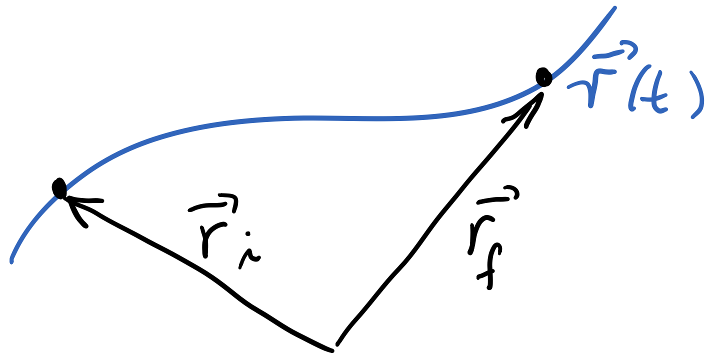
Lecture 11: Potential energy

- Conservative and non-conservative forces
- Potential energy
- Total mechanical energy
- Energy conservation

Conservative forces

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(\vec{r}) \cdot d\vec{r}$$



A force is called **conservative** if the work it does on an object as the object goes between two points is **independent of the path**.

⟶ The work done by a conservative force along any two paths between the same two points is **the same**.

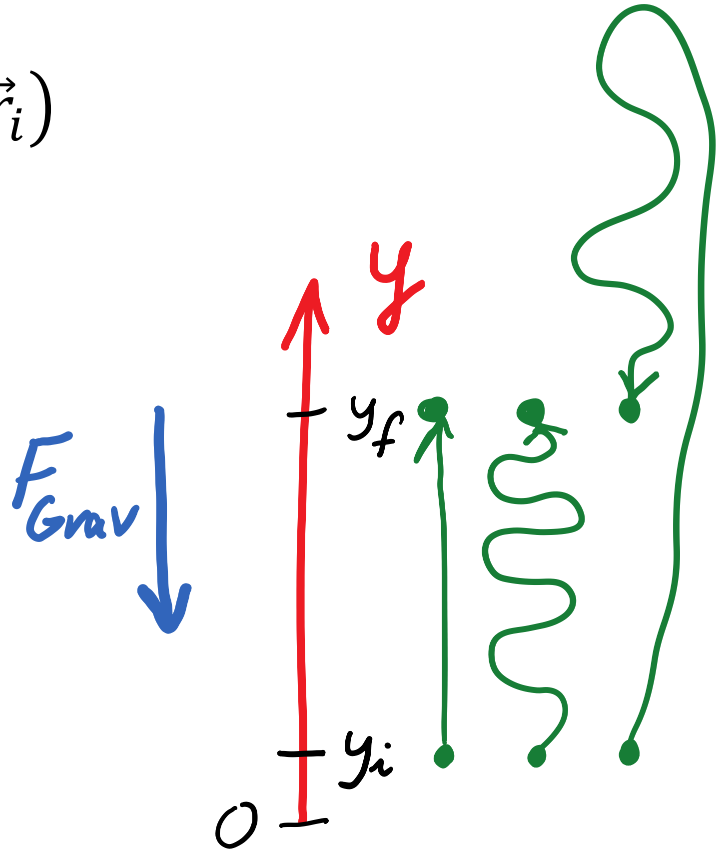
Example: Work done by gravity

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{Grav} \cdot d\vec{r} = -mg\hat{\jmath} \cdot (\vec{r}_f - \vec{r}_i)$$

$$W = -mg(y_f - y_i)$$

Depends only on y_i and y_f ,
not on path

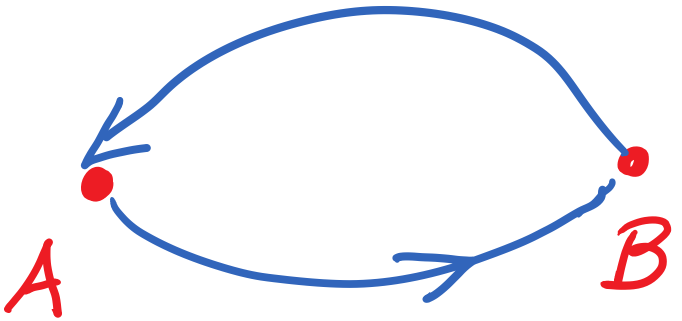
⟶ force of gravity is conservative



Properties of conservative forces:

Reverse path, get negative:

$$W_{A\to B} = -W_{B\to A}$$



Work done over a closed path is zero:

$$W_{A\to A} = \oint_A^A \vec{F} \cdot d\vec{r} = 0$$

Constant forces are conservative

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \vec{F} \cdot \int_{\vec{r}_i}^{\vec{r}_f} d\vec{r} = \vec{F} \cdot \left(\vec{r}_f - \vec{r}_i\right) = \vec{F} \cdot \vec{D}$$

$\vec{D} = \vec{r}_f - \vec{r}_i$ depends only on initial and final position, not path

Caution:

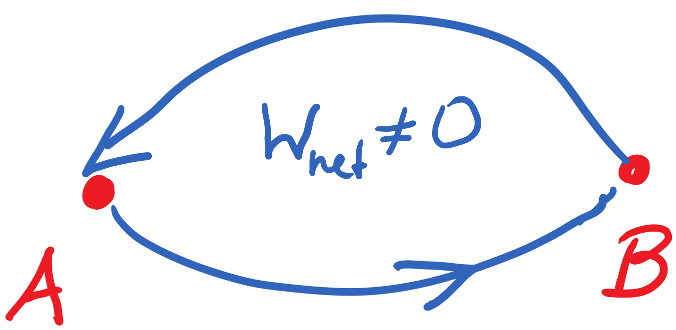
1. Force must be constant in magnitude and direction.
2. Not every conservative force has to be constant.

Non-conservative forces

If the work done by a force depends on the path, the force is non-conservative.

Different paths between initial and final point give different amount of work

Work for closed path is not zero.



HW: examine frictional force

Potential energy difference: definition

Work of conservative force $\vec{F}$ depends only on initial and final position, not on path $\longrightarrow$ each pair of points has unique value of W between them

Define: Difference in potential energy of force $\vec{F}$ between positions $\vec{r}_A$ and $\vec{r}_B$

$$\Delta U_{A\to B} = U(\vec{r}_B) - U(\vec{r}_A) = -W_{A\to B} = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{F}\cdot d\vec{r}$$

Potential energy: reference point

$$\Delta U_{A\to B} = U(\vec{r}_B) - U(\vec{r}_A) = U_B - U_A = -W_{A\to B}$$

Only differences in potential are meaningful

$\longrightarrow$ Choose arbitrary reference point $\vec{r}_0$ and assign it a value of potential energy U_0 that is *convenient*

