Lecture 13: Static Fluids

- Pressure
- Pascal's Principle
- Buyoancy force

Pressure

An object submerged in a fluid will experience a force acting on the surface.

Pressure p **=** Force magnitude per Area

$$\boxed{p = \frac{dF}{dA}}$$

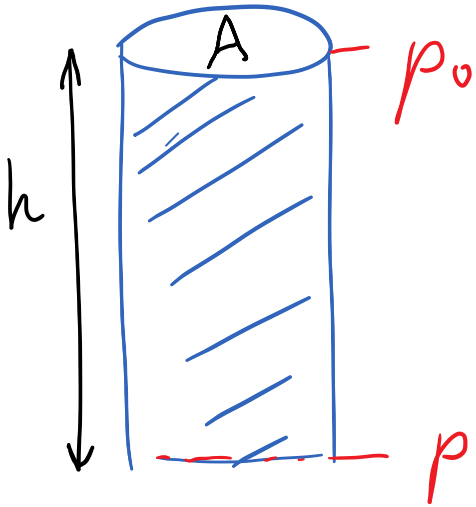
Unit: N/m^2 = Pa

Fluid at rest:

- at given depth, p is same in all directions.
- force due to pressure is perpendicular to all surfaces

Pressure increase with depth

Due to weight of column of fluid above



$$W = Mg = \rho V g = \rho A h g$$

$$\Delta p = \frac{W}{A} = \frac{\rho A h g}{A} = \rho g h$$

$$\boxed{p_{below} - p_{above} = \rho g h}$$

Atmospheric pressure

$$p_{atm} = 100kPa = 10^5 N/m^2$$

On 1cmx1cm: $10\text{N} \approx 1kg * g$
Above head (10cmx10cm): weight of 100kg

Demo: Magdeburg hemispheres

Magdeburg hemispheres

Otto von Guericke, 1654. 30 horses.

Magdeburg Hemispheres

$D = 50\,cm$

$p = \frac{F}{A}$ $\quad F = p \cdot A$ $\quad A = \pi \frac{D^2}{4}$

(cross section)

$$F \sim 10^5 \frac{N}{m^2} \cdot \pi \frac{(0.5m)^2}{4} \sim 2 \times 10^4 N$$

($\approx$ weight of a mass of 2000 kg)

Demo:

$D = 10\,cm$ $\quad F \approx 800N$ *

* and not perfect vacuum inside

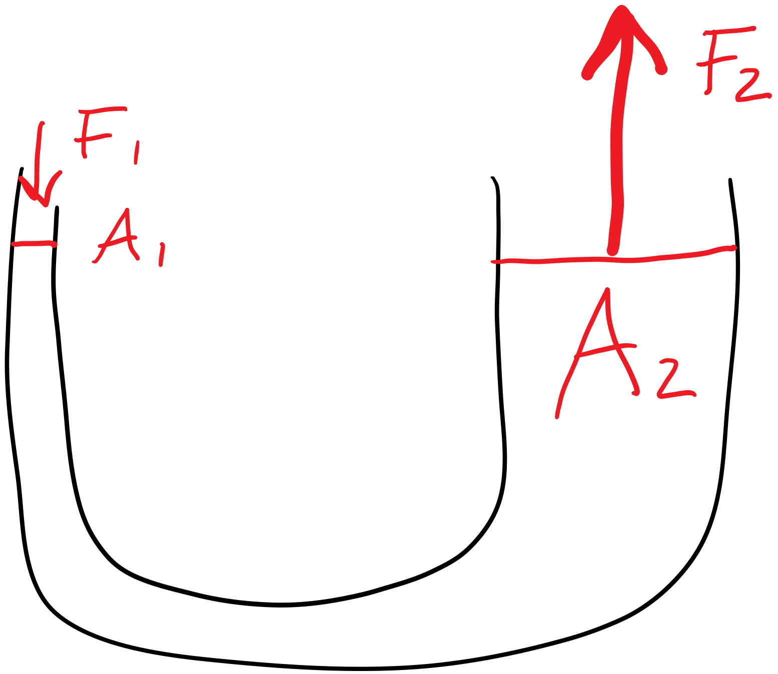
Pascal’s Principle

Pressure applied to a confined fluid increases the pressure throughout the fluid by the same amount.

