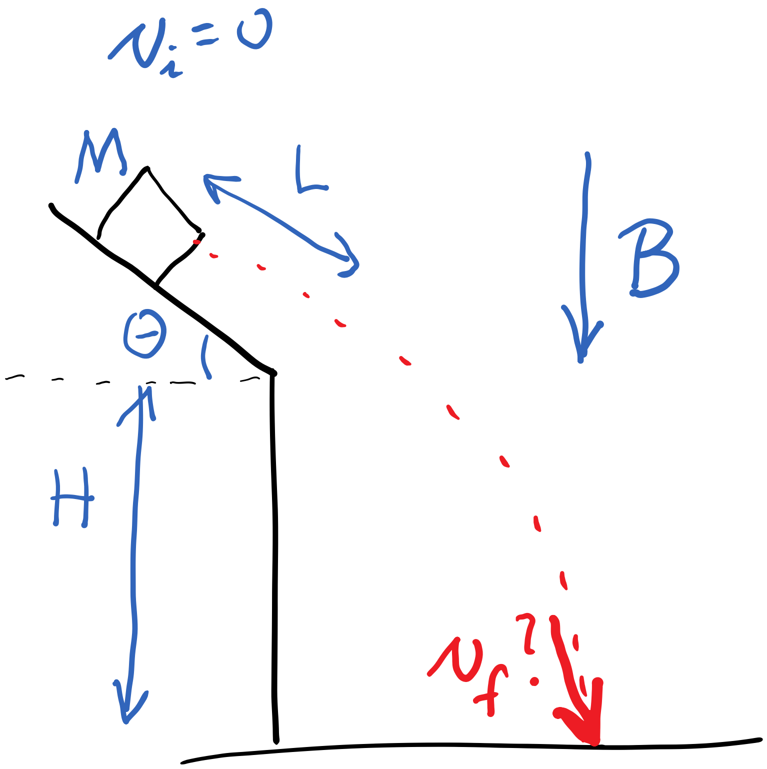
Lecture 12: Potential energy diagrams

- Problem with Work done by “other” forces
- Relationship between force and potential energy
- Potential energy diagrams

Example with other force



A block of mass M is at rest on an incline that makes an angle θ with the horizontal. It slides down a distance L and then flies of the edge a height H above the ground. Throughout its motion, a constant vertical blowing force of magnitude B is acting on the block.

Derive an expression for the speed with which the block hits the ground.

Relationship between force and potential energy

$$U(\vec{r}_B) - U(\vec{r}_A) = -W_{A\to B} = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{F}\cdot d\vec{r}$$

In one dimension: $\vec{F} = F_x(x)\hat{\imath}$

$$\Delta U = -\int F_x dx$$

$$F_x = -\frac{dU(x)}{dx}$$

$U_{grav} = mgy$ (y-axis up) $F_{grav,y} = -\frac{d}{dy}(mgy) = -mg$

In three dimensions: $U = U(x, y, z)$

$$F_x = -\frac{\partial U(x,y,z)}{\partial x}$$

$$F_y = -\frac{\partial U(x,y,z)}{\partial y}$$

$$F_z = -\frac{\partial U(x,y,z)}{\partial z}$$

Partial derivative $\frac{\partial}{\partial x}$ means: treat y and z like constants and only x like a variable