$$U(\vec{r}) - U(\vec{r}_0) = -W_{\vec{r}_0\to\vec{r}}$$

$$U(\vec{r}) = U(\vec{r}_0) - W_{\vec{r}_0\to\vec{r}}$$

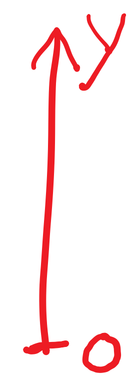
Potential energy of gravity*

*near Earth's surface

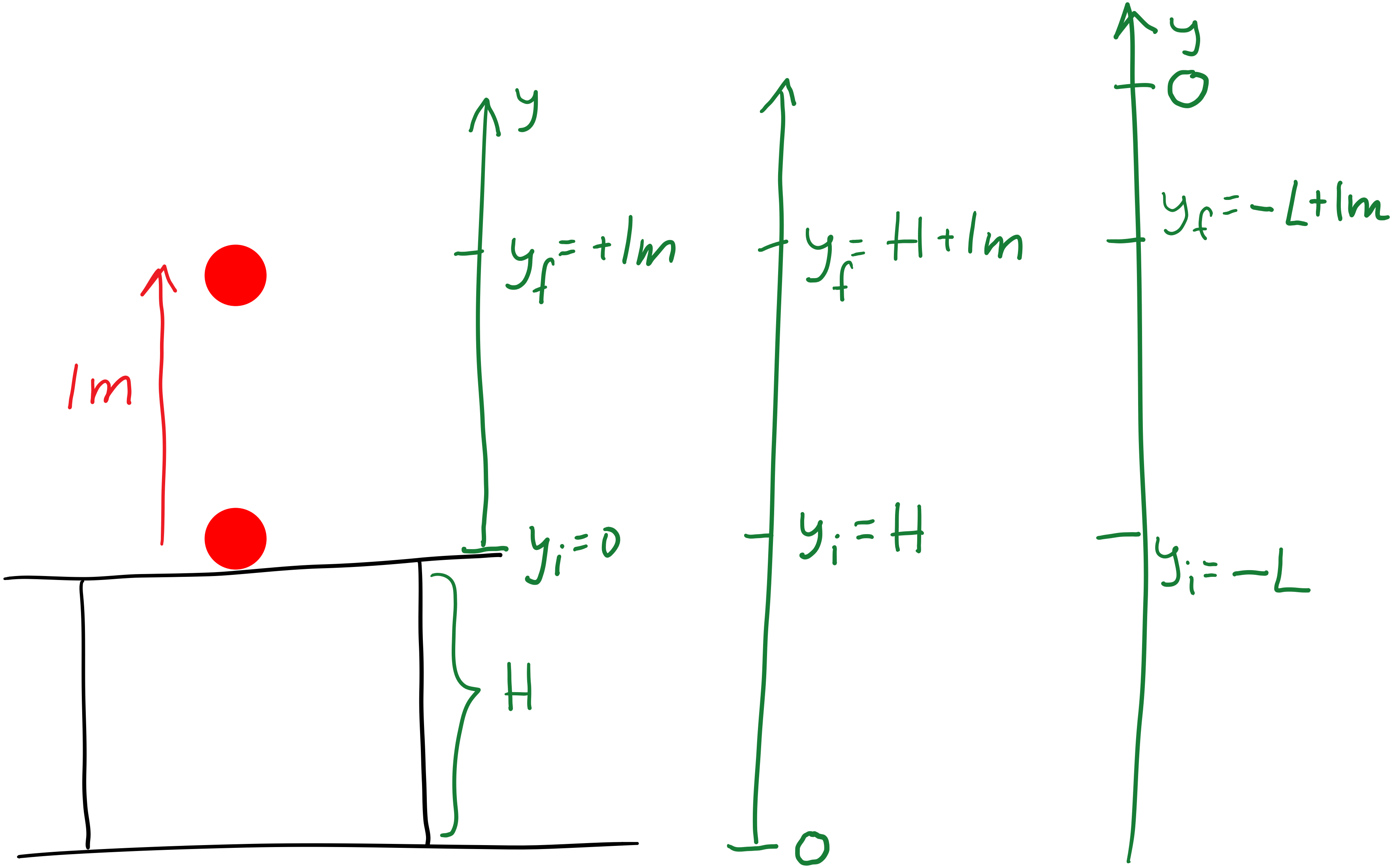
$$W_{grav} = -mg(y_f - y_i)$$ (y-axis vertically up)

$$U_{grav}(\vec{r}) - U_{grav}(\vec{r}_0) = -W_{grav\ y_o \to y} = -[-mg(y - y_0)]$$

Choose $\vec{r}_0 = 0$ and assign $U(\vec{r}_0) = 0$



$U_{grav}(\vec{r}) = mgy$ with y-axis up



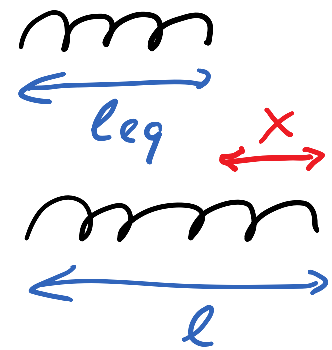
$$y_f - y_i = 1m$$

Potential energy of spring force

From lecture 10:

$$W_S = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$$

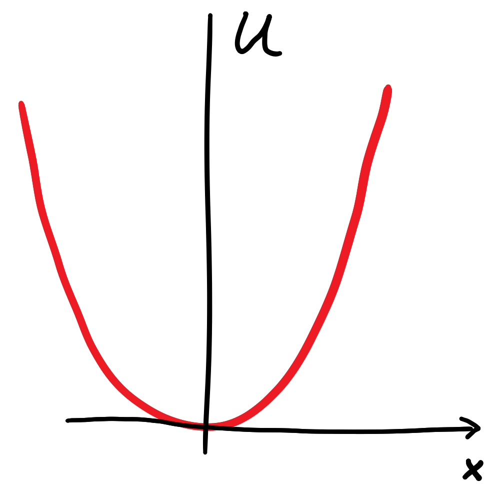
$$x = l - l_{eq}$$



$$\Delta U_s = -W_S = -\frac{1}{2}k\left(x_i^2 - x_f^2\right) = \frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