All points at the same level in a **contiguous** fluid have the same pressure.

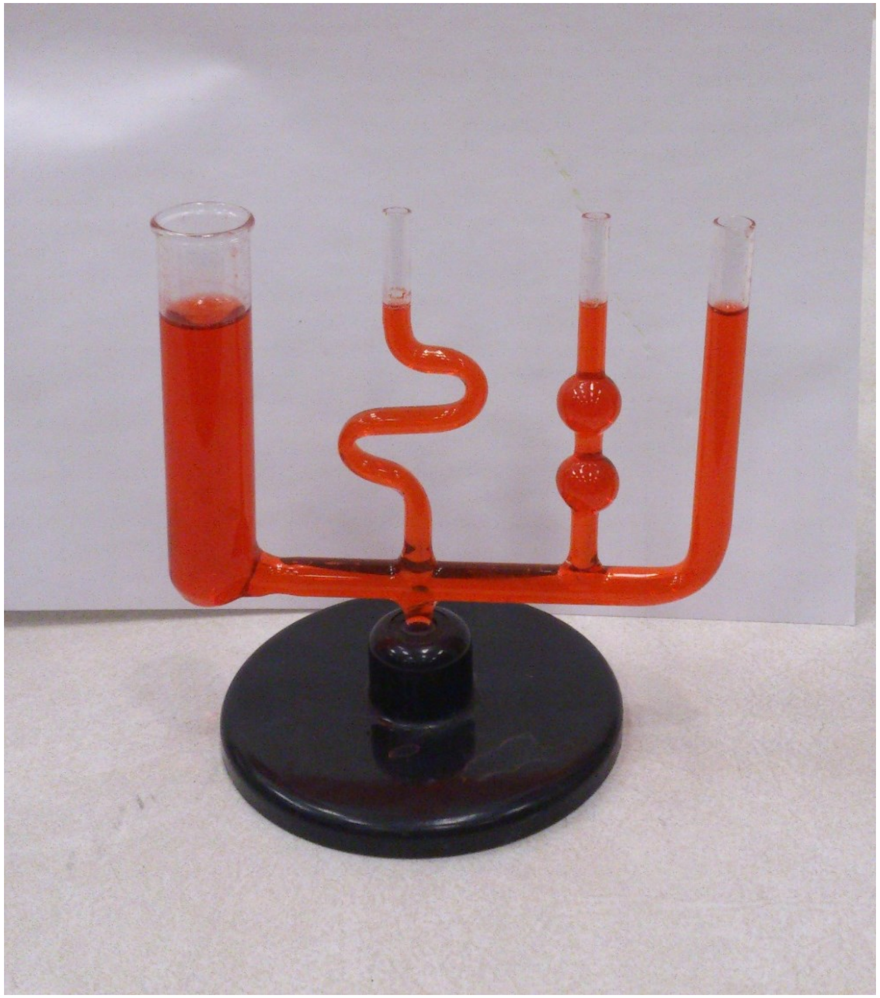
Applications of Pascal's Principle

Hydraulic lift

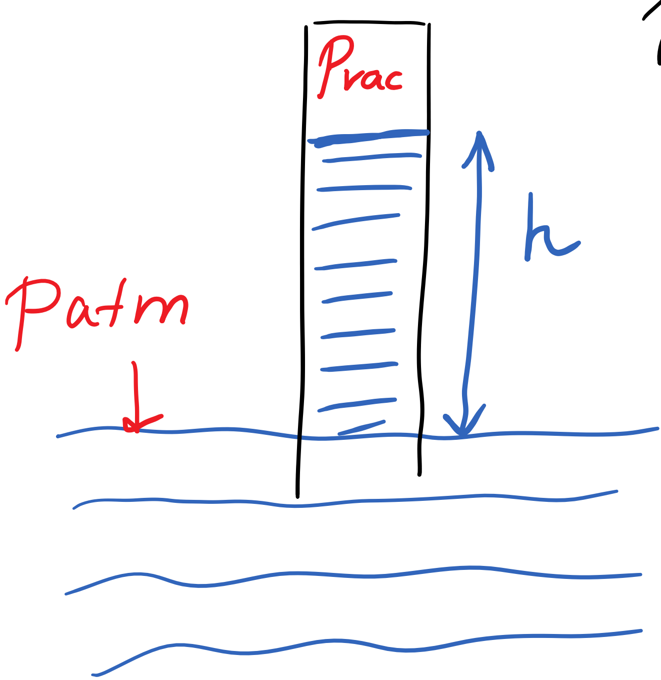


$$\Delta P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Demo: same water level in connected tubes of different shapes and cross sections



