

Lecture 25: Problem solving review

For test 3

Test Structure

In-person exam, 60 minutes

- 5 multiple choice questions, 10 points each
- 3 worked out problems, 50 points each, partial credit
- Equation sheet stapled to exam package
- **Test rooms:** <https://web.mst.edu/~vojtaa/engphys1/handouts/Testrooms.pdf>

Concepts: Rotation

- Relationship angular- linear quantities
- Rolling without slipping
- Moment of Inertia, parallel axis theorem
- Rotational kinetic energy
- Torque
- Angular dynamics
- Static equilibrium
- Angular momentum conservation

Concepts: Oscillation

- Position, velocity, acceleration in SHO
- Period of a simple pendulum
- Period of mass on a spring
- Physical pendulum
- Energetics

How to identify type of problem?

If object is not moving at all, or moving at constant velocity:
no acceleration, no angular acceleration, **Static Equilibrium**

If external forces and torques act on object(s):

- acceleration and angular acceleration can be obtained from **sum of forces/sum of torques**
- speed can be obtained from **Energy/Work**
- speed can be obtained from acceleration+kinematics only **if forces/torques are constant**

If **no** external torques act (e.g. rotational collisions):

Angular momentum conserved, mechanical energy changes

Energy problems

Identify motion of each object:

Only translating $\rightarrow K = K_{\text{trans}} = \frac{1}{2} M v^2$

Only rotating $\rightarrow K = K_{\text{rot}} = \frac{1}{2} I \omega^2$

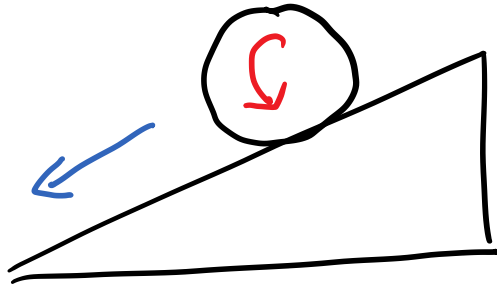
Both rotating and translating $\rightarrow K = K_{\text{trans}} + K_{\text{rot}}$
 $K = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$

