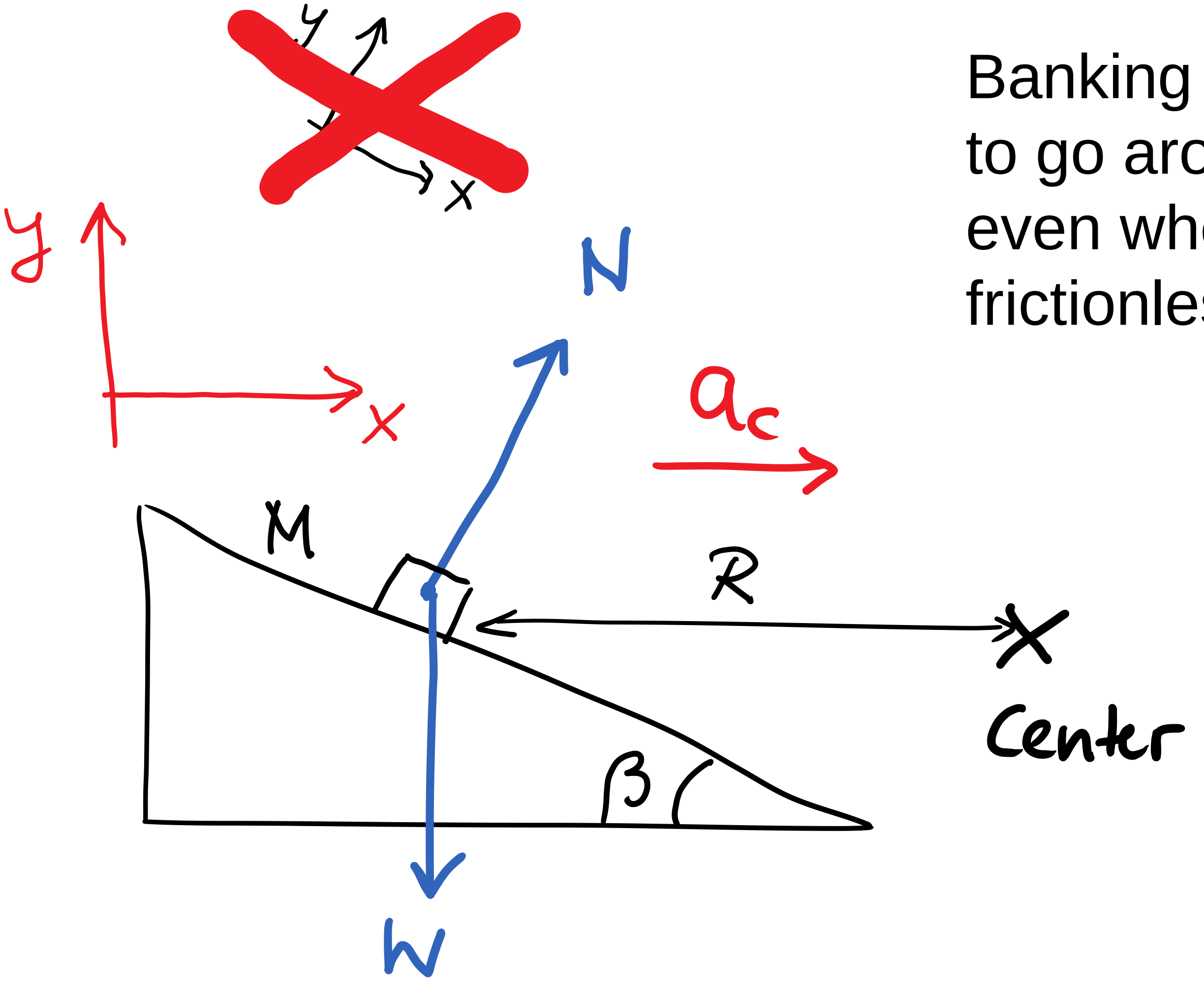
$$N + (-W) = 0$$

$$N = mg$$

Maximum speed if: $f_s = f_{s\,max} = \mu N = \mu mg$

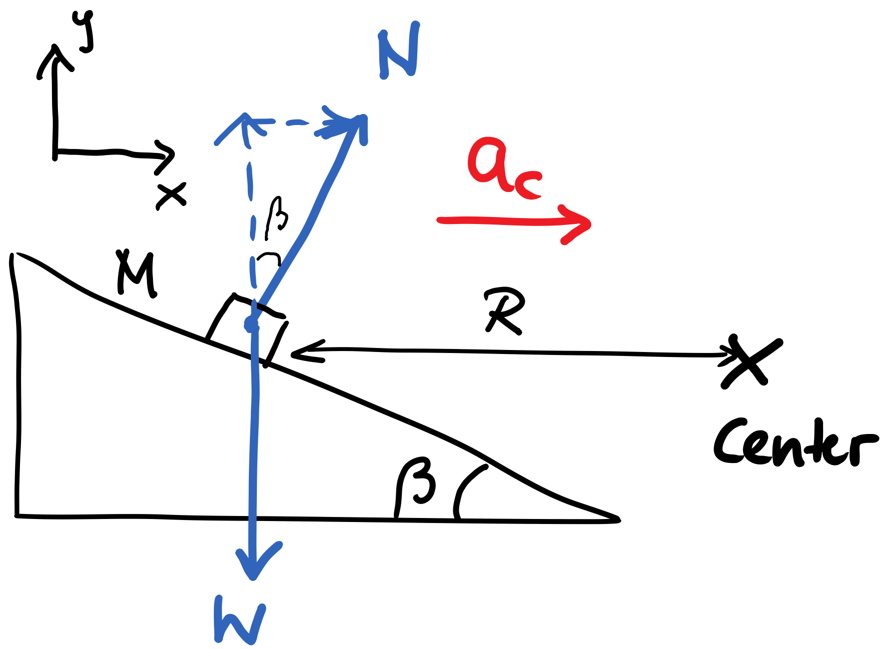
$$v_{max} = \sqrt{\mu g R}$$

Car in banked curve



Banking makes it possible to go around the curve even when the road is frictionless.