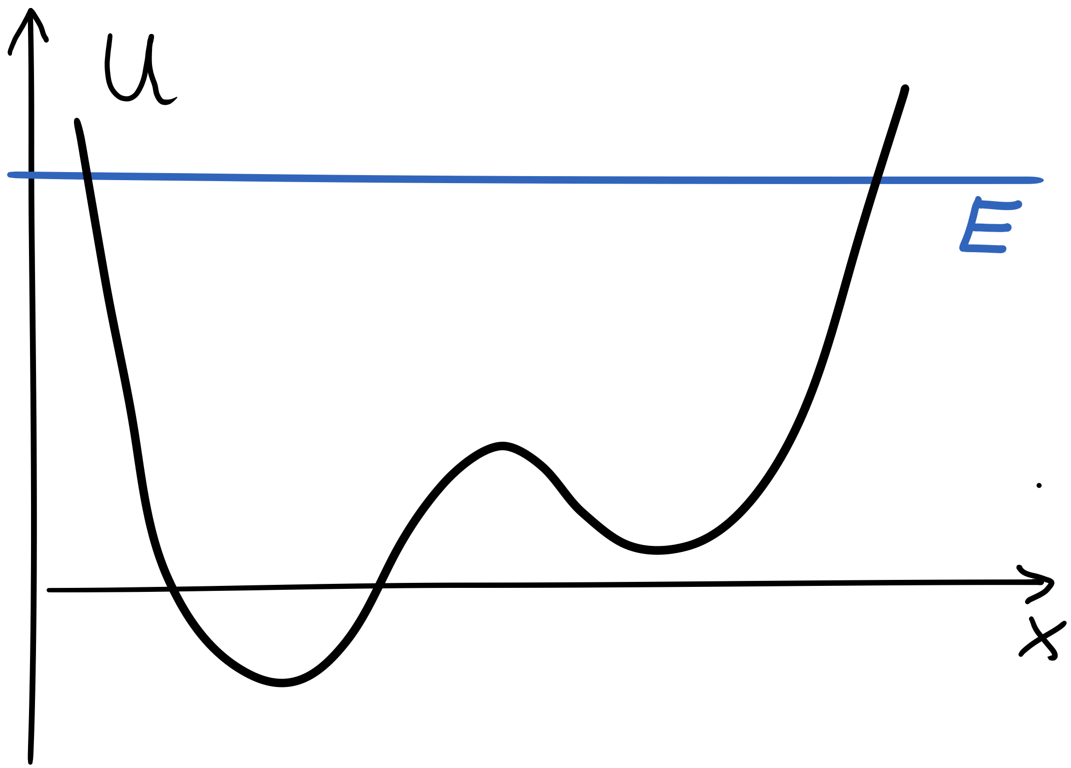
Example: $U(x,y,z) = xy^2z$

$$\frac{\partial U}{\partial x} = y^2 z, \quad \frac{\partial U}{\partial y} = 2xyz, \quad \frac{\partial U}{\partial z} = xy^2$$

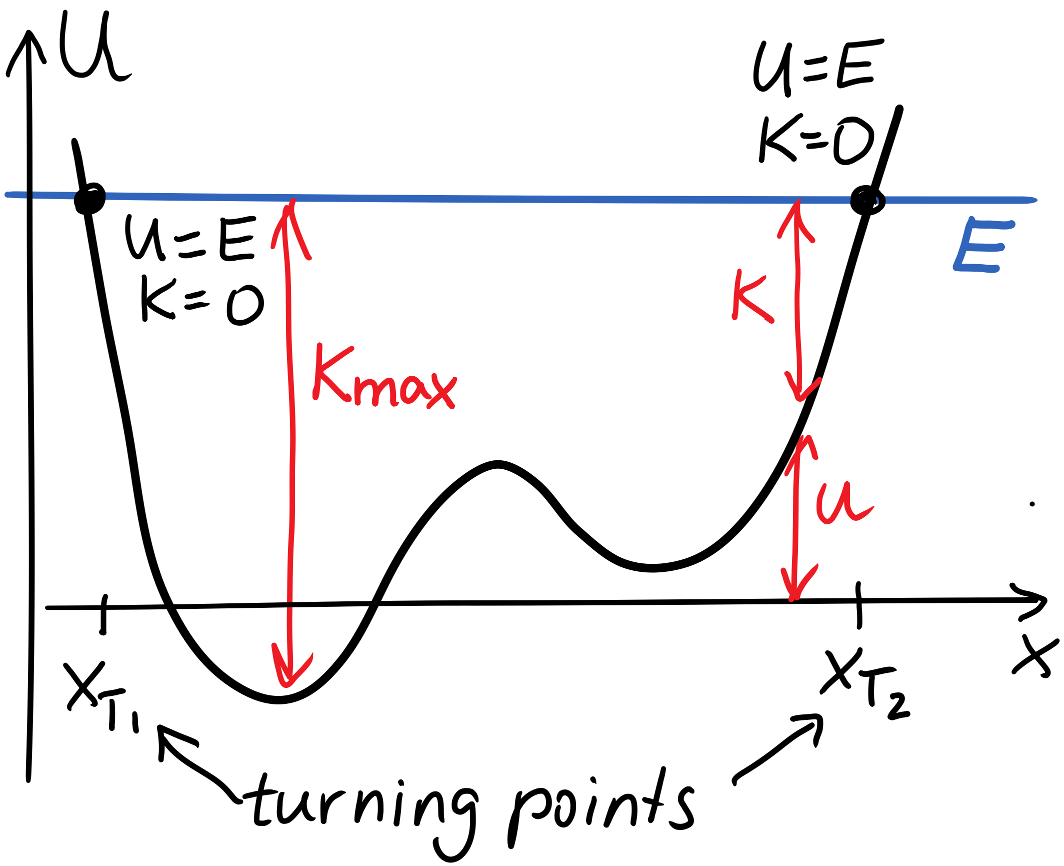
Motion in a potential energy well

Consider motion in one dimension under the influence of a single conservative force with potential energy $U(x)$.

