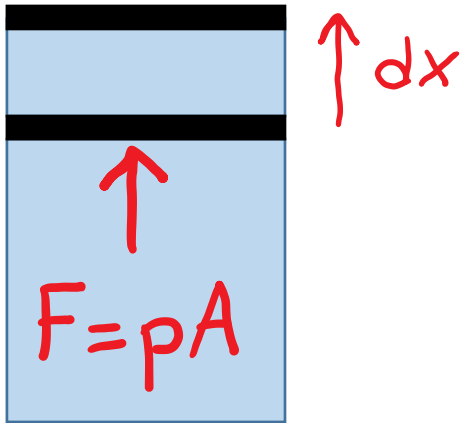


Lecture 29: 1st Law of Thermodynamics

- thermodynamic work
- 1st law of Thermodynamics
- equation of state of the ideal gas
- Isochoric, isobaric, and isothermal process in ideal gas

Thermodynamic work



$$\begin{aligned}dW &= F dx \\ &= pA dx \\ &\quad \underbrace{\hspace{1cm}} \\ &\quad dV\end{aligned}$$

$$dW = p dV$$

$$W = \int_i^f p(V, T) dV$$

work done **by** the gas

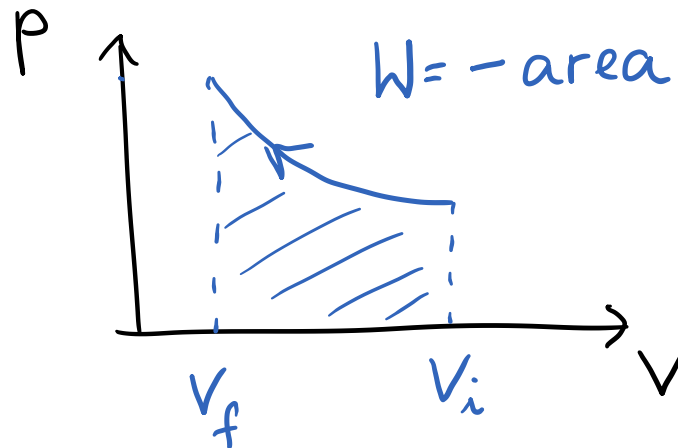
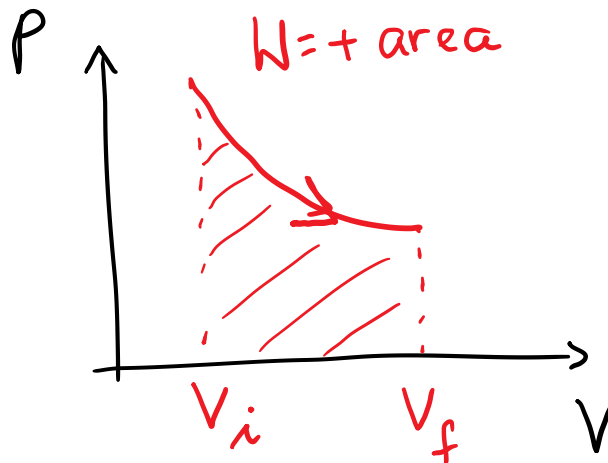
Work and p-V curve

$$W = \int_i^f p(V, T) dV$$

work done **by** the gas
 \Rightarrow area under the $p - V$ -curve

$W > 0$ if gas **expands** ($\Delta V > 0$)

$W < 0$ if gas is **compressed** ($\Delta V < 0$)



First Law of Thermodynamics

The internal energy of a system can change:

- Heat flow in or out of system
- Work done on or by gas

$$\Delta U = Q - W$$

Q : net heat flowing into the system

W : work done **by** the system

ΔU is completely determined by initial and final values of the state variables
(even though W and Q depend on the process!)

Internal energy

Internal energy U of the system
= kinetic energies of particles
+ potential energy of interaction

Value of U depends on state of system only.

