

Rec Sec Number _____

TEST 2 (4 pages)

and First Name: _____

For questions on this page, write the letter which you believe to be the best answer in the underlined space provided **to the left of the question number**. For problems on subsequent pages: your solution to a question with *OSE* in front of it must begin with an *Official Starting Equation*. The expression for the final result must be in system parameters and simplified as far as possible. Draw a box around your answer to each question. Neglect air resistance. Calculators and notes cannot be used during the test. If you have any questions, ask the proctor. **You must put your name on each page.**

Test Total = 200 / 200

A 1. (10 points) Sphere A of mass M and radius R , and sphere B of mass $\frac{1}{2}M$ and radius R are fully submerged in a pool of water. To keep them from rising, the spheres are anchored to the bottom of the pool by means of massless ropes. Which statement is true?

- A) Both spheres experience the same buoyancy force.
- B) Sphere A has a larger buoyancy force.
- C) The tension in the rope has a larger magnitude for sphere A than for sphere B.
- D) The tensions in the ropes are equal.

$B = \rho_{\text{fluid}} V_{\text{disp}} g$
same $R \rightarrow$ same V_{disp}

B 2. (10 points) The escape velocity from a planet with a certain mass and radius is V . The escape velocity from a planet of the same mass and one-fourth of the radius is

- A) V
- B) $2V$
- C) $4V$
- D) $16V$

$\frac{1}{2} m v_{\text{esc}}^2 - \frac{GMm}{R} = 0$
 $v_{\text{esc}} = \sqrt{\frac{2GM}{R}}$ $R \rightarrow \frac{1}{4}R$
 $v \rightarrow 2V$

C 3. (10 points) A bowling ball of mass $10M$ and a tennis ball of mass M are dropped from the top of the physics building. Neglecting air resistance, which statement is true?

- A) The force of gravity on each ball is the same.
- B) The bowling ball will hit the ground with a higher velocity than the tennis ball.
- C) The bowling ball will hit the ground with a higher kinetic energy than the tennis ball.
- D) Both balls will hit the ground with the same kinetic energy.

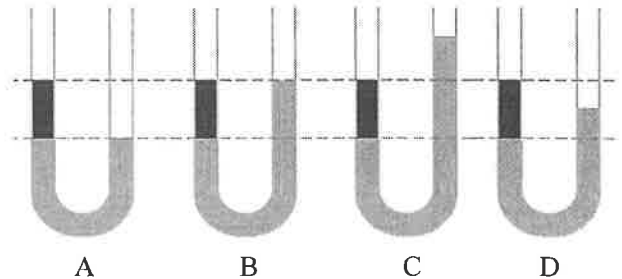
$mgh = \frac{1}{2} m v^2$
 $v = \sqrt{2gh}$
same v
 $K = \frac{1}{2} m v^2$
by $M \rightarrow$ by K

B 4. (10 points) A physics professor is walking along the sidewalk at 5 m/s . A skateboarder of mass M moving at 10 m/s in the same direction is coming up from behind and hits the professor. During the collision, the skateboarder grabs the professor, and they move together at 8 m/s in the original direction. What is the mass of the professor?

- A) $\frac{1}{3} M$
- B) $\frac{2}{3} M$
- C) $\frac{8}{5} M$
- D) $2 M$

$M \cdot 10 \frac{m}{s} + m \cdot 5 \frac{m}{s} = (M+m) 8 \frac{m}{s}$

A 5. (10 points) The figure shows four situations in which two different liquids, depicted in black and grey, respectively, are in a U-tube that is open to the atmosphere. The liquids do not mix. In which situation is it impossible for the liquids to be in static equilibrium?

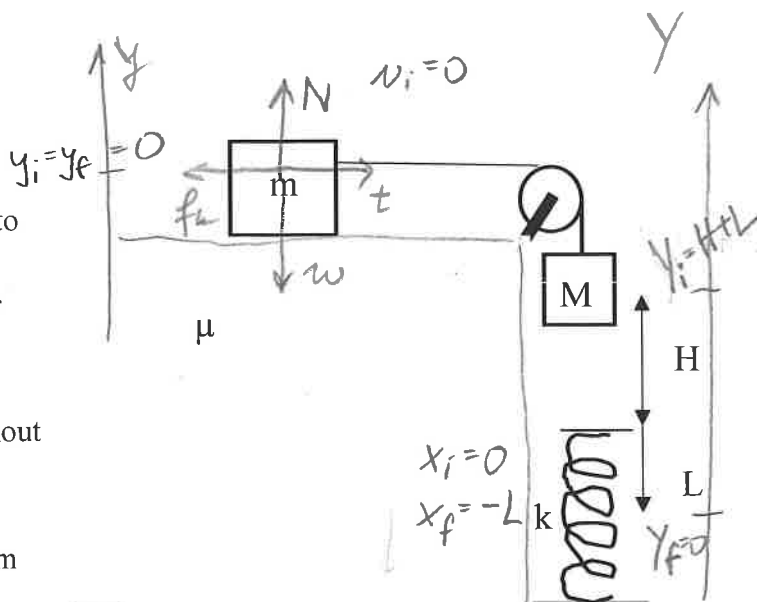


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6. A block of mass m is on a rough horizontal surface (coefficient of kinetic friction μ). A light string attaches it to another block of mass M that hangs over a massless, frictionless pulley. The blocks are released from rest. After moving downward for a distance H , the block of mass M encounters a vertical spring of spring constant k and compresses it a distance L . The string remains taut throughout the motion.

