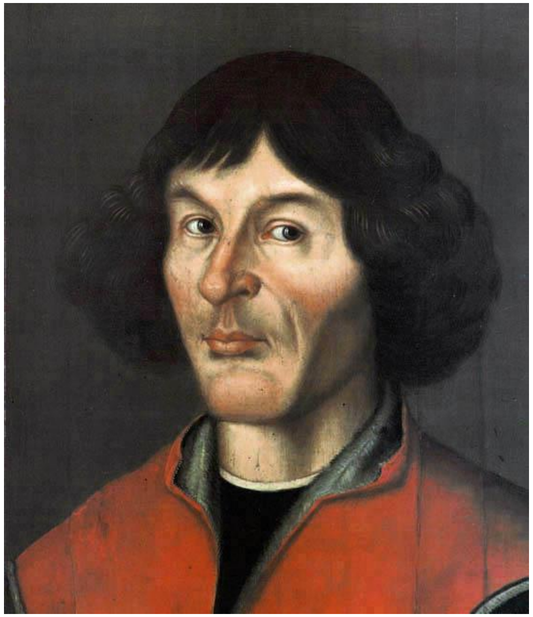
Lecture 15: Universal gravitation

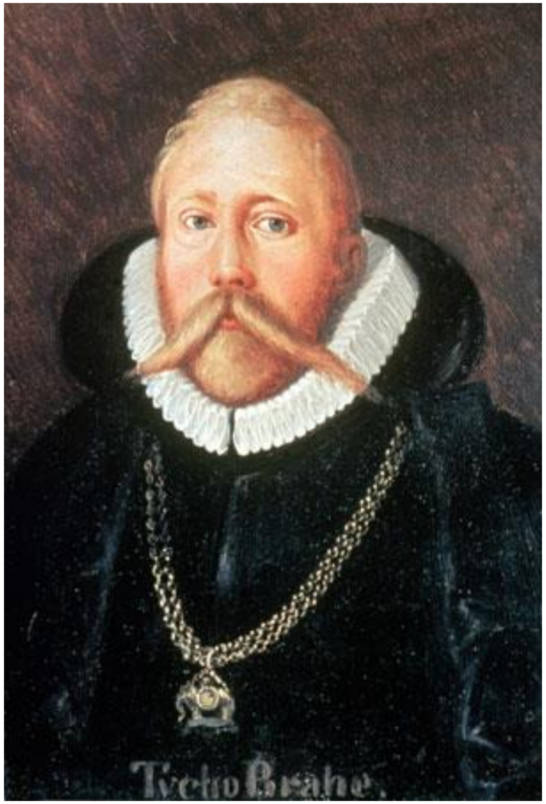
- Kepler’s Laws of planetary motion
- Newton’s law of universal gravitation
- Free fall acceleration on surface of a planet
- Satellite motion
- Gravitational Waves

Brief history of cosmology

Ptolemy (85 - 165)
geocentric theory

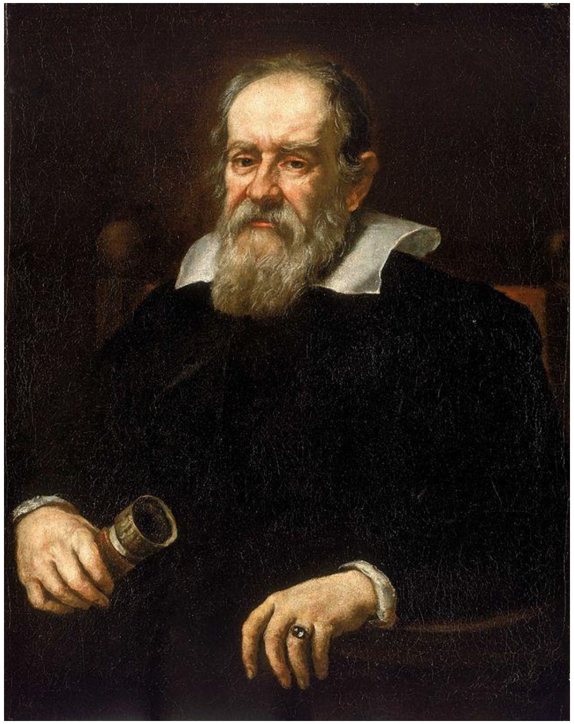


Nicolaus Copernikus
(1473-1543)
heliocentric theory



Tycho Brahe
(1546-1601)

