Lecture 30: 2nd Law of Thermodynamics

- Thermodynamic cycles

- 2nd law of Thermodynamics

- Carnot Cycle
Thermodynamic Cycles

System returns to initial state

\[ U_f = U_i \]

\[ \Delta U = Q - W = 0 \]

\[ Q = W \]
Work in cycles

\[ W = \text{area enclosed in the cycle} \]
Clockwise: more positive \( W \) than negative \( W \)
\[ W_{\text{net}} > 0 \]
Counter-clockwise: more negative \( W \) than positive \( W \)
\[ W_{\text{net}} < 0 \]
Heat engine

Hot reservoir $T_{\text{hot}}$

$Q_{+} > 0$ into system

Heat engine $\rightarrow$ Work

Cold reservoir $T_{\text{cold}}$

$Q_{-} < 0$ out of system

$Q_{\text{net}} = Q_{+} + Q_{-}$

$= Q_{+} - |Q_{-}|$
Efficiency

- Hot reservoir $T_{\text{hot}}$
- $Q_+ > 0$ into system
- Heat engine
- $Q_ - < 0$ out of system
- Cold reservoir $T_{\text{cold}}$

$$e = \frac{W_{\text{out}}}{Q_H} = \frac{\Omega_{\text{net}}}{Q_H}$$

Cycle

$$W = Q$$

$$e = \frac{Q_H + Q_C}{Q_H}$$

$$e = 1 - \frac{|Q_C|}{Q_H}$$
2nd Law of Thermodynamics

Clausius:
Heat flows naturally from a hot object to a cold object; heat will not flow spontaneously from a cold object to a hot object.

Kelvin-Planck:
No device is possible whose sole effect is to transform a given amount of heat completely into work.

impossible to construct perpetual motion machine of 2nd kind
Forbidden by 2nd Law

$$e = 1 - \frac{|Q_c|}{Q_H}$$

$$e = 1 \text{ impossible}$$
Carnot Cycle

1-2 isothermal expansion
2-3 adiabatic expansion
3-4 isothermal compression
4-1 adiabatic compression
Carnot Cycle

1-2 isothermal expansion
\[ \Delta U = 0 \]
\[ Q = W = \int p \, dV = \int nRT \, dV \]
\[ Q = nRT_H \ln \frac{V_2}{V_1} > 0 \text{ in} \]

2-3 adiabatic expansion
\[ Q = 0 \]
\[ \Delta U = -W \quad W = -nC_V (T_C - T_H) \]

3-4 isothermal compression
\[ \Delta U = 0 \quad Q = W = nRT_C \ln \frac{V_4}{V_3} < 0 \text{ out} \]

4-1 adiabatic compression
\[ Q = 0 \quad \Delta U = -W \quad W = -nC_V (T_H - T_C) \]
Efficiency of Carnot Cycle

1-2 isothermal expansion
\[ W = Q = nRT_H \ln \frac{V_2}{V_1} > 0 \]

2-3 adiabatic expansion
\[ Q = 0, \quad W = -nc_v (T_c - T_H) < 0 \]

3-4 isothermal compression
\[ Q = W = nRT_c \ln \frac{V_4}{V_3} < 0 \]

4-1 adiabatic compression
\[ Q = 0, \quad W = -nc_v (T_H - T_c) \]

Efficiency
\[ e = \frac{W_{\text{net}}}{Q_{\text{in}}} = \frac{nRT_H \ln \frac{V_2}{V_1} + nRT_c \ln \frac{V_4}{V_3}}{nRT_H \ln \frac{V_2}{V_1}} = 1 - \frac{T_c}{T_H} \ln \frac{V_3}{V_4} \]

with \( T \) and \( V \) = const. on adiabatic + some math

\[ e_{\text{Carnot}} = 1 - \frac{T_c}{T_H} \]
Carnot Cycle has maximum efficiency

Heat transfer during isothermal process reversible
No heat transfer during process that involves temperature change
   $\rightarrow$ Carnot cycle is reversible
If more efficient engine existed:
Couple hypothetical engine with reverse Carnot engine
Transforms amount of heat completely into work
Violates 2\textsuperscript{nd} Law
   $\rightarrow$ More efficient engine can not exist

\[ e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} \]

Maximum efficiency of any cycle operating between $T_C$ and $T_H$