Derive an expression for the speed of the blocks when the spring has been compressed a distance L , in terms of system parameters.



$$E_f - E_i = W_{other}$$

$$\frac{1}{2}(m+M)v_f^2 + mgy_f + Mgy_f + \frac{1}{2}kx_f^2 - \frac{1}{2}(m+M)v_i^2 + mgy_i + Mgy_i + \frac{1}{2}kx_i^2 = W_f$$

$$\frac{1}{2}(m+M)v_f^2 + \frac{1}{2}k(-L)^2 - Mg(H+L) = \underbrace{\mu N(H+L)}_{-1} \cos 180^\circ$$

Find N :

$$\Sigma F_y = f_{ky} + N_y + \cancel{t_y} + \cancel{w_y} = m\cancel{a_y}$$

$$N = mg$$

$$\frac{1}{2}(m+M)v_f^2 + \frac{1}{2}kL^2 - Mg(H+L) = -\mu mg(H+L)$$

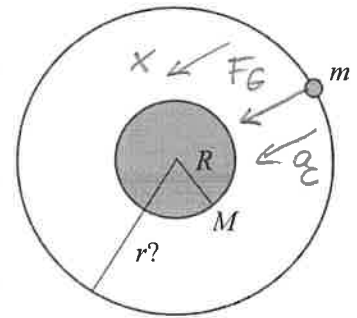
$$v_f = \sqrt{\frac{2}{m+M} \left[Mg(H+L) - \frac{1}{2}kL^2 - \mu mg(H+L) \right]}$$

$$v_f = \sqrt{\frac{2}{m+M} \left[(M - \mu m)g(H+L) - \frac{1}{2}kL^2 \right]}$$

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7. A defunct satellite that has become a piece of space junk of mass m is in a circular orbit around a planet of the mass M and radius R . It takes the satellite time T to perform one full revolution around the planet.



a) (5) Draw the free body diagram for the satellite and include all the required information to solve this problem.

b) (25) (OSE) Derive a symbolic expression for the radius r of the orbit, in terms of system parameters.

$$\Sigma F_x = F_{Gx} = m a_x$$

$$\frac{GMm}{r^2} = m \frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = v^2 = \frac{4\pi^2 r^2}{T^2}$$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$$

c) (20) Scientists want to launch a rocket from the surface of the planet to bring down the defunct satellite. Derive a symbolic expression for the minimum launch speed needed so that the rocket reaches a distance r from the center of the planet. Use r as a system parameter for this part!

v_{min} : if $v_f = 0$

$$E_f - E_i = W_{other}$$

$$E_i = E_f$$

$$\frac{1}{2} m_i v_i^2 - \frac{GMm_i}{R} = \frac{1}{2} m_i v_f^2 - \frac{GMm_i}{r}$$

$$\frac{1}{2} m_i v_i^2 = \frac{GMm_i}{R} - \frac{GMm_i}{r}$$

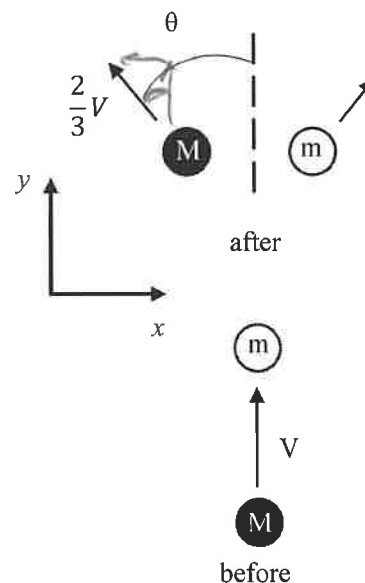
$$v_i = \sqrt{2GM \left(\frac{1}{R} - \frac{1}{r} \right)}$$



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Name: Solution

8. Two pucks are on a frictionless horizontal air hockey table. The white puck of mass m is initially at rest. The black puck of mass M is moving with a speed V in the positive y -direction and collides with the white puck. Immediately after the collision, the black puck is travelling with a speed $\frac{2}{3}V$ at an angle θ to the left of the positive y -axis, while the white puck is traveling at an **unknown speed** in an **unknown direction**.



a) OSE (40 points) Derive expressions for the **x- and y-components** of the white puck's velocity after the collision, in terms of system parameters.

$$\vec{J}_{\text{net}} = \vec{P}_f - \vec{P}_i$$

$$P_{ix} = P_{fx}$$

$$0 = -\frac{2}{3}MV \sin \theta + m v_x$$

$$P_{iy} = P_{fy}$$

$$MV + 0 = \frac{2}{3}MV \cos \theta + m v_y$$

$$m v_y = MV - \frac{2}{3}MV \cos \theta$$

$$v_x = \frac{2}{3} \frac{M}{m} V \sin \theta$$

$$v_y = \frac{M}{m} V \left(1 - \frac{2}{3} \cos \theta\right)$$

b) OSE (10 points) Derive an expression for the impulse delivered to the black puck by the white puck, **in unit vector notation**.

$$\vec{J}_{\text{net}} = \vec{P}_f - \vec{P}_i$$

$$\vec{P}_i = MV \hat{j}$$

$$\vec{P}_f = \frac{2}{3}MV [-\sin \theta \hat{i} + \cos \theta \hat{j}]$$

$$\vec{J} = \frac{2}{3}MV [-\sin \theta \hat{i} + \cos \theta \hat{j}] - MV \hat{j}$$

$$\vec{J} = MV \left[-\frac{2}{3} \sin \theta \hat{i} + \left(\frac{2}{3} \cos \theta - 1\right) \hat{j} \right]$$

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