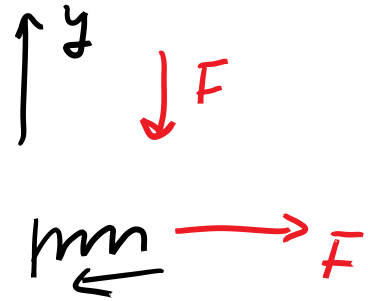
Lecture 6: Electric potential

Potential energy

Mechanics:

$F_{grav} = mg$ $\qquad$ $U_{grav} = mgy$

$F_{spring} = -kx$ $\qquad$ $U_{sp} = \frac{1}{2}kx^2$



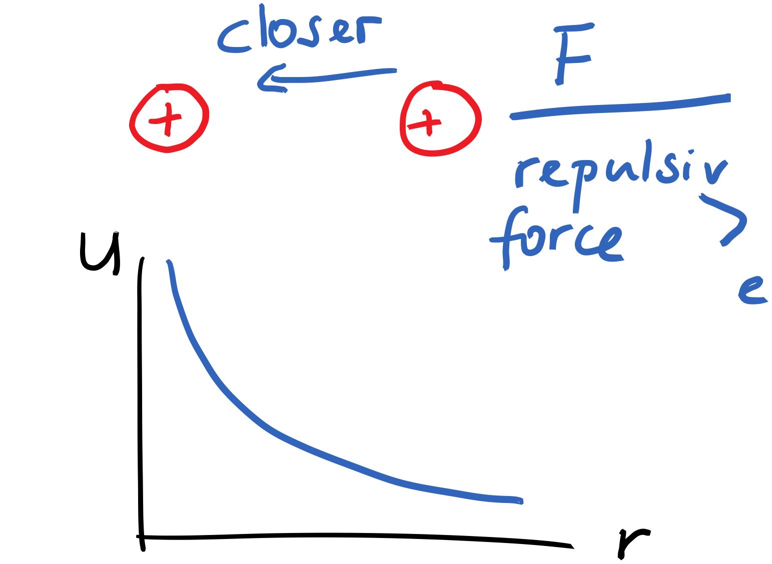
Potential energy increases in the direction opposite to the force

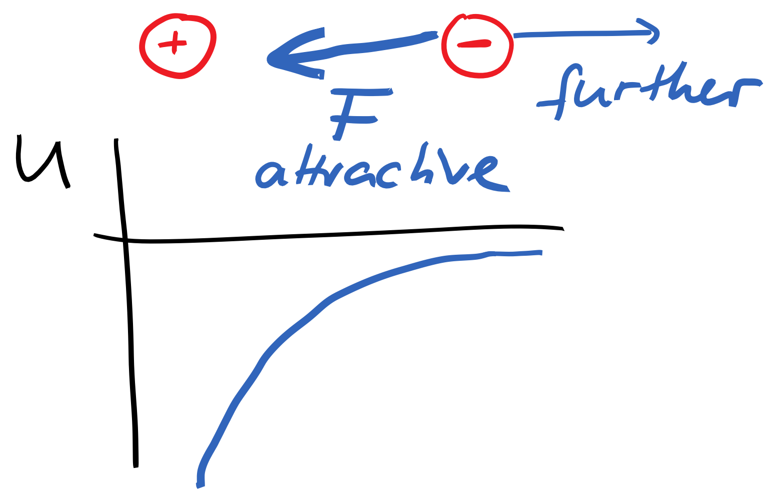
Electric potential energy

Two point charges:

$$F = k\frac{|\,qq'\,|}{r^2}$$

$$U = k\frac{qq'}{r}$$





Electric potential

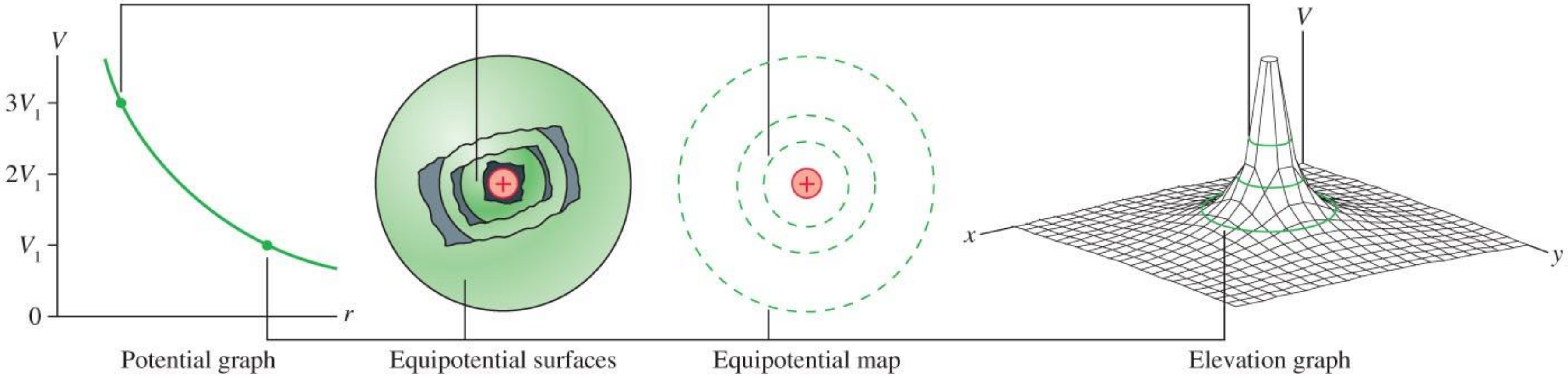
$$F = k\frac{|qq'|}{r^2} \xrightarrow[\frac{F \text{ on } q'}{q'}]{} E = \frac{k|q|}{r^2}$$