The longest straw… or: How high can you pump water by suction?



$$p_{vac} \approx 0$$

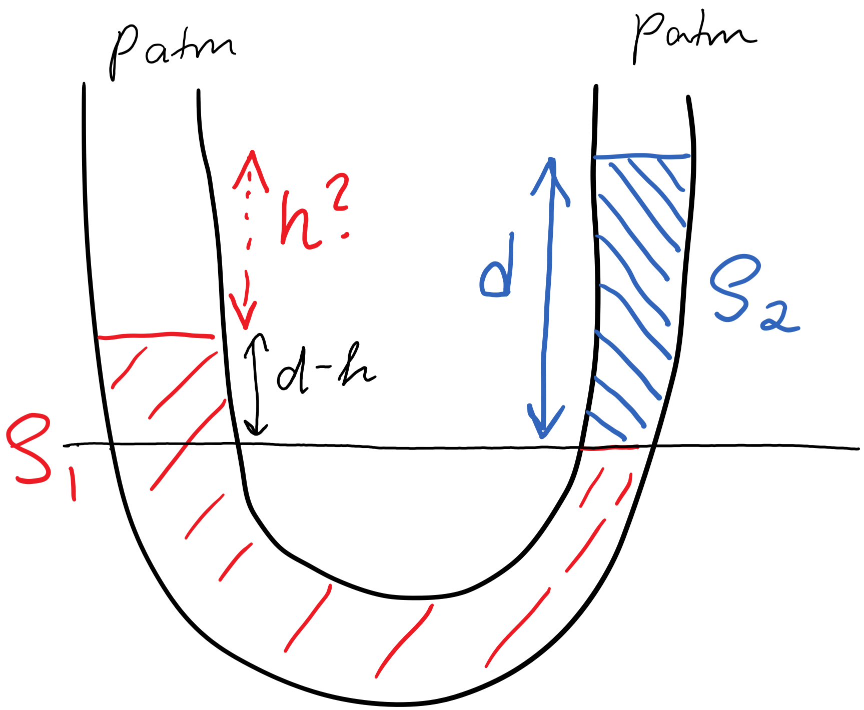
$$p_{below} - p_{above} = \rho g h$$

$$p_{atm} - 0 = \rho g h$$

$$h_{max} = \frac{p_{atm}}{\rho_{water} \cdot g}$$

$$h_{max} \approx 10\,m$$

Example 1



$$P_L = P_R$$

$$\cancel{P_{atm}} + \rho_1 \cancel{g} (d-h) = \cancel{P_{atm}} + \rho_2 \cancel{g} d$$