$W_{other} = 0\ ,\ E_f = E_i$



Kinetic and potential energy



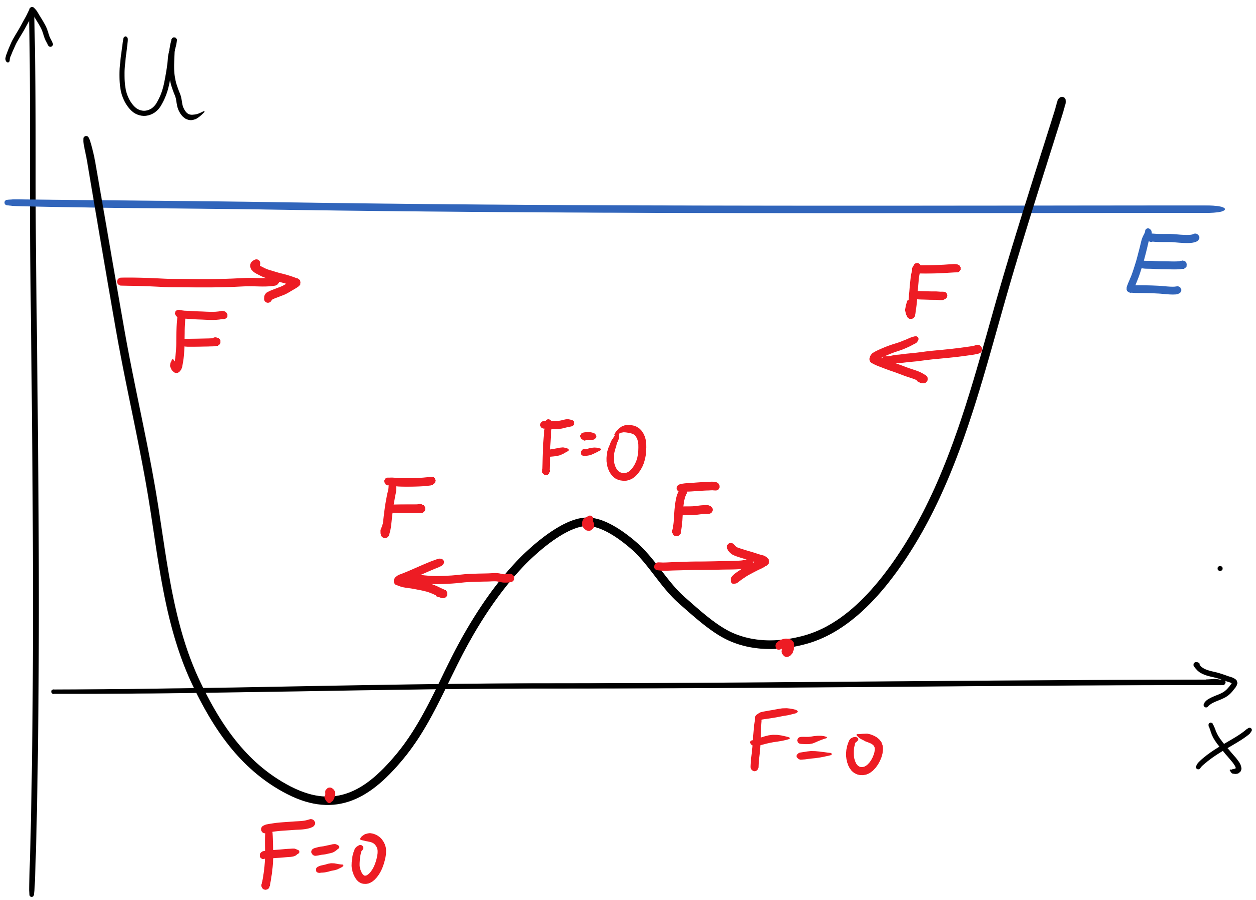
$$E = K(x) + U(x)$$
$$\Longrightarrow K(x) = E - U(x)$$