$$\boxed{U = k\frac{qq'}{r}} \xrightarrow[V = \frac{U}{q'}]{} \boxed{V = k\frac{q}{r}}$$

Electric potential energy of pair of point charges

Electric potential of point charge q

Electric Potential of a Point Charge



$$V = K\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

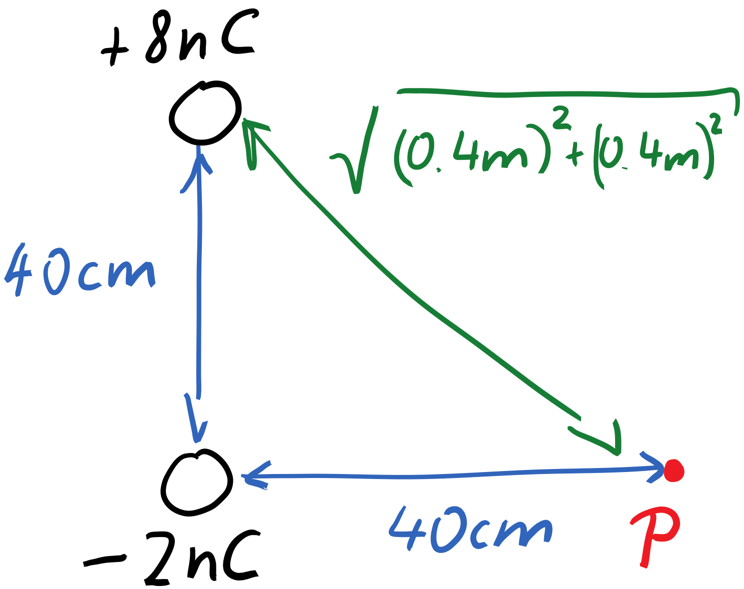
Electric potential at distance r from a point charge q

Unit: $1 \text{ Volt} = \dfrac{1 \text{ Joule}}{1 \text{ Coulomb}}$

Potential differences are created by separating positive and negative charges

Multiple charges

$$V = \sum_i V_i = \sum k \frac{q_i}{r_i} = k \sum \frac{q_i}{r_i}$$



$$V = V_1 + V_2$$

$$V = k\left(\frac{-2nC}{0.4m} + \frac{8nC}{\sqrt{2\cdot(0.4m)^2}}\right)$$

$$V = 9\times10^{9}\,\frac{Nm^2}{C^2}\left(\frac{-2\times10^{-9}C}{0.4m} + \frac{8\times10^{-9}C}{0.56m}\right)$$

$$V = 83.6V$$

Remember HW wk 2 # 4?

Energy conservation

$$K_i + U_i = K_f + U_f$$

$$U = qV$$

$$K_i + qV_i = K_f + qV_f$$

$$\boxed{K_f - K_i = -q(V_f - V_i)}$$

$\Delta U > 0$ $\Delta K < 0$ slow down

$\Delta U < 0$ $\Delta K > 0$ speed up

Example

v_i $\qquad\qquad\qquad\qquad$ v_f

$V_i = 500V$ $\qquad\qquad\qquad\qquad$ $V_f = 400V$

$\Delta V = V_f - V_i = -100V$

$\Delta K = -q\Delta V > 0$

positive q speeds up

v_i $\qquad\qquad\qquad\qquad$ v_f

negative q slows down

$\Delta K = -\underset{<0}{q}\,\underset{<0}{\Delta V} < 0$

Electron Volt

An electron moving through a potential difference of 1 Volt will gain kinetic energy of

$$\Delta K = -q\Delta V = -(-e)\cdot 1V$$

$$= \underbrace{1.6\times10^{-19}C\cdot 1V}_{\text{electron Volt}}$$

$$1eV = 1.6\times10^{-19}J$$

Example

What is the speed of an 8.7MeV proton?

$$K = \frac{1}{2} m v^2 \qquad m_p = 1.7 \times 10^{-27} kg$$

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \cdot 8.7 \times 10^6 \times 1.6 \times 10^{-19} J}{1.7 \times 10^{-27} kg}}$$

$$= 4.1 \times 10^7 \frac{m}{s}$$

very fast

⚠ Nothing can be faster than $3 \times 10^8 \frac{m}{s}$ speed of light in vacuum