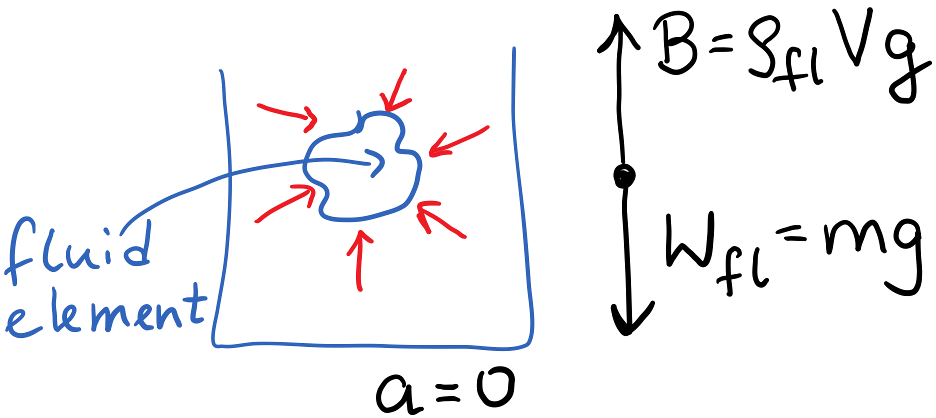
$$\rho_1 d - \rho_1 h = \rho_2 d$$

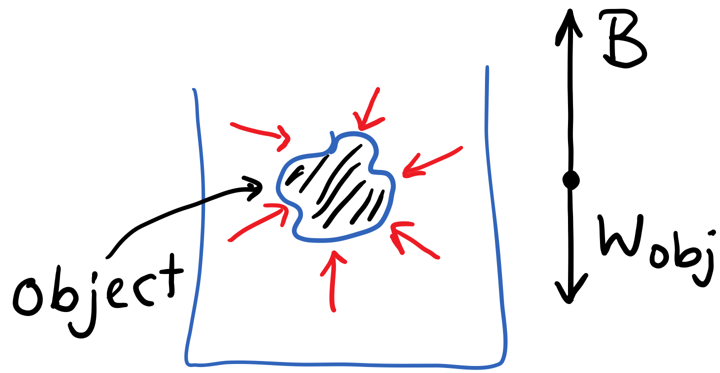
$$\rho_1 h = \rho_1 d - \rho_2 d$$

$$\boxed{h = \frac{\rho_1 - \rho_2}{\rho_1} d}$$

Buyoancy and Archimedes' Principle

An object fully or partially submerged in a fluid experiences an upward buoyancy force equal to the weight magnitude of the fluid displaced by the object.





