

Name: Solution September 21, 2022

Total Score: 120 /120

$$x = x_i + v_{ix}\Delta t + \frac{1}{2} a_x(\Delta t)^2$$

$$v_x = v_{ix} + a_x\Delta t$$

$$v_x^2 = v_{ix}^2 + 2 a_x(x - x_i)$$

$$y = y_i + v_{iy}\Delta t + \frac{1}{2} a_y(\Delta t)^2$$

$$v_y = v_{iy} + a_y\Delta t$$

$$v_y^2 = v_{iy}^2 + 2 a_y(y - y_i)$$

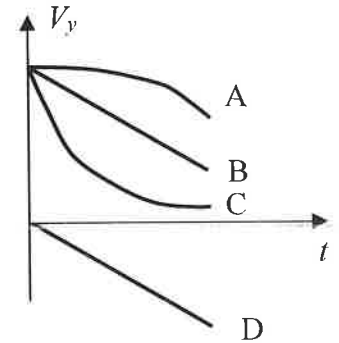
Free fall acceleration: $g = 9.8\text{m/s}^2$

Centripetal acceleration: $a_c = \frac{v^2}{R}$

$$v = \frac{2\pi R}{T}$$

___ 1.(5) A particle is moving along a straight line. The slope of the position vs time graph at a particular time gives the particle's

- A) instantaneous velocity at that time B) average speed
 C) instantaneous acceleration at that time D) displacement

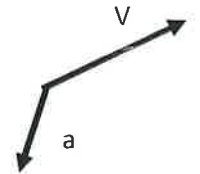


___ 2.(5) Raisin the cat drops a ball from her cat tree. The v_y-t graph of the ball's motion is given by which plot letter in the graph at the right? The y -axis is directed upwards.

- A) A B) B C) C D) D

___ 3. (5) A particle is moving with velocity V . At a particular instant, it experiences an acceleration a as shown in the figure. We know that the particle is:

- A) only speeding up B) speeding up and changing direction of motion.
 C) only slowing down D) slowing down and changing direction of motion



___ 4. (5) A ball is kicked from the ground with an initial velocity V at 30° above the horizontal. At the top of its trajectory, the ball's velocity and acceleration are:

- A) at right angles to one another B) in opposite directions
 C) in the same direction D) both zero

___ 5. (5) An object is moving in a circle of a given radius at constant speed. Which is true about the object?

- A) Its velocity is constant. B) Its acceleration is zero.
 C) Its acceleration is directed perpendicular to the velocity vector.
 D) The faster the object, the smaller its acceleration.

___ 6. (5) A particle rotates in a circle with centripetal acceleration a . If the period is halved without changing the radius, the new acceleration will be

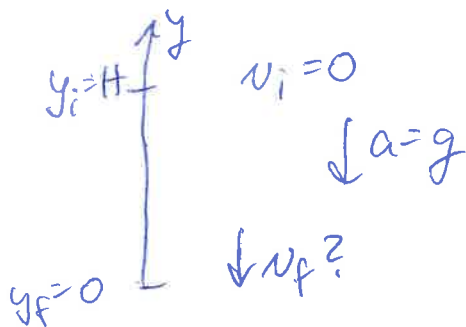
- A) $\frac{1}{4} a$ B) $\frac{1}{2} a$ C) $2 a$ D) $4 a$

$T \rightarrow \frac{1}{2} T$
 $v \rightarrow 2v$

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7.(25) A physics professor is paddling a kayak on the river when a rock, dropped from a bluff vertically down by a careless person, hits the water next to her. The height of the bluff above the river is $H=45$ m.

a) (5) In the space provided, draw a complete diagram for the rock with all information needed to solve the tasks below. Remember, any quantity used in the calculation must be defined in the diagram.



b) (10) Find the speed with which the rock hit the water. Derive a symbolic expression and calculate a numerical answer.

$$v_y^2 = v_{iy}^2 + 2a_y(y - y_i)$$

$$v_y^2 = 2(-g)(0 - H)$$

$|v_y| = \sqrt{2gH}$

$$|v_y| = \sqrt{2 \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 45 \text{ m}}$$

$$= 29.7 \frac{\text{m}}{\text{s}}$$

c) (10) How much time ΔT did it take the rock from the top of the bluff to the water? Derive a symbolic expression and calculate a numerical answer.

$$y = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$0 = H + \frac{1}{2} (-g) (\Delta t)^2$$

$\Delta t = \sqrt{\frac{2H}{g}}$

$$\Delta t = \sqrt{\frac{2 \cdot 45 \text{ m}}{9.8 \text{ m/s}^2}} = 3 \text{ s}$$

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8. (25) In the figure, the magnitudes of the vectors are $A=2$ and $B=3$. The angle θ equals 30° .