Small $U \Longrightarrow$ large K
Large $U \Longrightarrow$ small K

K is maximum where
U is minimum

At turning points: $U = E, K = 0$

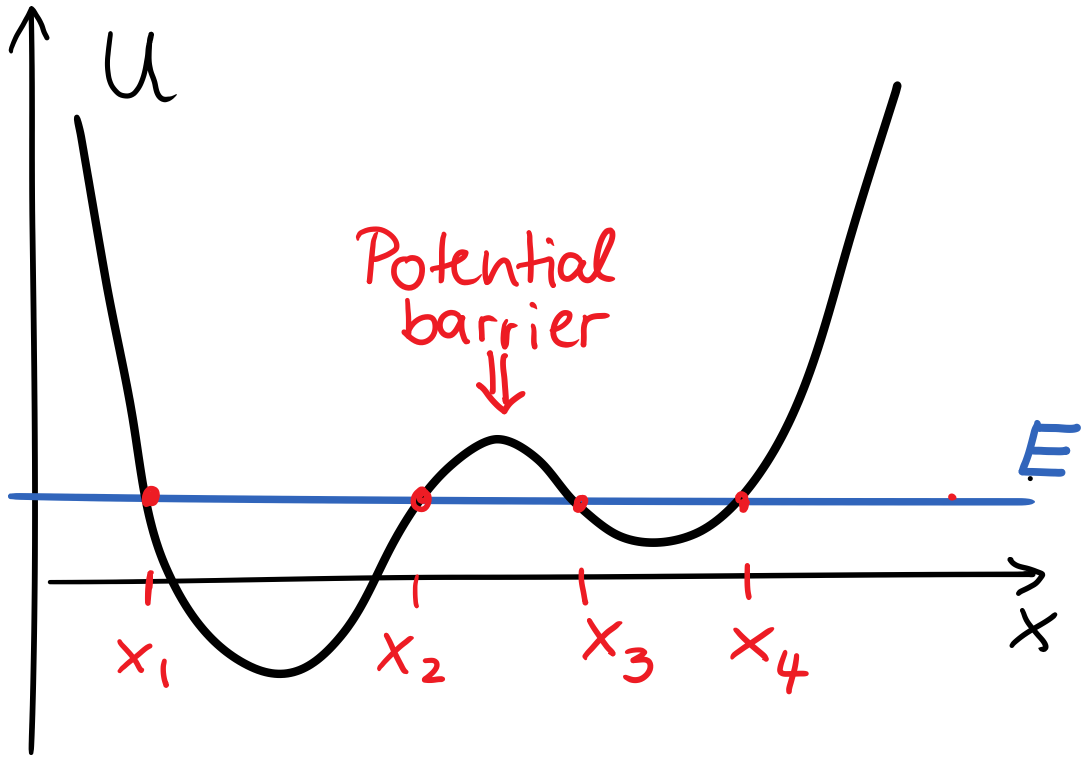
Force and potential energy



$$F_x = -\frac{dU(x)}{dx}$$

The force is the negative slope of potential energy graph

Different total mechanical energy

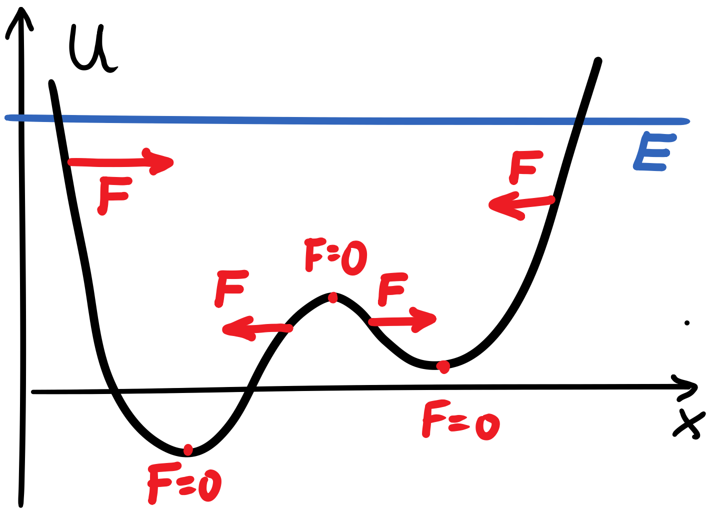


Motion possible between x_1 and x_2 or between x_3 and x_4

Not possible between x_2 and x_3 because $U(x) > E$

(K can not be negative)

Equilibrium

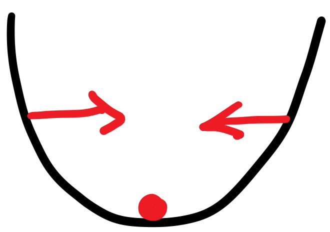


Equilibrium: $F_x = 0$

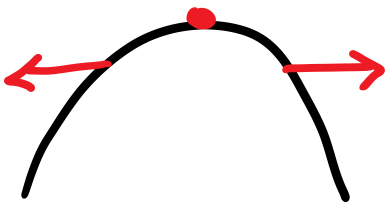
$$F_x = -\frac{dU(x)}{dx} = 0$$

$U(x)$ has local minimum or maximum

Minimum = stable



Maximum = unstable

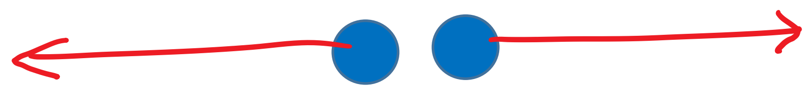


Example: Diatomic Molecule

Diatomic molecule: two atoms separated a distance r
What should we expect about force between atoms?

Too close: repulsive force

The shorter the distance,
the greater the force



Too far: attractive force

Force decreases with distance,
goes to zero for $r \to \infty$

Distance **just right:** equilibrium

Example: Diatomic Molecule

Diatomic molecule: two atoms separated a distance r