Lecture 21:
Torque

- Cross product
- Torque
- Relationship between torque and angular acceleration
- Problem solving

What causes rotation?

Demo: bolt and wrench

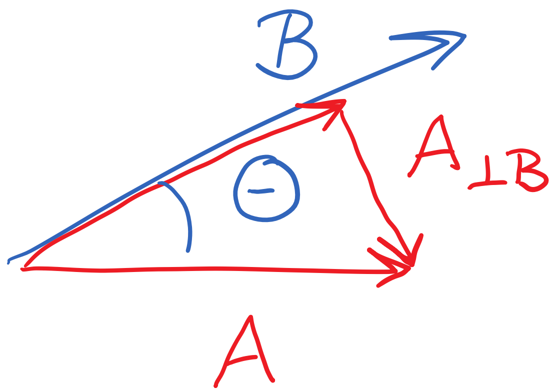
Need:

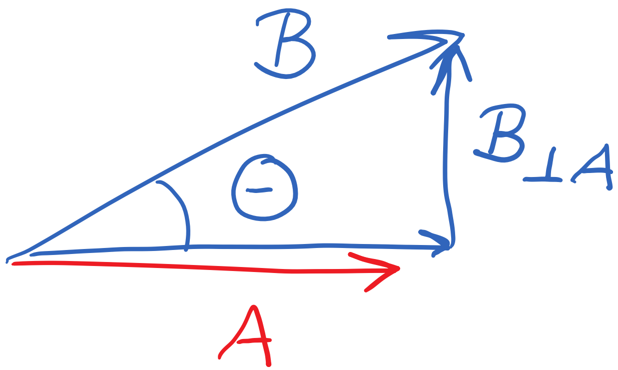
- Force
- Distance
- Perpendicular component

Vector cross product: magnitude

$$\vec{A} \times \vec{B} = \vec{C}$$

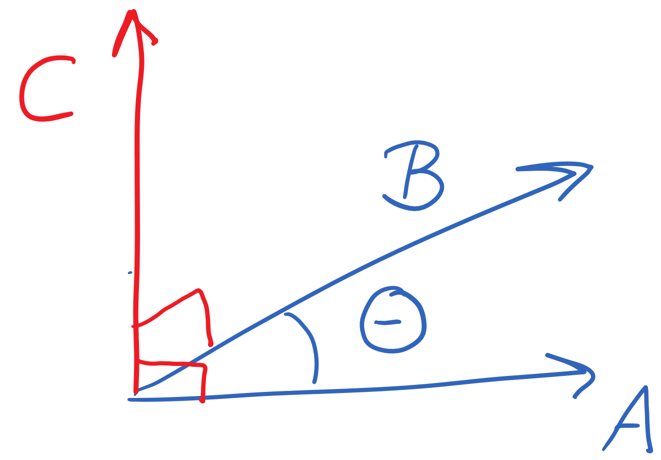
$$C = AB\,sin\theta = A_{\perp B}B = AB_{\perp A}$$





