Lecture 18:
Linear momentum and energy.
Center of mass motion.

- Multi-step problems
- Elastic Collisions
- Center of mass motion
- Rocket propulsion

Momentum and energy in multi-step problems

In a quick collision:

- total linear momentum is conserved $\vec{P}_f = \vec{P}_i$
- total mechanical energy is usually **NOT** conserved

$$E_{\boldsymbol{f}} \neq E_{\boldsymbol{i}}$$

Before or after collision:

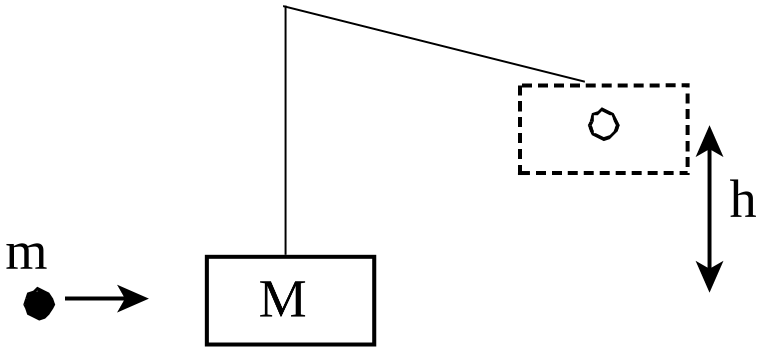
Mechanical energy may be conserved, or
change in mechanical energy may be obtained from

$$E_{\boldsymbol{f}} - E_{\boldsymbol{i}} = W_{other}$$

Example: Ballistic pendulum

A bullet of mass m and unknown speed is fired into a block of mass M that is hanging from two cords. The bullet gets stuck in the block, and the block rises a height h.

What was the initial speed of the bullet?



Energy in collisions

In a quick collision:

- total linear momentum is conserved $\vec{P}_f = \vec{P}_i$
- total mechanical energy is usually **NOT** conserved

$$E_f \neq E_i$$

because non-conservative forces act (deforming metal)

➞ Inelastic collision

Perfectly inelastic: objects stick together after collision

Elastic collision: mechanical energy is conserved

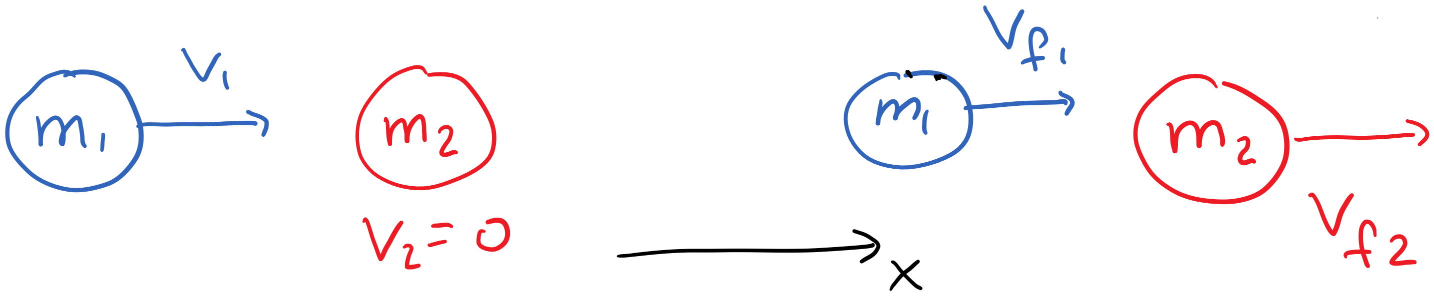
Elastic collisions

Mechanical energy is conserved if only conservative forces act during the collision.

Total linear momentum conserved: $\vec{P}_f = \vec{P}_i$

Total mechanical energy conserved: $E_{\boldsymbol{f}} = E_{\boldsymbol{i}}$

Example: elastic head-on collision with stationary target



x-component of momentum conservation:

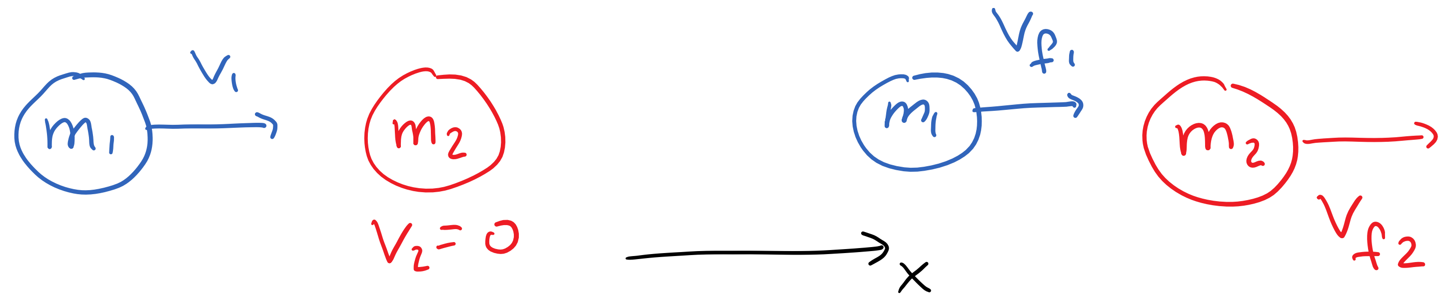
$$p_{1ix} + \cancelto{0}{p_{2ix}} = p_{1fx} + p_{2fx}$$

$$m_1 v_1 = m_1 v_{f\,1x} + m_2 v_{f\,2x}$$

Energy conservation:

$$E_f = E_i$$

$$½ m_1 v_{f1}^2 + ½ m_2 v_{f2}^2 = ½ m_1 v_1^2$$



After some algebra:

$$v_{f1x} = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$v_{f2x} = \frac{2m_1}{m_1 + m_2} v_1$$

Special cases:

$m_1 \ll m_2$ Ping-pong ball hits stationary cannon ball

$m_1 \gg m_2$ Cannon ball hits ping-pong ball

$m_1 = m_2$ Newton's cradle

$$v_{f1x} = \frac{m_1 - m_2}{m_1 + m_2} v_1 \qquad v_{f2x} = \frac{2m_1}{m_1 + m_2} v_1$$

$m_1 \ll m_2$ Ping-pong ball hits stationary cannon ball

$v_{f1x} \approx \frac{-m_2}{m_2} v_1 = -v_1$ $v_{f2x} \approx 2\frac{m_1}{m_2} v_1$

$m_1 \gg m_2$ Cannon ball hits ping-pong ball

$v_{f1x} \approx \frac{m_1}{m_1} v_1 = v_1$ $v_{f2x} \approx \frac{2m_1}{m_1} v_1 = 2v_1$

$m_1 = m_2$ Newton's cradle

$v_{f1x} = 0$ $v_{f2x} = \frac{2m}{m+m} v_1 = v_1$

[*Rosencrantz and Guildenstern are dead*]

General elastic collisions:
two-dimensional, off-center,
particles move away at angles

3 equations:

x-component of Momentum Conservation:

$$P_{fx} = P_{ix}$$

y-component of Momentum Conservation:

$$P_{fy} = P_{iy}$$

Conservation of mechanical energy:

$$E_{\boldsymbol{f}} = E_{\boldsymbol{i}}$$

Center of Mass: Definition

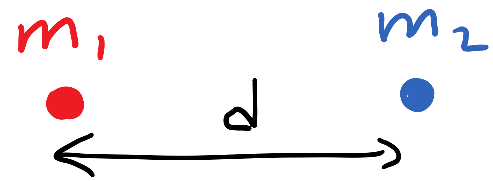
$$M_{tot}\vec{r}_{CM} = \sum_n m_n \vec{r}_n$$

$$X_{CM} = \frac{1}{M_{tot}} \sum_n m_n x_n \qquad Y_{CM} = \frac{1}{M_{tot}} \sum_n m_n y_n$$

Continuous object: integration
If object has line of symmetry: CM lies on it.
Same amount of mass on both sides.