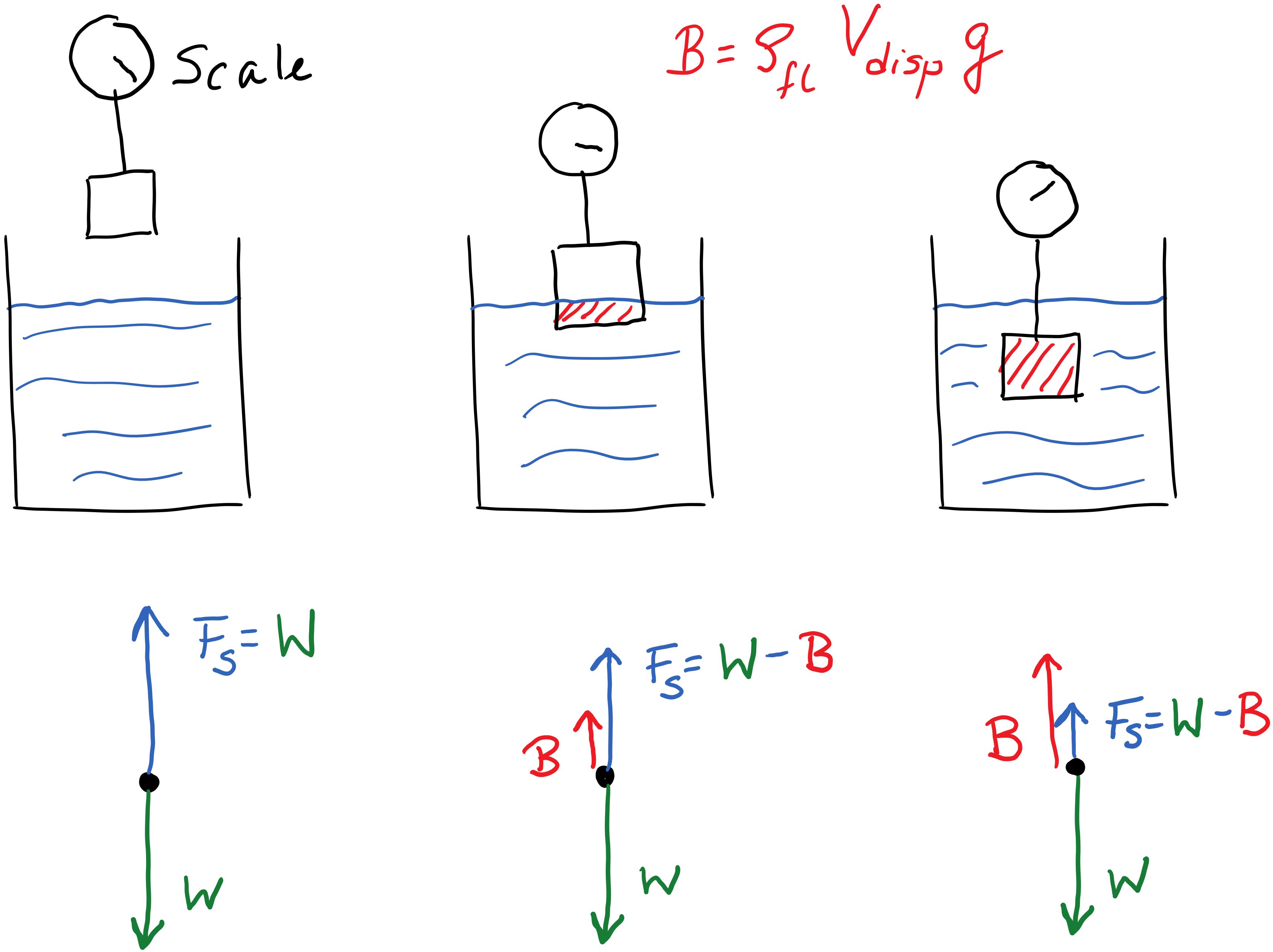
$$\boxed{B = \rho_{fluid} V_{disp} g}$$

Consequences of Archimedes' Principle

Density of object less than density of fluid:
Object floats

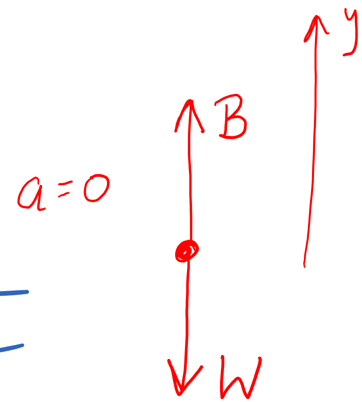
Density of object larger than density of fluid:
Object sinks

Demo: Buyoancy force


Scale
$B = \rho_{fl} V_{disp} g$
$F_s = W$
W
$F_s = W - B$
B
W
B
$F_s = W - B$
W

Example 2

A ball has a uniform mass density of ⅓ the density of water. What fraction of the ball's volume is below the water line?



$$\sum F_y = B_y + W_y = m\cancel{a_y}^{\,0}$$

$$B - W = 0$$

$$B = W$$

$$\rho_w V_{disp}\, g = mg = \rho_{ball} V g$$

$$\rho_w V_{disp} = \frac{1}{3}\rho_w V$$

$$\frac{V_{disp}}{V} = \frac{\frac{1}{3}\rho_w}{\rho_w} = \frac{1}{3}$$

Example 3

A cube of side length *L* is placed in water and an object with twice the cube's weight is placed on top of it. Because the density of water is ρ and the cube has a uniform density of ¼ρ, a portion of the cube remains above the waterline. If the cube stays in a level orientation, what is the difference between the pressure at the cube's lower (submerged) surface and atmospheric pressure, i.e., what is the gauge pressure at the lower surface?

$p - p_0$?