No slipping: relate v and ω

$$v = \omega R$$

Identify other energies

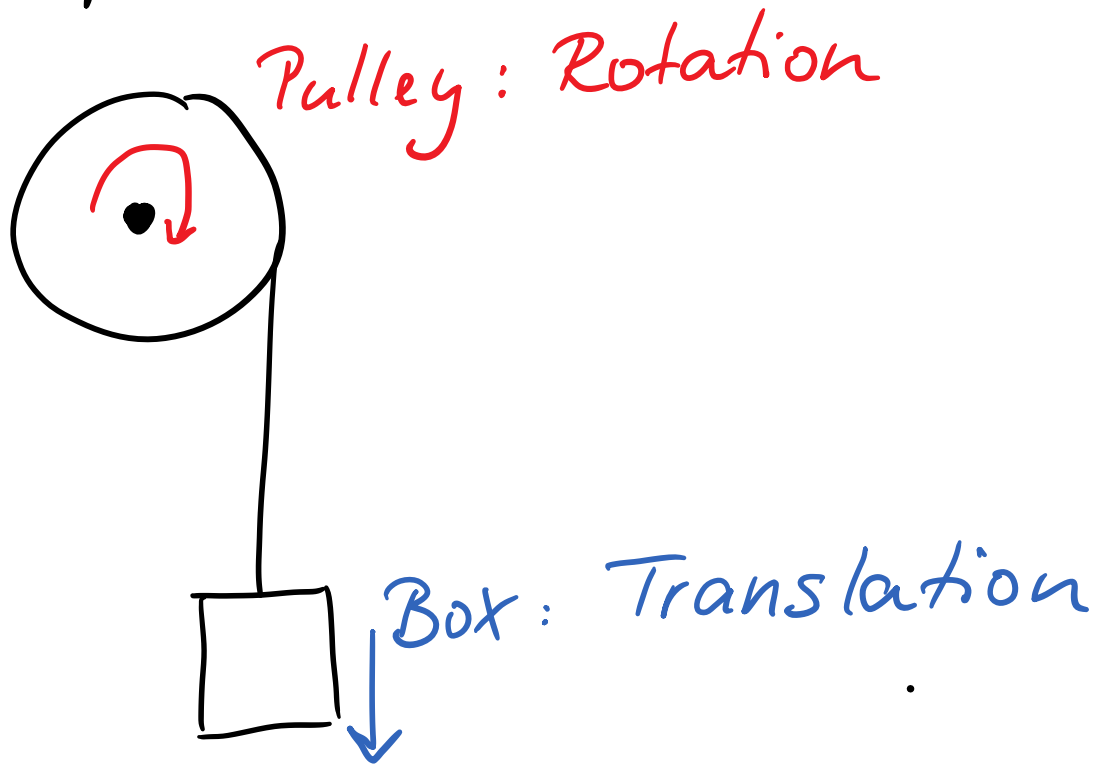
Ball rolling down incline



Translation + Rotation

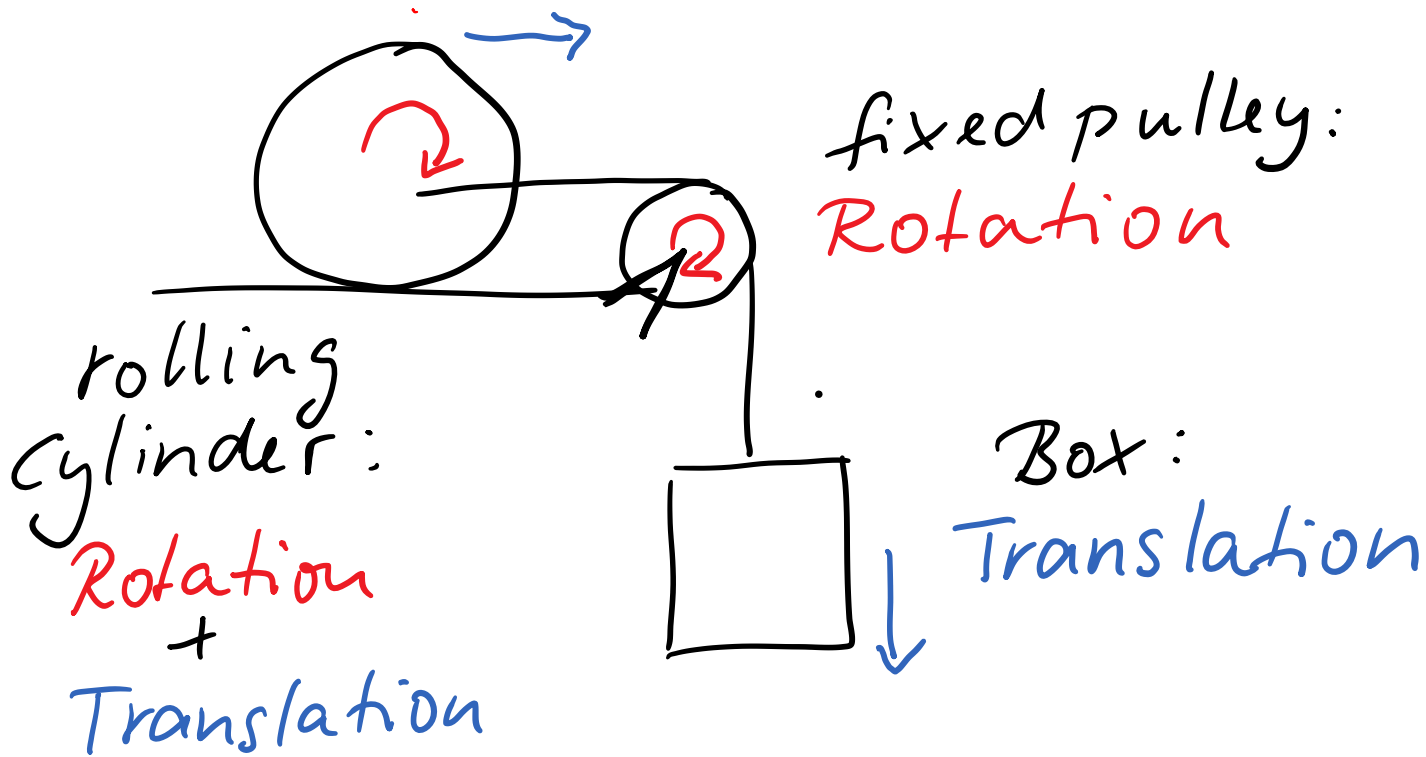
+ potential energy of gravity

Box suspended from fixed pulley,
rope unwinds, box descends:



+ potential energy of gravity for box

HW 10.87 :



+ potential energy of gravity for box

Example 1

You have a pumpkin of mass M and radius R . The pumpkin has the **shape of a sphere**, but it is **not uniform** inside; so you do not know its moment of inertia.