State is characterized by state variables such as:
temperature, pressure, volume, phase

Ideal gas

- particles do not interact with each other
- only elastic collisions between particles and with walls

Real gas can be treated as ideal if:
low density, low pressure, high temperature

State variables are related by:

$$pV = nRT$$

n number of moles

R universal gas constant

$R=8.315\text{J}/(\text{mol}\cdot\text{K})$

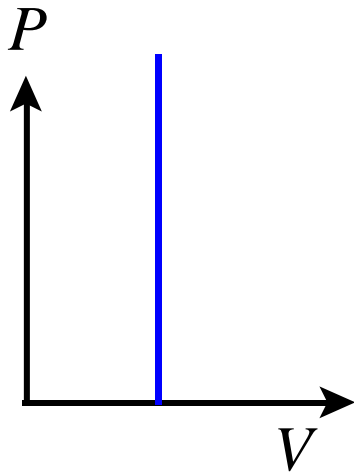
No interactions between particles

$\Rightarrow U$ consists only of kinetic energies of particles

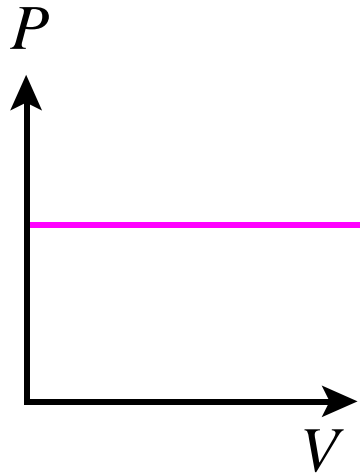
$\Rightarrow U$ depends only on temperature: $U = U(T)$ *

* Only true
for ideal gas!