Vector cross product: Direction

$$\vec{A} \times \vec{B} = \vec{C}$$



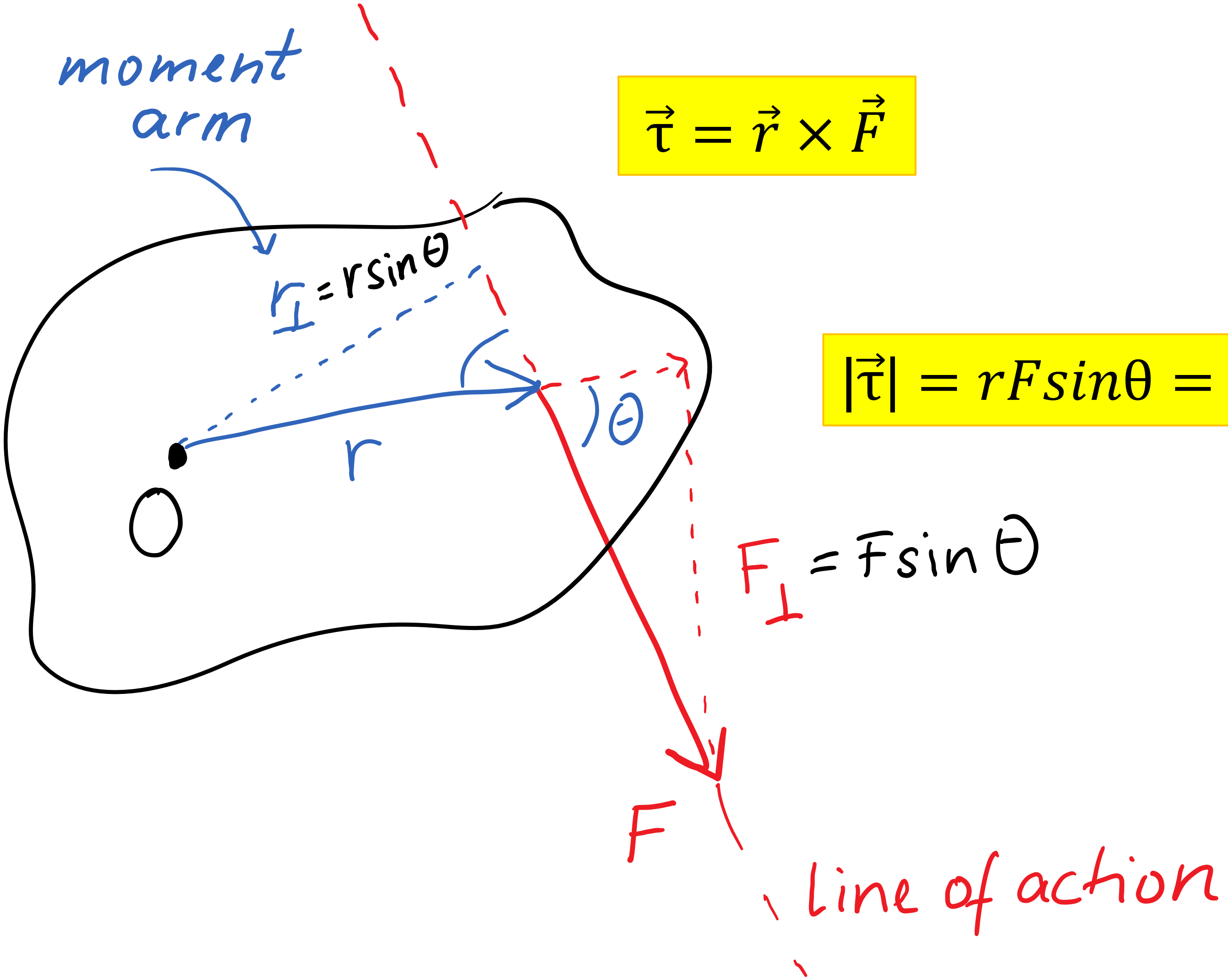
$\vec{C}$ is perpendicular to both $\vec{A}$ and $\vec{B}$

Direction: right hand rule

thumb × index finger = middle finger

Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$



$$|\vec{\tau}| = rF\sin\theta = rF_\perp = r_\perp F$$

Direction of torque

Right hand rule: $\vec{r} \times \vec{F} = \vec{\tau}$

thumb × index finger = middle finger

or easier:

If force tends to produce rotation in the positive z-direction, τ_z is *positive*:

$$\tau_z = +r\,F\sin(\theta)$$

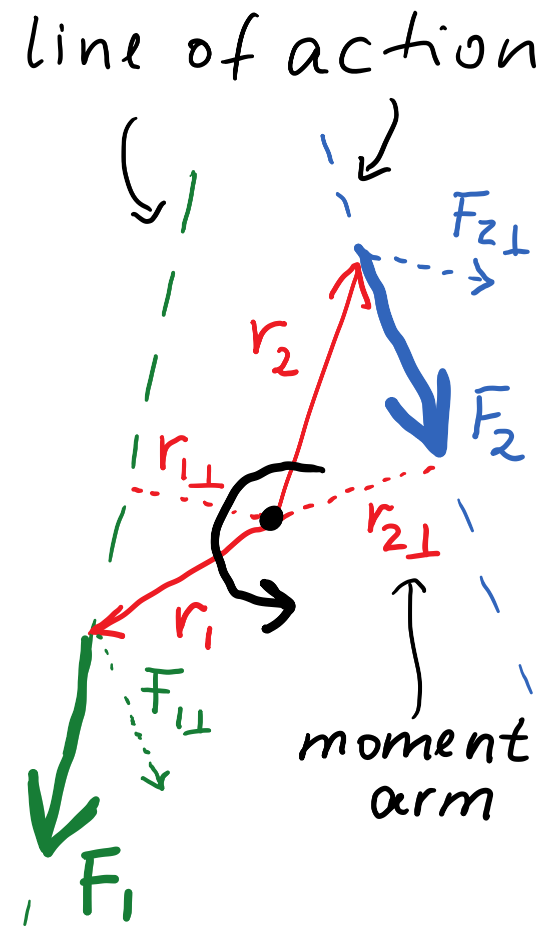
If force tends to produce rotation in the negative z-direction, τ_z is *negative*:

$$\tau_z = -\,r\,F\sin(\theta)$$

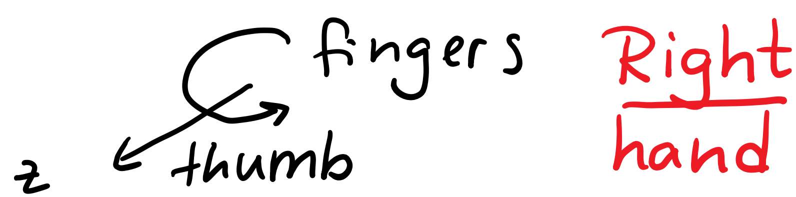
Indicate positive z-direction through curved arrow.

$$\tau_{1z} = +F_1 r_{1\perp} = +F_{1\perp} r_1$$

$$\tau_{2z} = -F_2 r_{2\perp} = -F_{2\perp} r_2$$



curved arrow ≙ positive z-direction



Angular acceleration of rigid object

Rigid object that can rotate about z-axis.
I_z moment of inertia about z-axis

$$\sum \tau_z = I_z \alpha_z$$

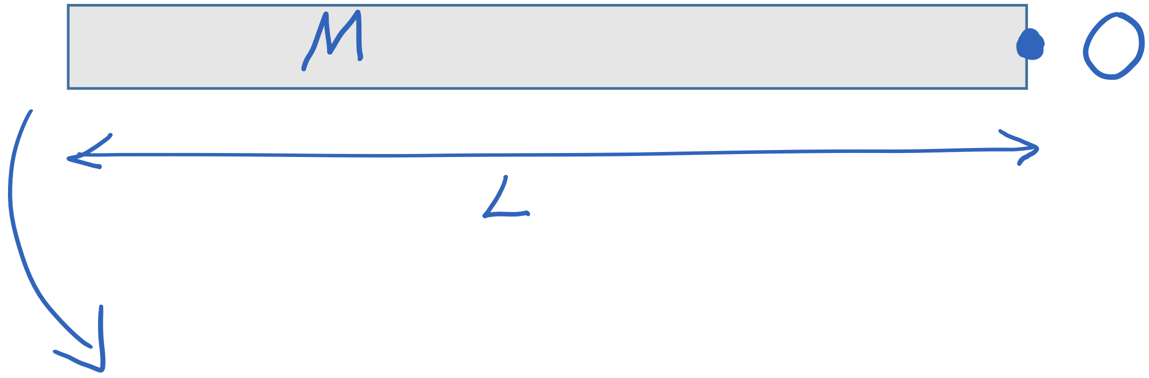
Compare to $\Sigma F_x = m a_x$

Begin with extended free-body diagram that shows forces and where they act on the object

Example 1:

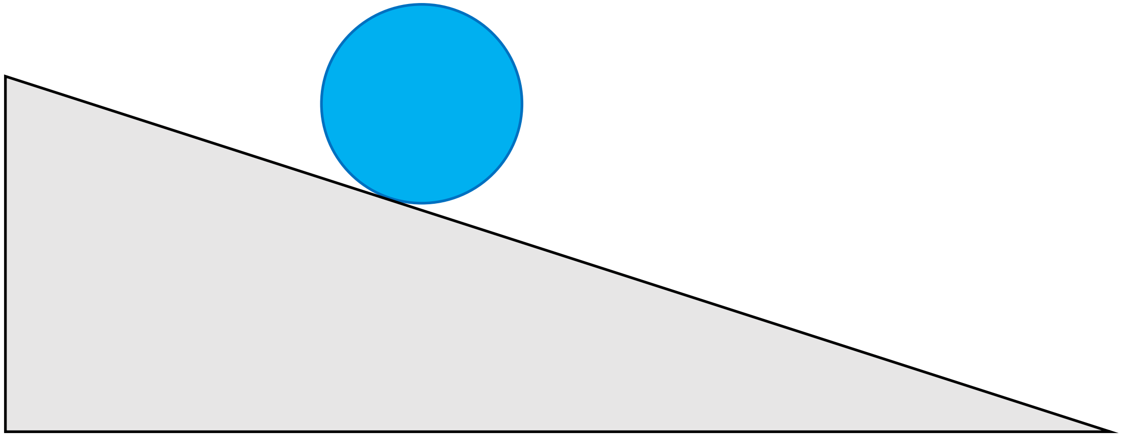
A uniform bar of length L and mass M can freely rotate about frictionless horizontal axis *O* at its end. The bar is initially in a horizontal position, is released from rest, and swings down under the influence of gravity. What is the initial angular acceleration of the bar just after it is released from rest?

$I_{bar} = ⅓\, M\, L^2$ about *O*

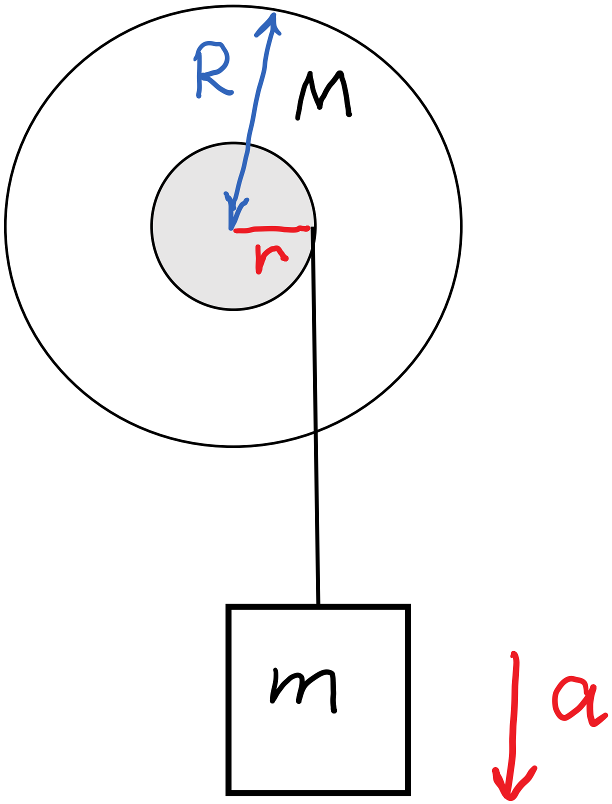


Example 2: Rolling w/o slipping

An object of mass M, radius R and moment of inertia I is rolling without slipping down incline that makes an angle θ with the horizontal. Derive an expression for the object's linear acceleration.



Example 3: Coupled objects



A small disk of radius r is glued onto a large disk of radius R that is mounted on a fixed axle through its center. The combined moment of inertia of the disks is I. A string is wrapped around the edge of the small disk, and a box of mass m is tied to the end of the string. The string does not slip on the disk.

Find the acceleration of the box after it is released from rest.