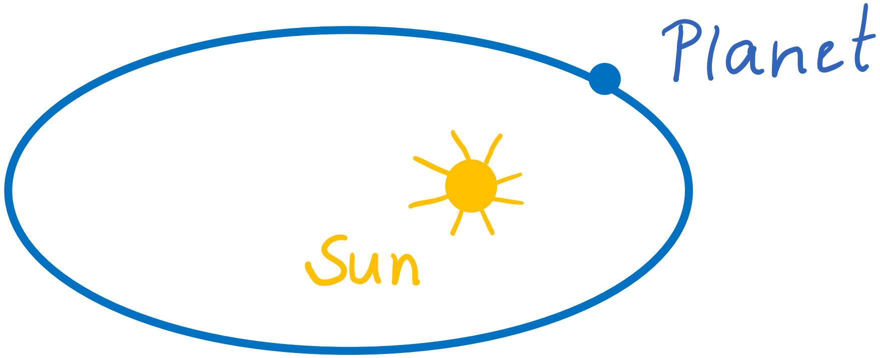
Johannes Kepler
(1571-1630)
Laws of planetary motion



Galileo Galilei (1564 - 1642)
telescope
discovered moons orbiting Jupiter
observed phases of Venus

Kepler's Laws: 1st Law

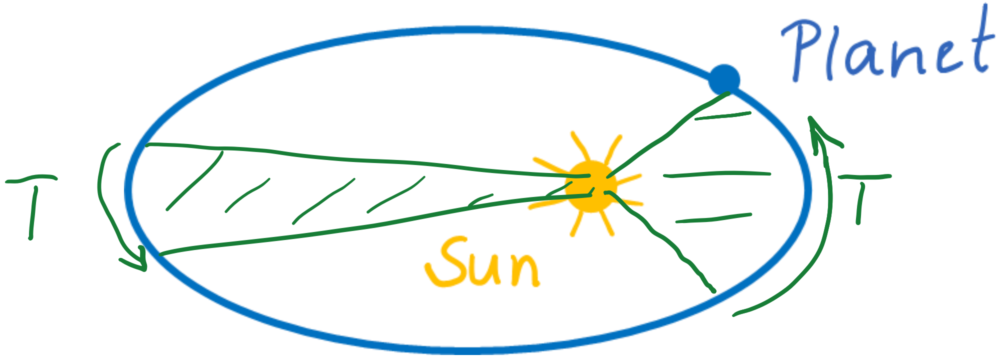
The planets move in elliptical orbits with the sun at one focus of the ellipse.



http://www.walter-fendt.de/ph6en/keplerlaw1_en.htm

Kepler's 2nd Law

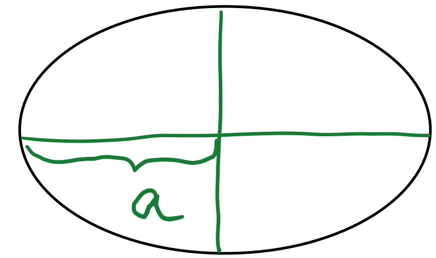
A line drawn between the sun and a planet sweeps out equal areas in equal intervals of time



