Fall 2023 Physics 1135 Print LAST Name: Solution

Rec Sec Number ____ **TEST 3 (4 pages)** and First Name: ____________

For questions on this page, write the letter which you believe to be the best answer in the underlined space provided **to the left of the question number**. For problems on subsequent pages: your solution to a question with *OSE* in front of it must begin with an *O*fficial *S*tarting *E*quation. The expression for the final result must be in system parameters and simplified as far as possible. Draw a box around your answer to each question. Neglect air resistance. Calculators and notes cannot be used during the test. If you have any questions, ask the proctor. **Put your name on each page.**

Test Total = 200/ 200

D 1. (10 points) A hoop, a disk, and a uniform sphere of same mass and radius are placed at the top of an incline and released from rest. They are rolling without slipping. Which one is first to reach the bottom of the incline?

A) all at the same time B) hoop C) disk D) sphere

D 2. (10 points) A rotating carousel has a horse near the outer edge and a lion halfway out from the center. Which of the following is true?

A) Horse and lion have the same linear velocity.
B) The horse has a larger angular velocity than the lion.
C) The lion has a larger linear velocity than the horse.
D) Horse and lion have the same angular velocity.

C 3. (10 points) If the amplitude of the simple harmonic oscillator is doubled, its period is

A) halved B) doubled C) unchanged D) increased by a factor of four.

B 4. (10 points) A block on a spring is undergoing simple harmonic motion with amplitude A. When it is one-third of the way to the maximum distance from its equilibrium position, what fraction of its total mechanical energy is potential energy?

A) ⅓ B) ⅑ C) ⅔ D) 8/9

A 5. (10 points) A simple pendulum on the surface of the Earth has period *T*. On a different planet, the same pendulum has period 2*T*. The free-fall acceleration on this planet is

A) ¼g B) ½g C) 2g D) 4g

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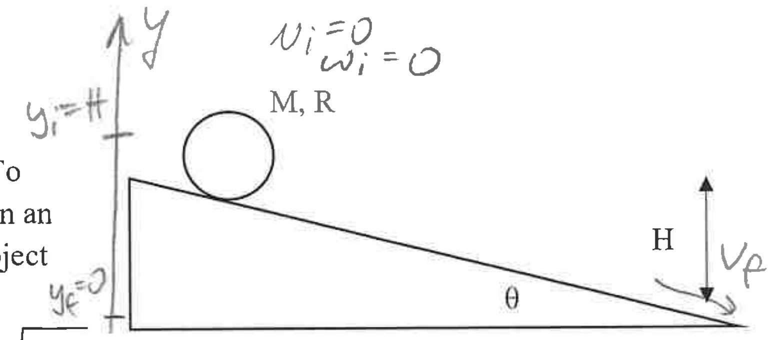
6. (50 pts) You have an object of mass M and radius R. To determine the moment of inertia, you roll the object down an incline that makes an angle θ with the horizontal. The object starts from rest and rolls without slipping. When it has descended a vertical height H, it has acquired a speed of $\sqrt{\frac{6}{5}gH}$.



Use energy methods to derive an expression for the moment of inertia of the object, in terms of M and R.

$$E_f - E_i = \cancel{W_{other}}^{\,0}$$

$$E_i = E_f$$

$$\frac{1}{2}M\cancel{v_i^2}^{\,0} + \frac{1}{2}I\cancel{\omega_i^2}^{\,0} + Mgy_i = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 + Mg\cancel{y_f}^{\,0}$$

$$MgH = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2$$

No slipping: $v = \omega R$

$$MgH = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\frac{v_f^2}{R^2}$$

$$MgH = \frac{1}{2}M\left(\frac{6}{5}gH\right) + \frac{1}{2}\frac{I}{R^2}\left(\frac{6}{5}gH\right)$$

$$M = \frac{3}{5}M + \frac{3}{5}\frac{I}{R^2}$$

$$\frac{2}{5}M = \frac{3}{5}\frac{I}{R^2}$$

$$\boxed{I = \frac{2}{3}MR^2}$$

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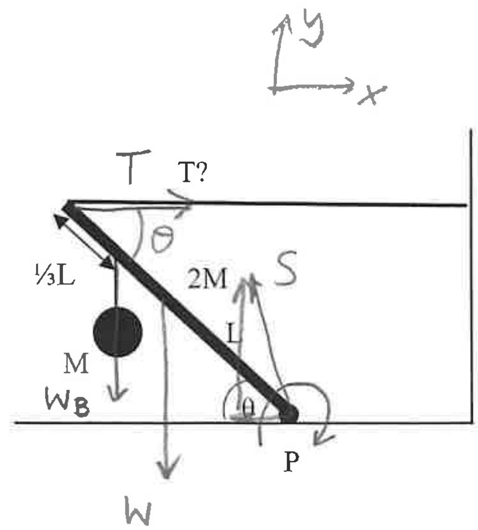
7. (50 points) A uniform post of mass $2M$ and length L is pivoted at the ground at point P. The post makes an angle θ with respect to the horizontal. A horizontal rope is tied to the post at the top end. A ball of mass M hangs from the post one-third of the length from the top end, as shown in the figure.



