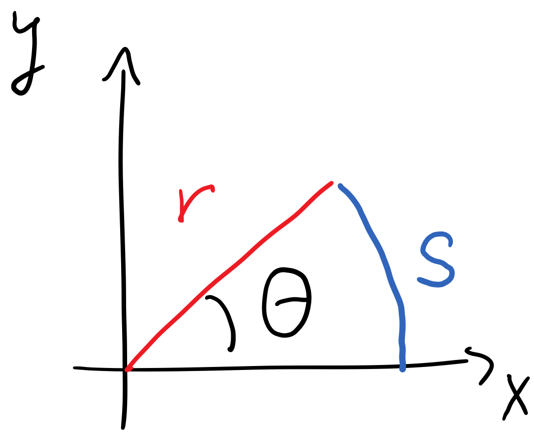
Lecture 20:
Rotational kinematics and energetics

- Angular quantities
- Rolling without slipping
- Rotational kinetic energy
- Moment of inertia
- Energy problems

Angle measurement in radians

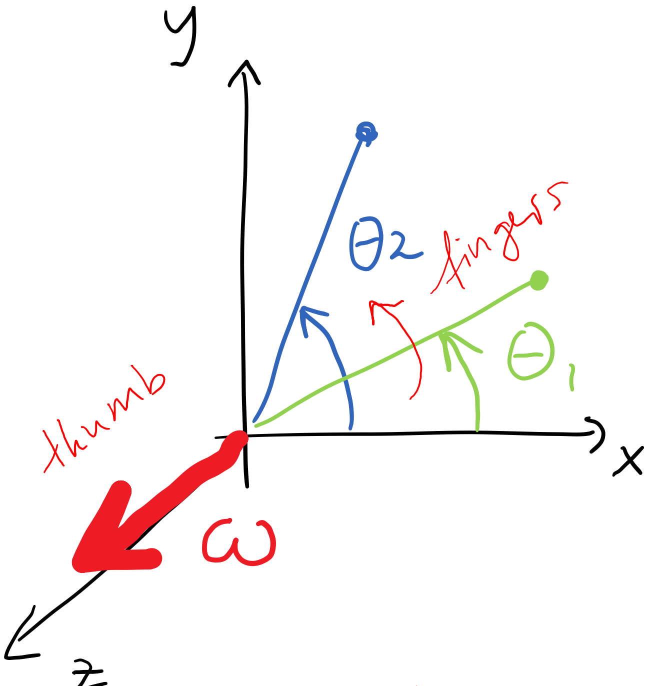


θ(in radians) **=** s/r

s is an arc of a circle of radius r

Complete circle:
$s = 2\pi r$, $\theta = 2\pi$

Angular Kinematic Vectors



right hand
thumb rule

Position: θ

Angular displacement:

$$\Delta\theta = \theta_2 - \theta_1$$

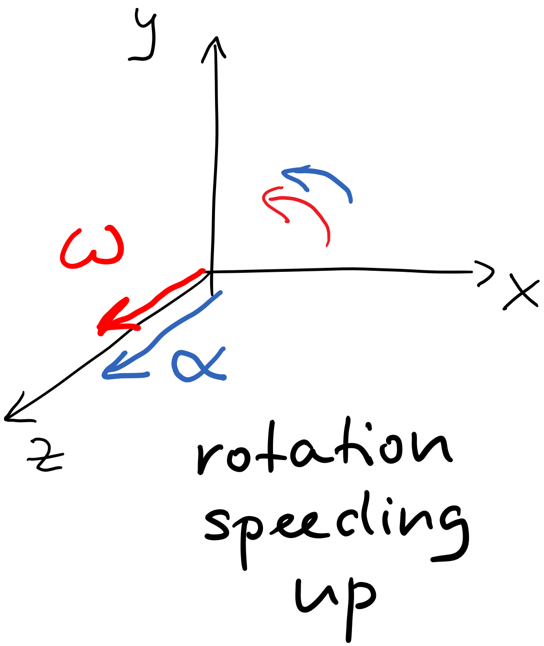
Angular velocity:

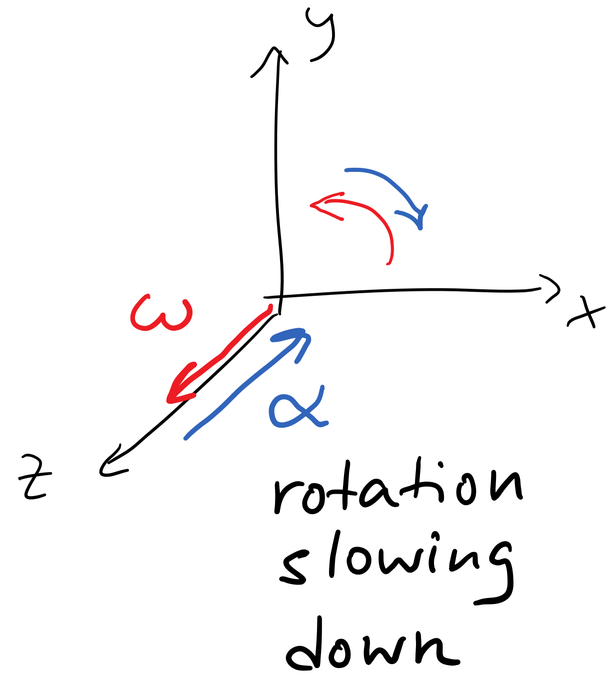
$$\omega_z = \frac{d\theta}{dt}$$

Angular velocity vector perpendicular to the plane of rotation

Angular acceleration

$$\alpha_z = \frac{d\omega_z}{dt}$$





Angular kinematics

For constant angular acceleration:

Compare constant a_x:

$$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$$

$$x = x_0 + v_{0x}t + ½a_x t^2$$

$$v_x = v_{0x} + a_x t$$

$$\omega_z = \omega_{0z} + \alpha_z t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$\omega_z{}^2 = \omega_{0z}{}^2 + 2\,\alpha_z(\theta - \theta_0)$$

$\theta \longleftrightarrow x$

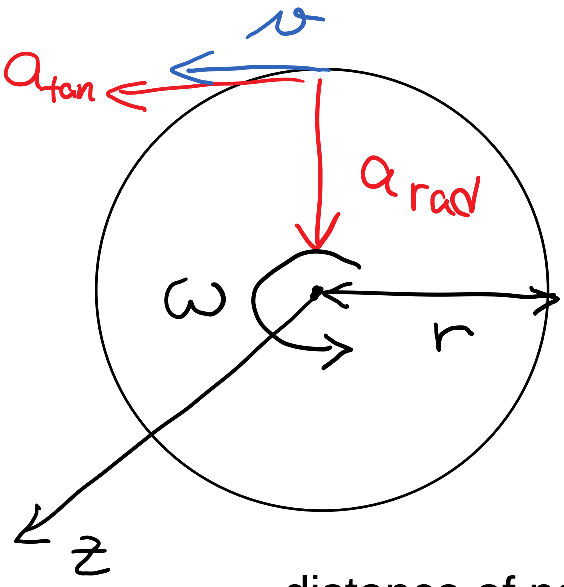
$\omega_z \longleftrightarrow v_x$

$\alpha_z \longleftrightarrow a_x$

Relationship between angular and linear motion

linear velocity tangent to circular path:

$$v = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt}$$



$$\boxed{v = \omega r}$$

$$\boxed{a_{tan} = \alpha r}$$

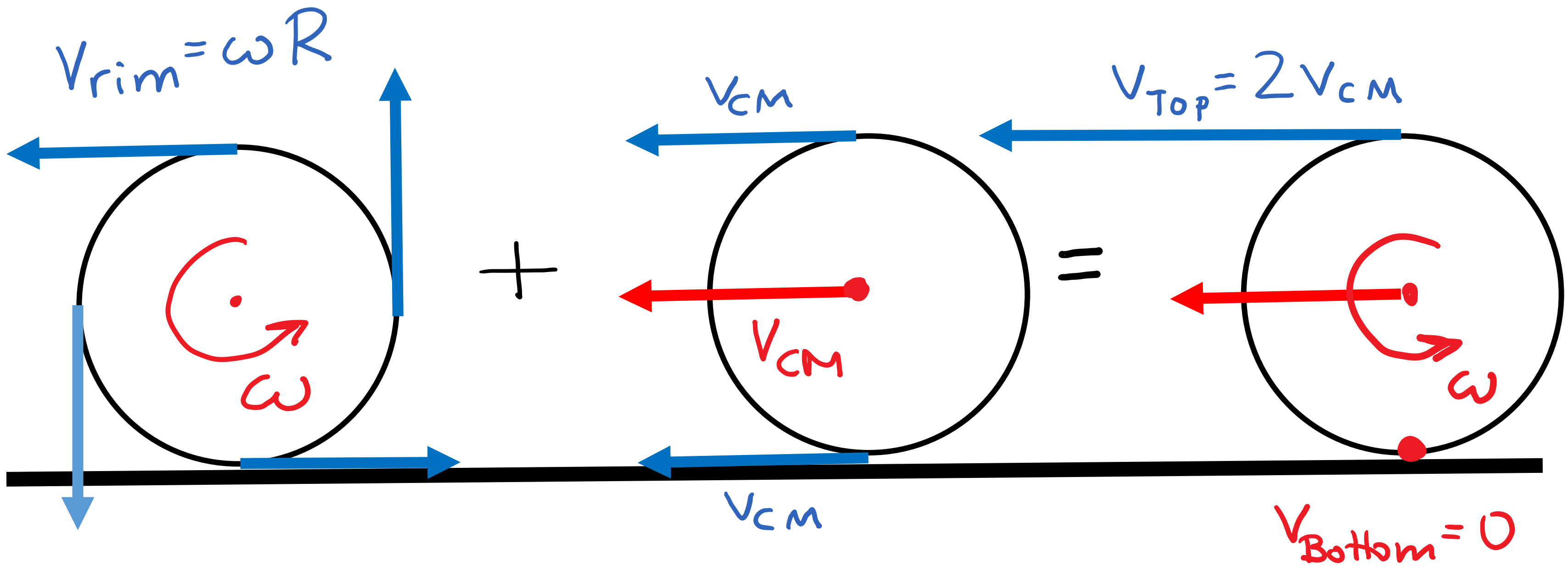
$$a_{rad} = \frac{v^2}{r} = \omega^2 r$$

r distance of particle from rotational axis

Angular speed ω is the same for all points of a rigid rotating body!

Rolling without slipping

Rotation about CM + Translation of CM = Rolling w/o slipping



If $v_{rim} = v_{CM}$: $v_{bottom} = 0$ no slipping

$\boxed{v_{CM} = \omega R}$

$\boxed{a_{CM} = \alpha R}$

Rotational kinetic energy

$$K_{rotation} = \sum ½ m_n v_n^2 = \sum ½ m_n \, (\omega_n r_n)^2$$

$$= \sum ½ m_n \, (\omega \, r_n)^2 = ½ \, [\, \sum m_n r_n^2] \, \omega^2$$

$$\boxed{K_{rot} = ½ \, I \, \omega^2}$$

with $\boxed{I = \sum_n m_n r_n{}^2}$ Moment of inertia

Moment of inertia

$$I = \sum_n m_n {r_n}^2$$

r_n perpendicular distance from axis

Continuous objects:

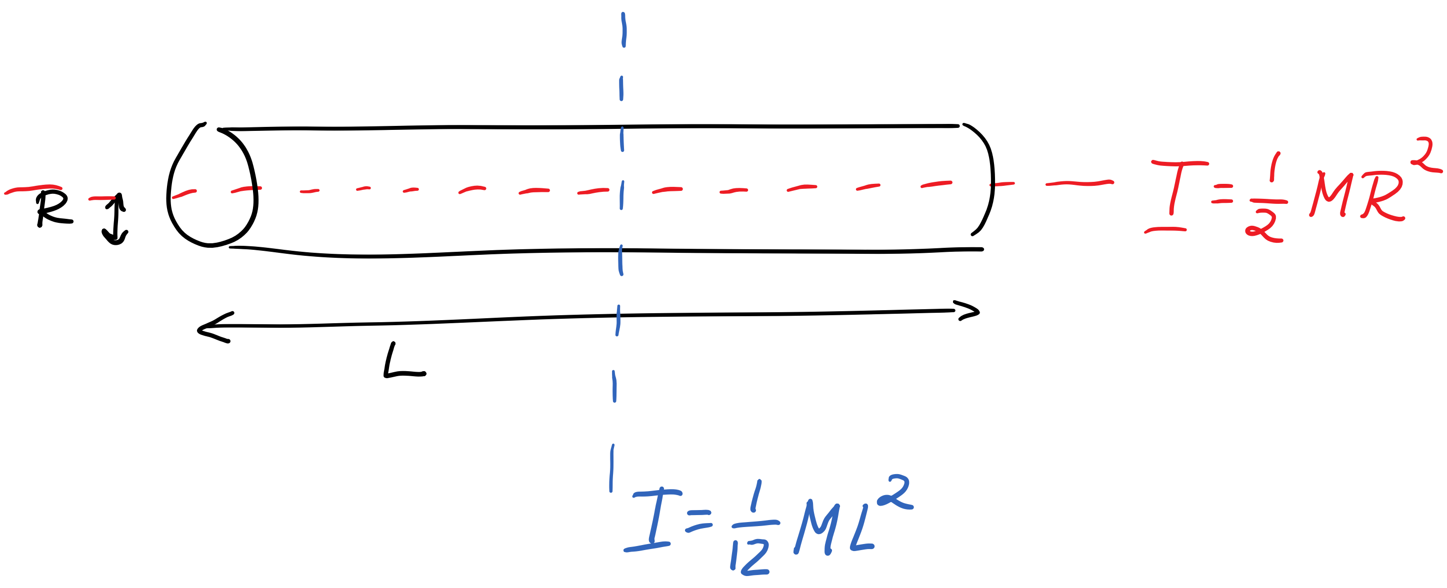
$$I = \sum_n m_n {r_n}^2 \longrightarrow \int r^2 dm$$

Table p. 291

* No calculation of moments of inertia by integration in this course.

Properties of the moment of inertia

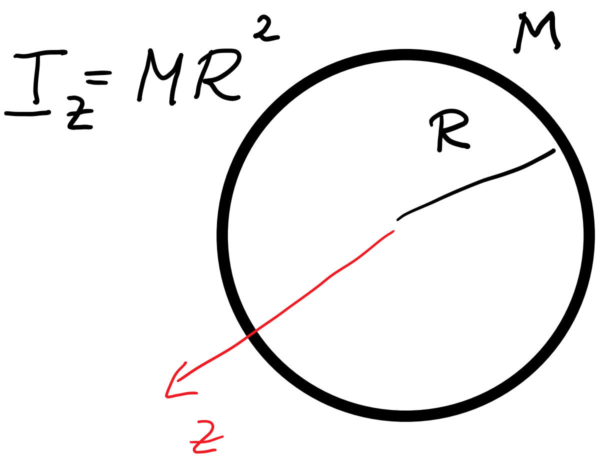
1. Moment of inertia I depends upon the axis of rotation. Different I for different axes of same object:

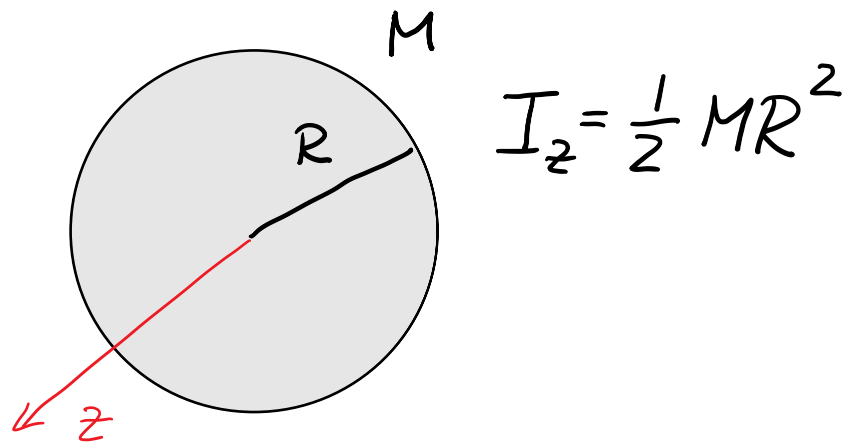


Properties of the moment of inertia

2. The more mass is farther from the axis of rotation, the greater the moment of inertia.

Example:
Hoop and solid disk of the same radius R and mass M.



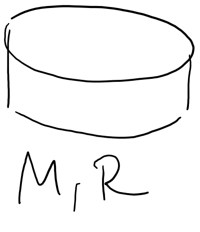


Properties of the moment of inertia

3. It does not matter where mass is along the rotation axis, only radial distance r_n from the axis counts.

Example:

I for cylinder of mass M and radius R: $I = ½MR^2$
same for long cylinders and short disks

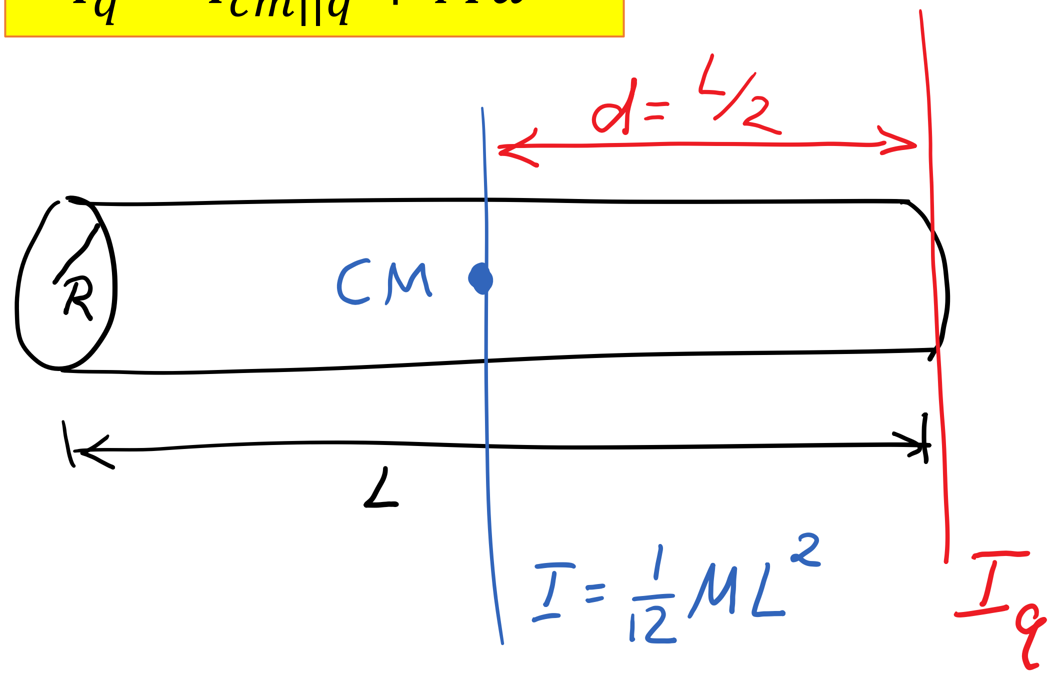


Parallel axis theorem

distance $CM \leftrightarrow q$

$$I_q = I_{cm||q} + Md^2$$

Example:



$$I_q = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \underbrace{\frac{1}{12}ML^2 + \frac{1}{4}ML^2}_{\frac{1+3}{12}ML^2} = \frac{1}{3}ML^2$$

Rotation and translation

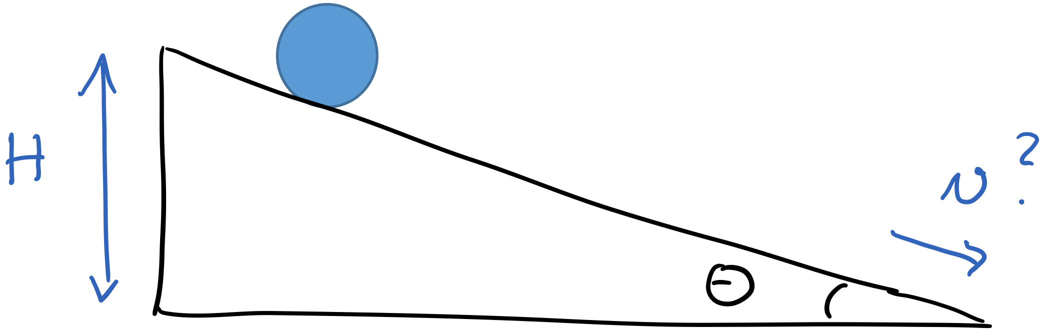
$$K = K_{trans} + K_{rot} = ½ M v_{CM}^2 + ½ I \omega^2$$

with $$v_{CM} = \omega R$$

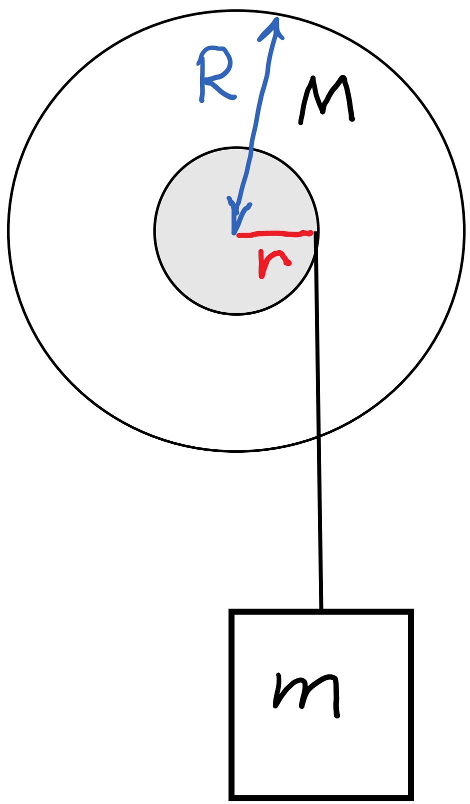
Hoop-disk-race

Demo: Race of hoop and disk down incline

Example: An object of mass M, radius R and moment of inertia I is released from rest and is rolling down incline that makes an angle θ with the horizontal. What is the speed when the object has descended a vertical distance H?



Example with coupled objects



A small disk of radius r is glued onto a large disk of radius R that is mounted on a fixed axle through its center. The combined moment of inertia of the disks is I. A string is wrapped around the edge of the small disk, and a box of mass m is tied to the end of the string. The string does not slip on the disk. The box is released from rest.

Find the speed of the box after it has descended a distance d.