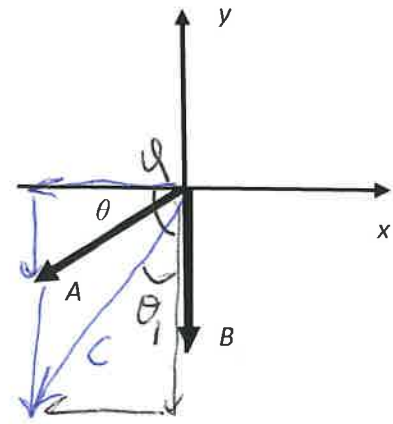
a) Calculate the vector components A_x , A_y , B_x , B_y .

$$A_x = -A \cos \theta = -2 \cdot \cos 30^\circ = -1.73$$

$$A_y = -A \sin \theta = -2 \cdot \underbrace{\sin 30^\circ}_{1/2} = -1$$

$$B_x = 0$$

$$B_y = -B = -3$$



b) The vector $\vec{C} = \vec{A} + \vec{B}$. Sketch vector \vec{C} in the diagram and calculate its components, magnitude, and direction.

$$C_x = A_x + B_x = -1.73$$

$$C_y = A_y + B_y = -1 - 3 = -4$$

$$C = \sqrt{C_x^2 + C_y^2} = 4.36$$

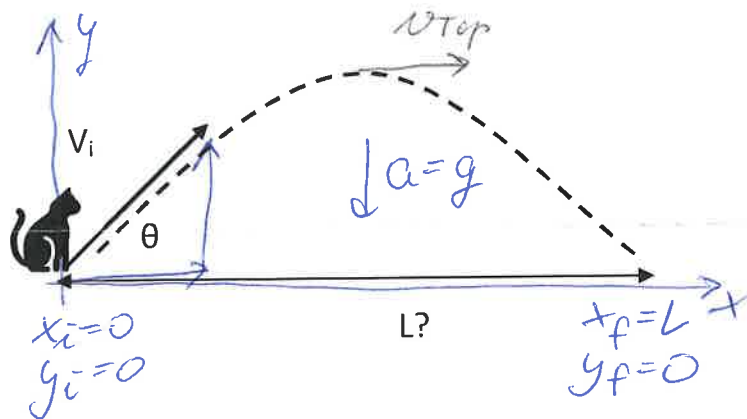
$$\theta_1 = \arctan \left| \frac{C_x}{C_y} \right| = 23.4^\circ$$

$$\text{or } \varphi = \arctan \left| \frac{C_y}{C_x} \right| = 66.6^\circ$$

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9. (40) Frodo the cat jumps off the floor with an initial velocity of magnitude V_i directed at an angle θ with respect to the horizontal and lands back on the floor some unknown horizontal distance away.

a) Complete the diagram on the right with all information necessary to solve the parts below.



b) Derive a symbolic expression for the time Δt it takes Frodo to land back on the floor, in terms of V_i , θ and g (or a subset thereof).

$$y = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$0 = 0 + v_i \sin \theta \Delta t + \frac{1}{2} (-g) (\Delta t)^2$$

$$v_i \sin \theta \Delta t = \frac{1}{2} g (\Delta t)^2$$

for $\Delta t \neq 0$:

$$\Delta t = \frac{2 v_i \sin \theta}{g}$$

c) Derive a symbolic expression for the horizontal distance L Frodo covers during his jump, in terms of V_i , θ , g , and Δt from part b (or a subset thereof).

$$x = x_i + v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

$$L = v_i \cos \theta \Delta t$$

or

$$L = \frac{2 v_i^2 \sin \theta \cos \theta}{g}$$

d) What are the x - and y -components of Frodo's velocity when he is at the highest point of his jump?

$$v_x = v_{ix} + a_x \Delta t = v_i \cos \theta$$

$$v_y = 0, \text{ highest point}$$

40/40 points for this page