Center of Mass: Example

$$X_{CM} = \frac{1}{M_{tot}} \sum_n m_n x_n$$



Center of mass and momentum

$$M_{tot}\vec{r}_{CM} = \sum_n m_n \vec{r}_n$$

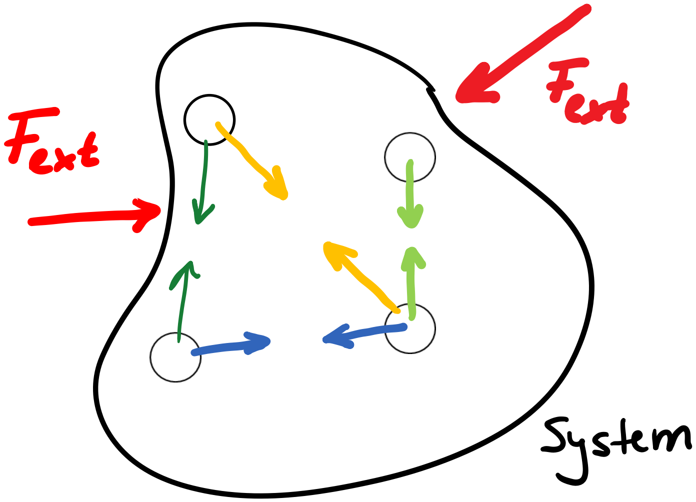
$$M_{tot}\frac{d\vec{r}_{CM}}{dt} = M_{tot}\vec{v}_{CM} = \sum_n m_n \vec{v}_n = \sum_n \vec{p}_n = \vec{P}$$

$$M_{tot}\frac{d\vec{v}_{CM}}{dt} = M_{tot}\vec{a}_{CM} = \frac{d\vec{P}}{dt} = \sum \vec{F}$$

Center of mass and external forces

$$M_{tot}\vec{a}_{CM} = \frac{d\vec{P}}{dt} = \sum \vec{F}$$

Particles in system interact
Internal forces occur in
action-reaction pairs, cancel.
Only external forces remain.

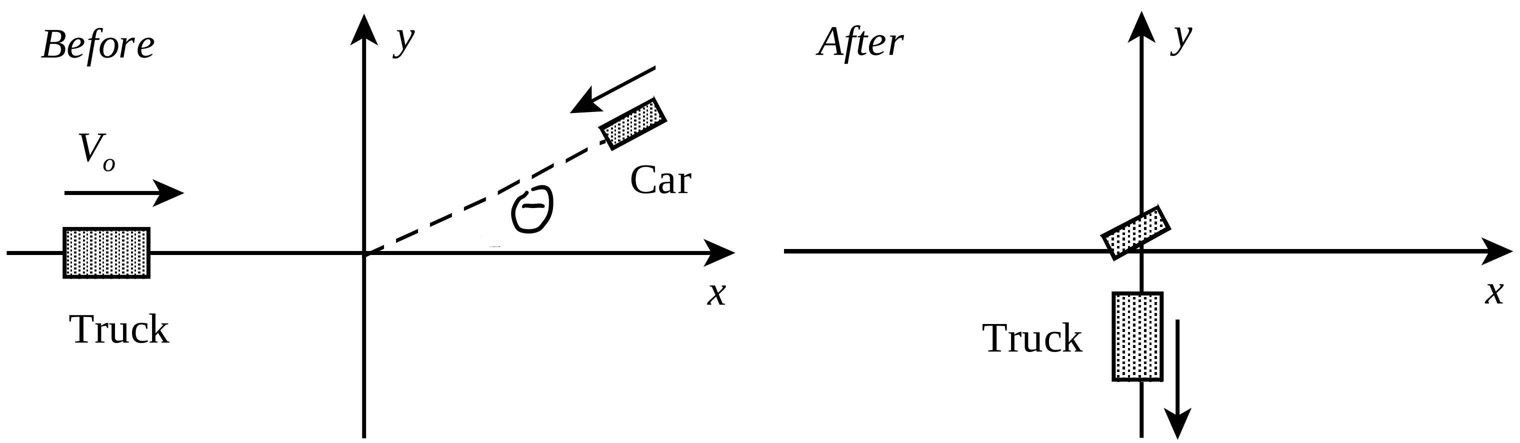


$$\boxed{\sum \vec{F}_{ext} = M_{tot}\vec{a}_{CM}}$$

[Demo: Center-of-mass motion]

Discussion question

A truck is moving with velocity V_o along the positive x-direction. It is struck by a car which had been moving towards it at an angle θ with respect to the x-axis. As a result of the collision, the car is brought to a stop and the truck ends up sliding in the negative y-direction. The truck is twice as heavy as the car. (Example from last lecture) Find the **x-component of the velocity of the center of mass** of truck and car before the collision.



Another discussion question

You find yourself in the middle of a frictionless frozen lake. How do you get to the shore?

Throw something

Same principle as rocket motion

Demo: rocket cart