Choose $x = 0$ as reference point,
assign $U_S(x = 0) = 0$

$$U_{spring} = ½kx^2$$



Total mechanical energy

$$\Delta K = W_{net} = W_{conservative} + W_{other}$$

$$K_f - K_i + (-W_{cons}) = W_{other}$$

$$K_f - K_i + \left(U_f - U_i\right) = W_{other}$$

$$(K_f + U_f) - (K_i + U_i) = W_{other}$$

Total mechanical energy of a system: $E = K + U$

$$E_f - E_i = W_{other}$$

$$E = K + U \qquad E_f - E_i = W_{other}$$

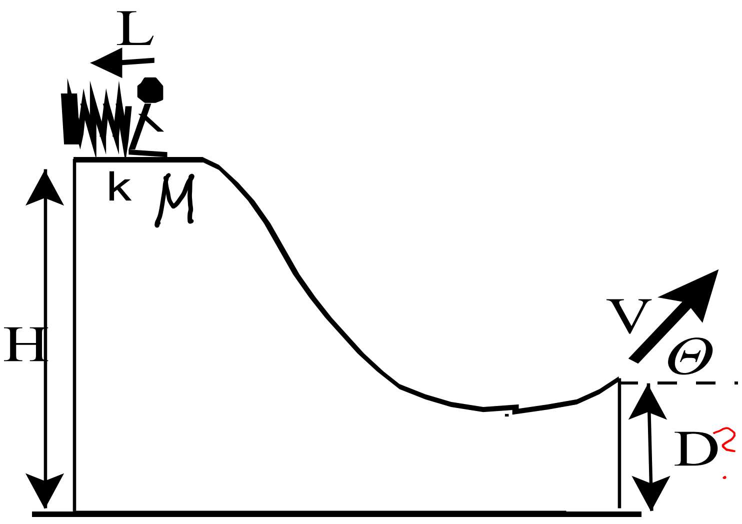
If only conservative forces act: $W_{other} = 0$

$$E_f = E_i$$

Total mechanical energy is conserved.

Example

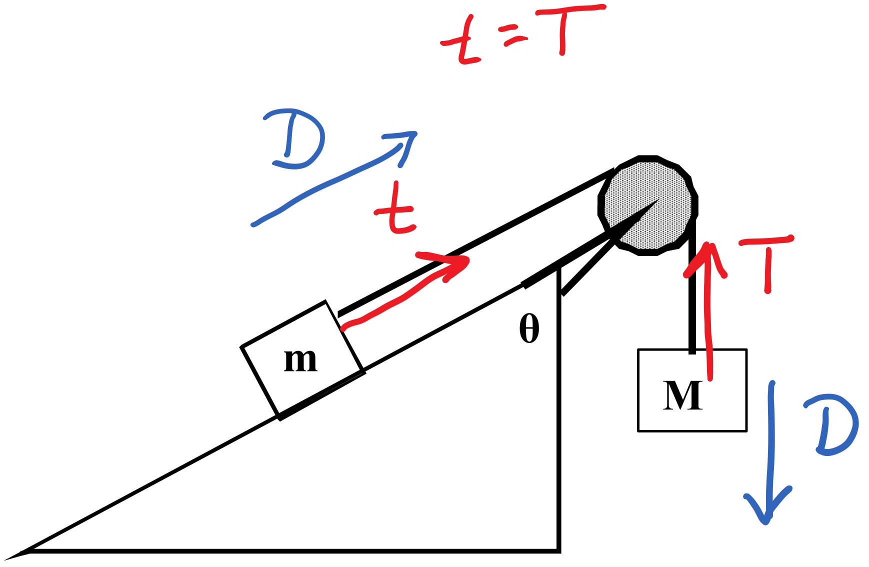
In a new Olympic discipline, a ski jumper of mass M is launched by means of a compressed spring of spring constant k. At the top of a frictionless ski jump at height H above the ground, he is pushed against the spring, compressing it a distance L. When he is released from rest, the spring pushes him so he leaves the lower end of the ski jump with a speed V at a positive angle θ with respect to the horizontal.



Determine the height D of the end of the ski jump in terms of given system parameters.

Tension in coupled objects

Net work done by tension in coupled system is zero



$$W_t = \vec{t} \cdot \vec{D} = tD$$

$$W_T = \vec{T} \cdot \vec{D} = -TD$$

$$W_t + W_T = 0$$

Example with coupled objects

A block of mass *m* is on a frictionless incline that makes an angle θ with the vertical. A light string attaches it to another block of mass *M* that hangs over a massless frictionless pulley. The blocks are then released from rest, and the block of mass *M* descends. What is the blocks' speed after they move a distance *D*?