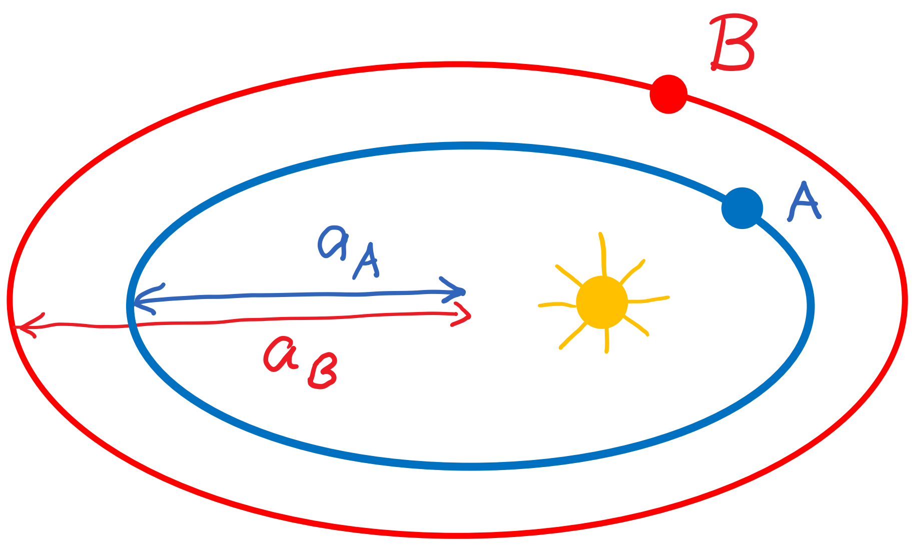
http://www.walter-fendt.de/ph6en/keplerlaw2_en.htm

Kepler's 3rd law

The square of a planet's orbital period is proportional to the cube of the semi-major axis length.