In order to determine the moment of inertia, you decide to roll the pumpkin down an incline that makes an angle θ with the horizontal. The pumpkin starts from rest and rolls without slipping. When it has descended a vertical height H , it has

acquired a speed $V = \sqrt{\frac{5}{4}gH}$.

Use energy methods to derive an expression for the moment of inertia of the pumpkin.

Forces and torques

Draw extended free-body diagram

Identify motion of object:

Object can rotate →

$$\sum \tau_z = I \alpha_z$$

Object can translate →

$$\sum \vec{F} = M \vec{a}$$

Object can do both →

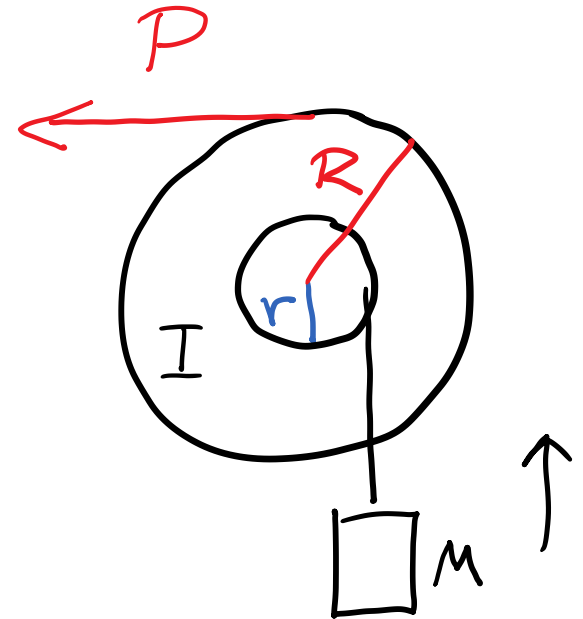
$$\begin{aligned} \sum \tau_z &= I \alpha_z \\ \sum \vec{F} &= M \vec{a} \end{aligned}$$

No slipping: relate α and a $a = \alpha R$

Example 2

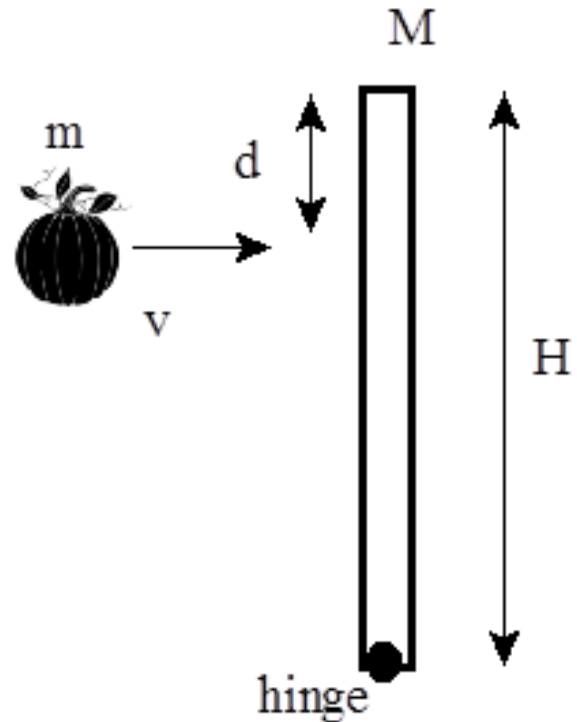
A yo-yo shaped device (moment of inertia about center is I) is mounted on a horizontal frictionless axle through its center is used to lift a load of mass M . The outer radius of the device is R , the radius of the hub is r . A constant horizontal force of magnitude P is applied to a rope wrapped around the outside of the device, the box, which is suspended from a rope wrapped around the hub, accelerates upwards. The ropes do not slip.

Derive an expression for the acceleration of the box.



Example 3

In a pumpkin throwing contest, a small pumpkin of mass m is moving horizontally with speed v when it hits a vertical pole of length H and mass M that is pivoted at a hinge at its foot. The pumpkin hits the pole a distance d from its upper end and becomes impaled on a long nail that is sticking out of the pole. The pumpkin is small enough to be treated as a point mass.



Derive an expression for the angular speed of the system after the collision.

A statics example

A box of weight $\frac{1}{2}W$ hangs from the top end of a uniform post that is pivoted on the ground at an angle θ with respect to the vertical. A horizontal rope is tied to the post a quarter of the way from the top end. The length of the post is L and its weight is W . The tension in the horizontal rope is $2W$.

Derive an expression for angle θ in terms of system parameters. Simplify your answer.