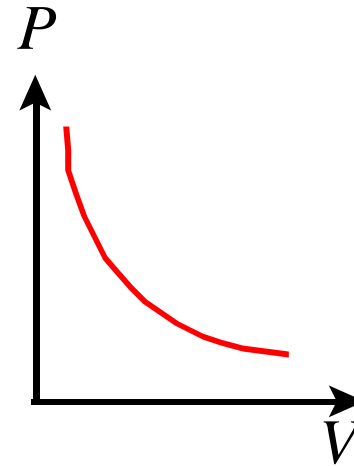
Important processes for the ideal gas



$V = \text{const.}$
isochoric



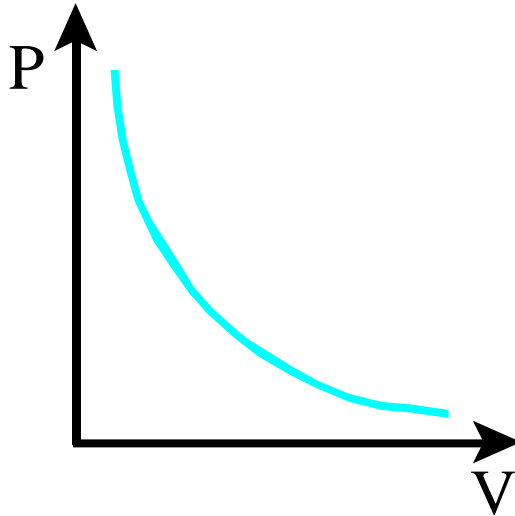
$p = \text{const.}$
isobaric



$T = \text{const.}$
isothermal

$$p \cdot V = nRT = \text{const}$$
$$p \sim \frac{1}{V}$$

Adiabatic Process



$$Q = 0$$

$$pV^\gamma = \text{const}$$

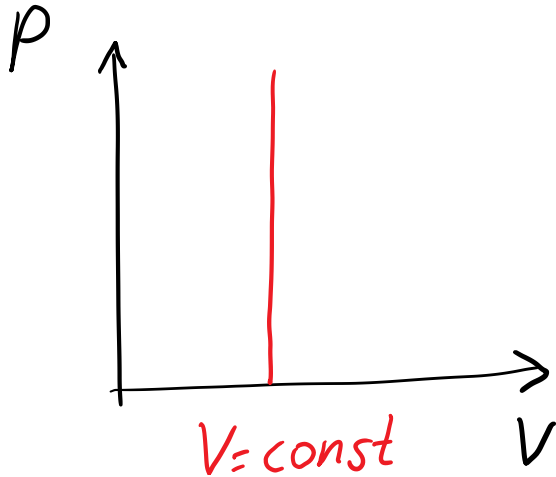
$$\gamma = c_p/c_v$$

$$\gamma = 1.67 \text{ (monoatomic)}$$

$$\text{or } 1.40 \text{ (diatomic)}$$

$\gamma > 1$ steeper than isothermal

Isochoric process for the ideal gas



$$W = \int p \, dV = 0$$

$$Q = n c_v \Delta T$$

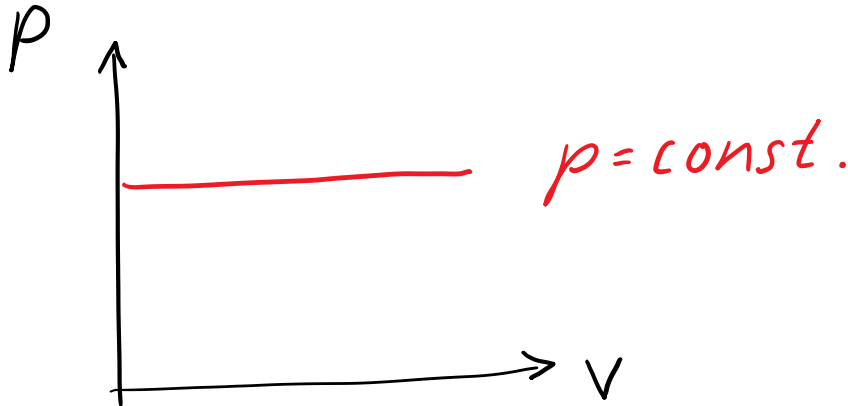
↑ specific heat at constant volume

monoatomic $c_v = \frac{3}{2} R$

diatomic $c_v = \frac{5}{2} R$

$$\Delta U = Q - W = n c_v \Delta T$$

Isobaric process for the ideal gas



$$\Delta U = Q - W$$

$$\Delta U = n C_p \Delta T - p \Delta V$$

$$W = \int p dV$$

$$= p \int dV$$

$$W = p \Delta V$$

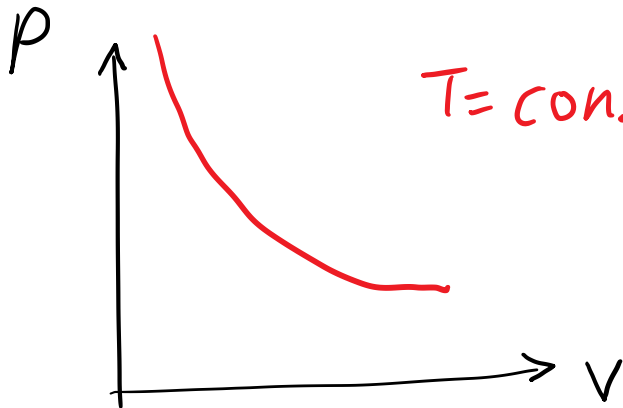
$$Q = n C_p \Delta T$$

↑
specific heat at constant pressure

$$C_p = \frac{5}{2} R \text{ monoatomic}$$

$$C_p = \frac{7}{2} R \text{ diatomic}$$

Isothermal process for the ideal gas



$$W = \int p dV$$

$$pV = nRT$$

$$p(V, T) = \frac{nRT}{V}$$

$$W = \int_i^f \frac{nRT}{V} dV = nRT \int_i^f \frac{dV}{V}$$

\uparrow
 $T = \text{const}$

$$W = nRT \ln \frac{V_f}{V_i}$$

$$U = U(T) \Rightarrow \text{if } T = \text{const}, U = \text{const} \Rightarrow \Delta U = 0$$

$$\Delta U = Q - W = 0$$

$$Q = W = nRT \ln \frac{V_f}{V_i}$$

Isobaric vs isochoric process

Consider two process between same two temperatures:

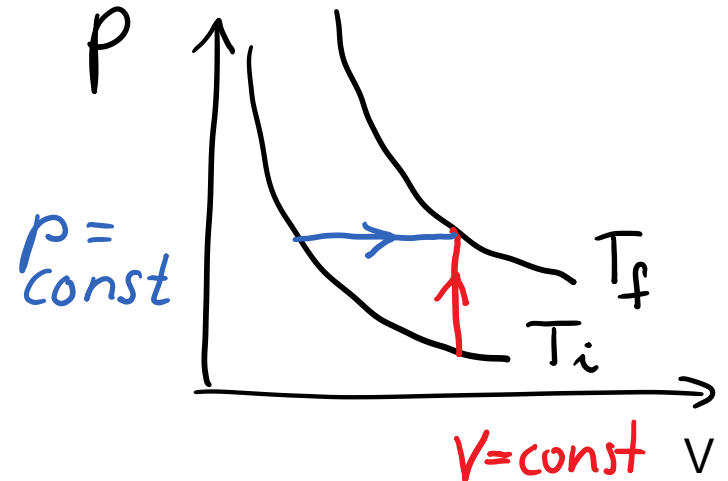
Isochoric: $\Delta U = nc_V\Delta T$

Isobaric: $\Delta U = nc_p\Delta T - p\Delta V$

$$nc_V\Delta T = nc_p\Delta T - p\Delta V$$

with $p\Delta V = nR\Delta T$:

$$c_p - c_V = R$$



To increase U by same amount as in isochoric process, more Q needed for isobaric process because gas is doing work

Difference between c_V and c_p

$$c_p - c_V = R$$

Monoatomic gas: $c_V = \frac{3}{2}R, c_p = \frac{5}{2}R$

Diatomic gas: $c_V = \frac{5}{2}R, c_p = \frac{7}{2}R$

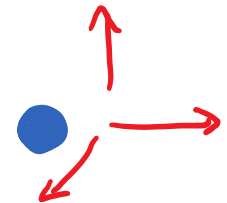
Mono- vs diatomic gas

Monoatomic gas: $c_V = \frac{3}{2}R, c_p = \frac{5}{2}R$

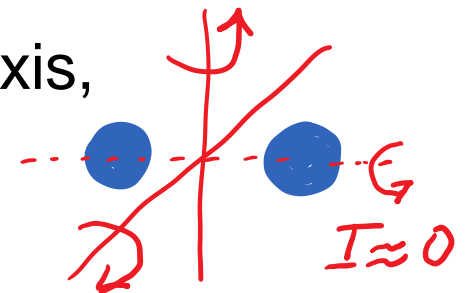
Diatomic gas: $c_V = \frac{5}{2}R, c_p = \frac{7}{2}R$

Each degree of freedom in the kinetic energy contributes $\frac{1}{2}R$ to specific heat

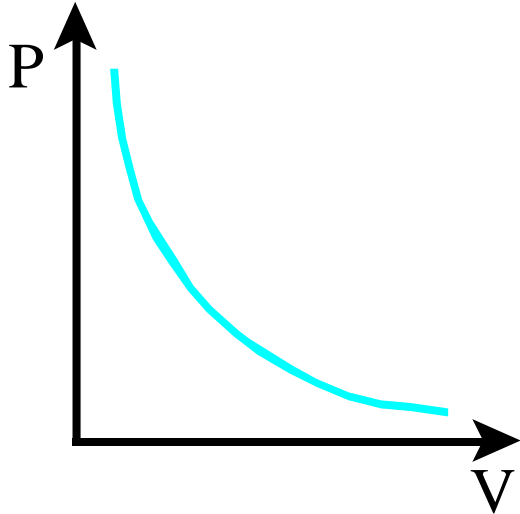
Monoatomic gas: 3 directions for translation



Diatomic gas: 3 translations, 3 rotations, but very small moment of inertia about one axis, this rotation does not contribute



Adiabatic Process



$$Q = 0 \rightarrow \Delta U = -W$$

$$dU = -p dV$$

$$nC_v dT = -\frac{nRT}{V} dV$$

$$c_v \frac{dT}{T} = -R \frac{dV}{V}$$

$$c_v \ln \frac{T}{T_0} = -R \ln \frac{V}{V_0}$$

$$\gamma = \frac{C_p}{C_v}$$

$$\left(\frac{T}{T_0}\right) = \left(\frac{V}{V_0}\right)^{-\frac{R}{C_v}} = \left(\frac{V}{V_0}\right)^{1-\gamma}$$

$$C_p - C_v = R$$

$$-\frac{R}{C_v} = 1 - \frac{C_p}{C_v}$$

$$pV = nRT$$

$$\gamma = 1.67 \text{ mono atomic}$$
$$1.40 \text{ di}$$

$$TV^{\gamma-1} = \text{const}$$
$$\Rightarrow PV^{\gamma} = \text{const}$$