a) (10 points) Complete the diagram with all information necessary to solve parts b and c below.

b) (OSE) (20 pts) Derive an expression for the tension T in the horizontal cable in terms of system parameters.
(Hint: Torques about P)

$$\sum \tau_z = \cancel{\tau_{S_z}}^{0} + \tau_{T_z} + \tau_{W_z} + \tau_{W_{B_z}} = 0$$

$$+TL\sin\theta - 2Mg\frac{L}{2}\cos\theta - Mg\frac{2}{3}L\cos\theta = 0$$

$$TL\sin\theta = MgL\cos\theta\,(\underbrace{1+2/3}_{5/3})$$

$$\boxed{T = \frac{5}{3}Mg\cot\theta}$$

c) (OSE) (20 pts) Derive expressions for the horizontal and vertical components of the support force at the pivot point in terms of system parameters. **Treat the tension T as a system parameter for this part.**

$$\sum F_x = T_x + \cancel{W_{B_x}}^{0} + \cancel{W_x}^{0} + S_x = 0$$

$$T + S_x = 0 \longrightarrow \boxed{S_x = -T}$$

$$\sum F_y = \cancel{T_y}^{0} + W_{B_y} + W_y + S_y = 0$$

$$-Mg - 2Mg + S_y = 0 \qquad \boxed{S_y = 3Mg}$$

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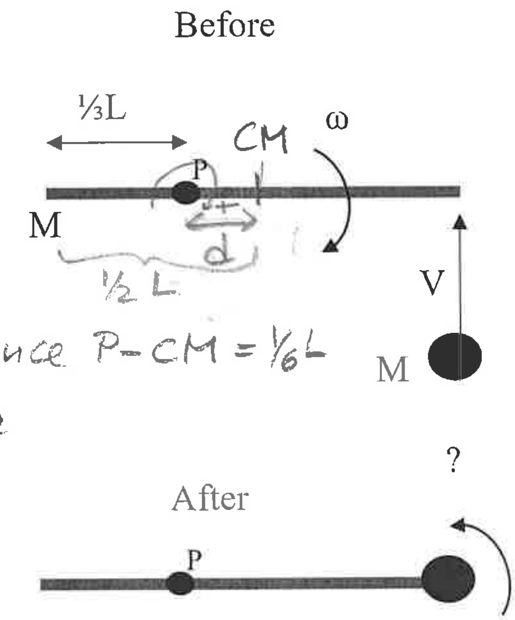
8. (50 points) A uniform beam of mass M and length L is pivoted at point P one-third L from the left end. The beam is rotating with an angular speed ω in the clockwise direction. A putty ball of mass M, traveling with speed V, hits the beam at its right end at a 90-degree angle, as shown in the figure, and sticks to the beam.



a) (10 points) Derive an expression for the moment of inertia of the beam about point P.

$I_P = I_{CM} + M d^2$ $\qquad$ d distance P–CM = $\frac{1}{6}L$

$I_P = \frac{1}{12} M L^2 + M\left(\frac{1}{6}L\right)^2 = \underbrace{\left(\frac{1}{12} + \frac{1}{36}\right)}_{\frac{3+1}{36} = \frac{1}{9}} M L^2$

$$\boxed{I_P = \frac{1}{9} M L^2}$$

b) (40 points) Derive an expression for the angular speed of the beam-putty system immediately after the collision.

Treat the putty as a point mass.

$\sum \cancel{\vec{\tau}_{ext}}^{\,0} = \frac{d\vec{L}}{dt}$

$L_{iz} = L_{fz}$

$I \omega_{iz} - M V \frac{2}{3} L = I_{tot}\, \omega_{fz}$

$\frac{1}{9} M L^2 \omega_i - \frac{2}{3} M V L = \left(\frac{1}{9} M L^2 + \underbrace{M\left(\frac{2}{3}L\right)^2}_{\text{putty = point mass}}\right) \omega_{fz}$

$\frac{1}{9} M L^2 \omega_i - \frac{2}{3} M V L = \underbrace{\left(\frac{1}{9} + \frac{4}{9}\right)}_{5/9} M L^2 \omega_{fz}$

$$\boxed{\omega_f = \left| \frac{\frac{1}{9} \cancel{M} L^2 \omega_i - \frac{2}{3} \cancel{M} V L}{\frac{5}{9} \cancel{M} L^2} \right| = \left| \frac{\omega_i}{5} - \frac{6}{5}\frac{V}{L} \right|}$$

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