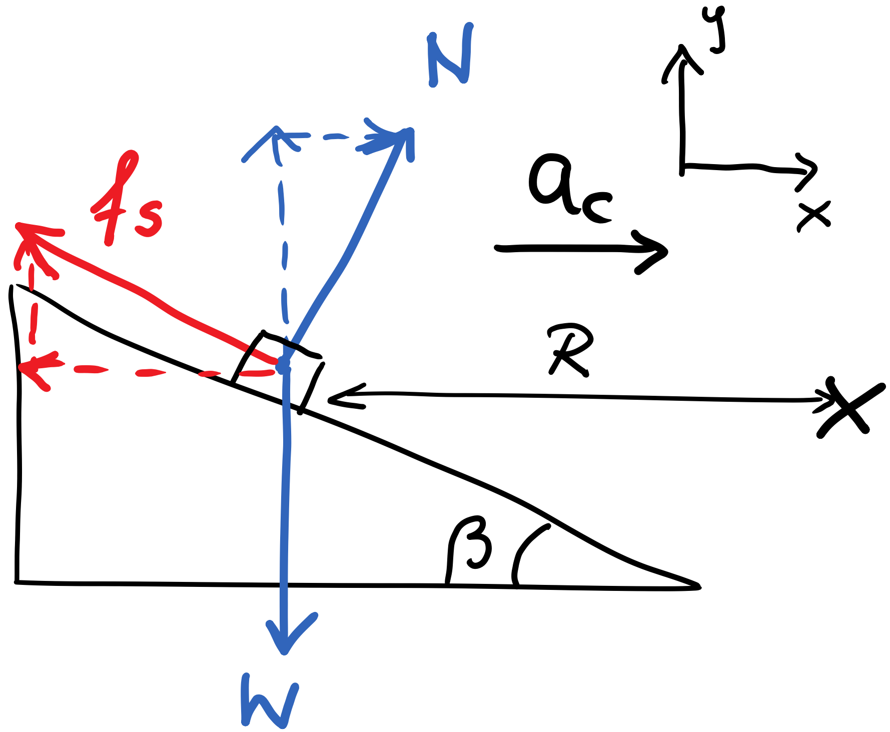
Car in banked curve: design speed



$v = v_D$: no friction

Car in banked curve with friction

Going slower than design speed

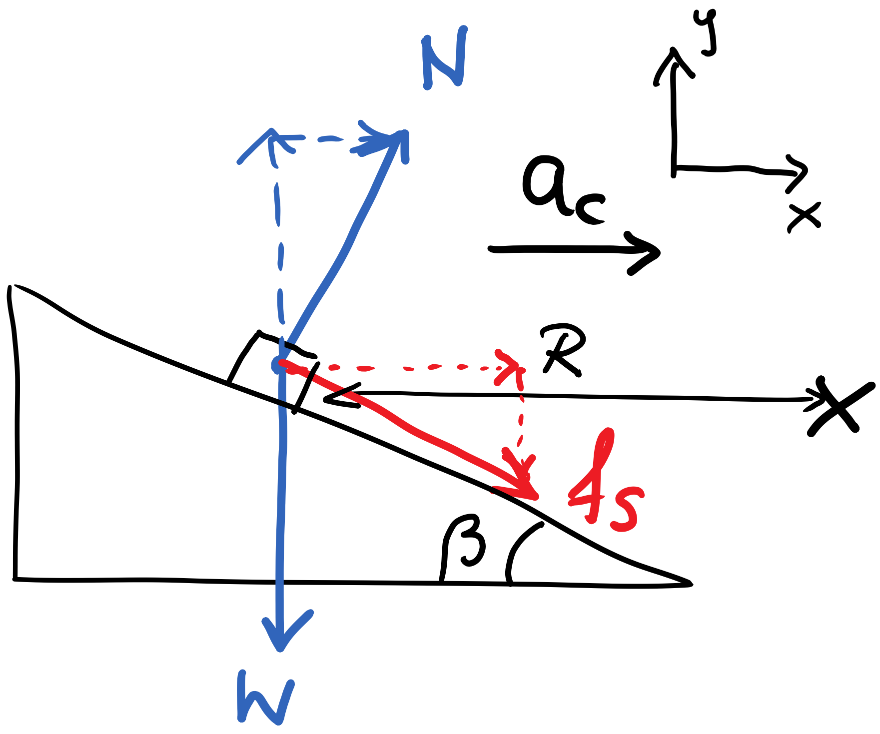


$v < v_D$ f_s up incline

Find minimum speed in HW

Car in banked curve with friction

Going faster than design speed



$v > v_D$ f_s down incline

Find maximum speed in HW