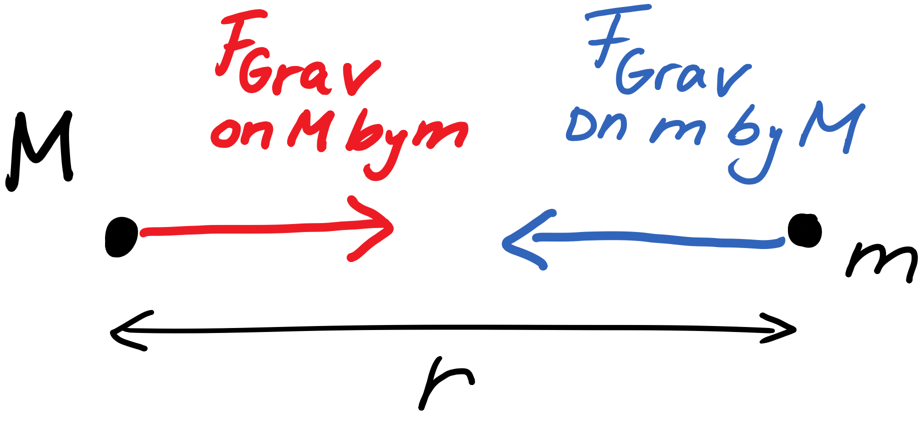
$$T^2 \sim a^3$$



$$\frac{T_A^2}{T_B^2} = \frac{a_A^3}{a_B^3}$$

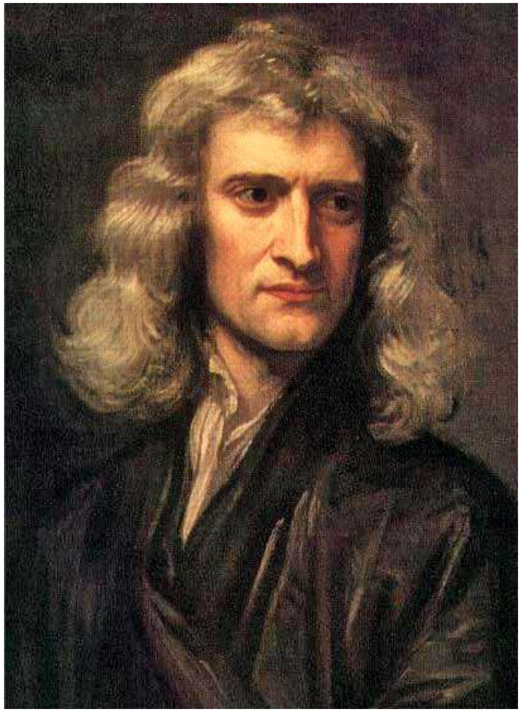
Newton's Law of Universal Gravitation

$$F_{grav} = \frac{GmM}{r^2}$$



$$G = 6.67 \times 10^{-11} Nm^2/kg^2$$

Universal gravitational constant

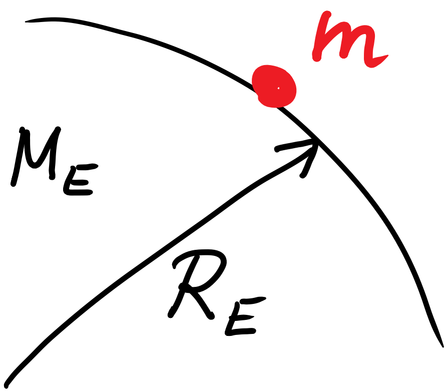


Sir Isaac Newton
(1643 - 1727)

Gravitation near Earth's surface

$$F_{grav} = \frac{GmM}{r^2}$$

But we used $W = F_{grav} = mg$?



Force which Earth exerts on object of mass m located close to Earth's surface:

$$F_{grav} = \frac{GmM_{Earth}}{{R_{Earth}}^2} = mg$$

$$g = \frac{GM_{Earth}}{{R_{Earth}}^2}$$

⟶ Find free-fall acceleration on any planet of mass M and radius R

Find mass of Earth

$$g = \frac{GM_{Earth}}{{R_{Earth}}^2}$$

If G and R_{Earth} known:

$$M_{Earth} = \frac{g{R_{Earth}}^2}{G}$$

G: Cavendish balance

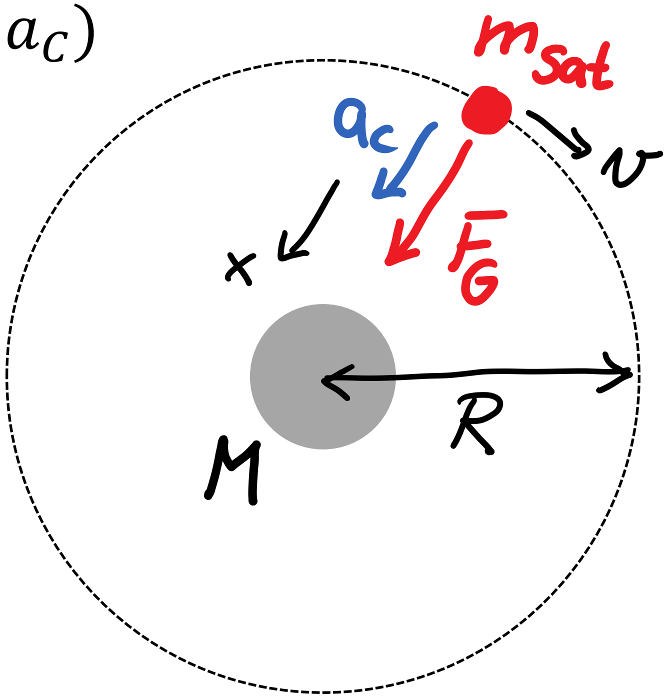
R_E: Eratosthenes 200BC

Satellite Motion

$$\sum F_x = F_{grav,x} = m_{sat}a_x = m_{sat}(+a_C)$$

$$\frac{G m_{sat} M}{R^2} = m_{sat}\frac{v^2}{R}$$

$$\frac{GM}{R} = v^2 = \left(\frac{2\pi R}{T}\right)^2$$



$$T^2 = \frac{4\pi^2}{GM}R^3$$

Kepler's 3rd Law

Example 2

A satellite of mass **m** is orbiting with period **T** in a circular orbit a distance **h** above the surface of a planet. The radius of the planet is **R**.
Find the free-fall acceleration on the planet's surface.