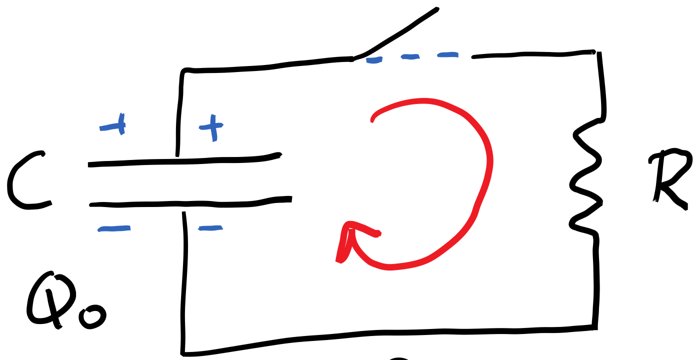
Lecture 19: RC Circuits

- Charging and discharging capacitors through a resistor

Discharging a capacitor



switch closed at $t = 0$

→ charge flows

→ capacitor discharges

$$\Delta V_C = \frac{Q_0}{C}$$

Loop rule: $\sum \Delta V_i = 0$

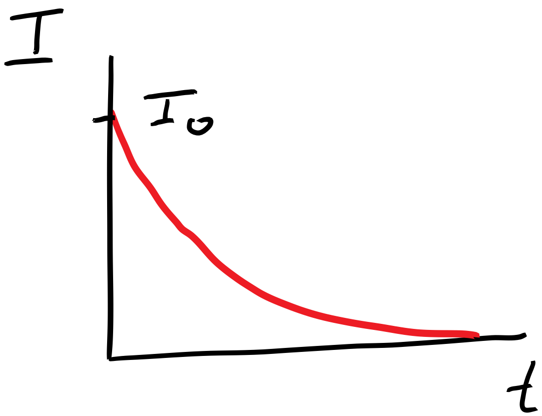
$$\Delta V_C + \Delta V_R = 0$$

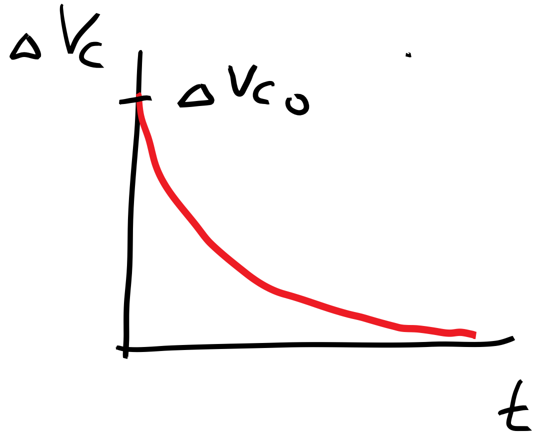
$$\Delta V_C - IR = 0$$

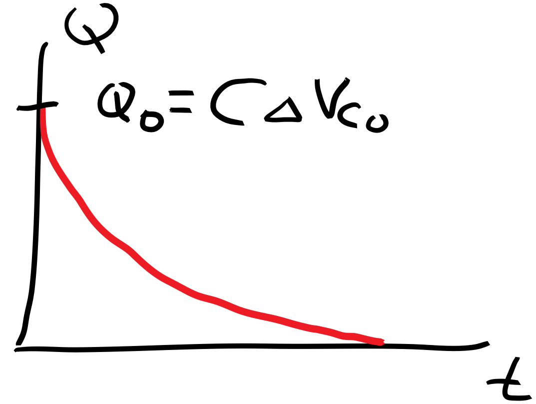
$t = 0$: $\Delta V_{C0} = I_0 R$ $\qquad I_0 = \dfrac{\Delta V_{C0}}{R}$

As t increases: I, ΔV_C decrease until capacitor is discharged and $Q = 0$

Discharging a capacitor







$$I = I_0 e^{-\frac{t}{RC}}$$

$$\Delta V_c = \Delta V_{c_0} e^{-t/RC}$$

$$Q = Q_0 e^{-t/RC}$$

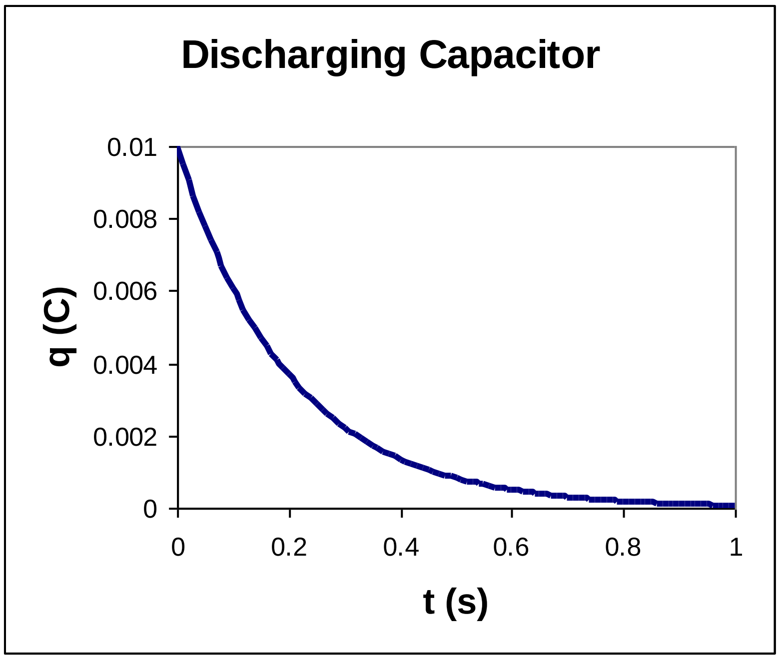
$\tau = RC$ time constant

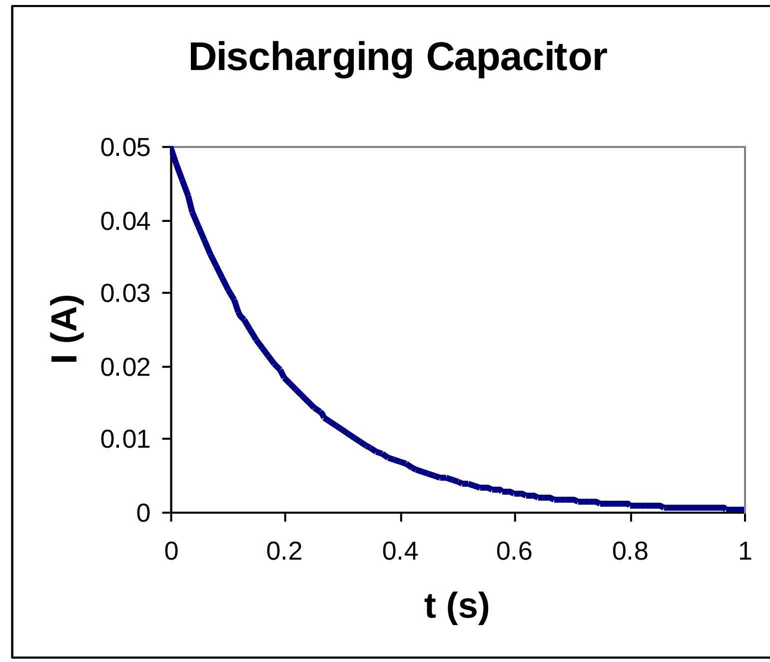
Discharging a capacitor:

Thanks to Dr. Pringle for the use of these slides

$$q(t) = Q_0\, e^{-\frac{t}{RC}}$$

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

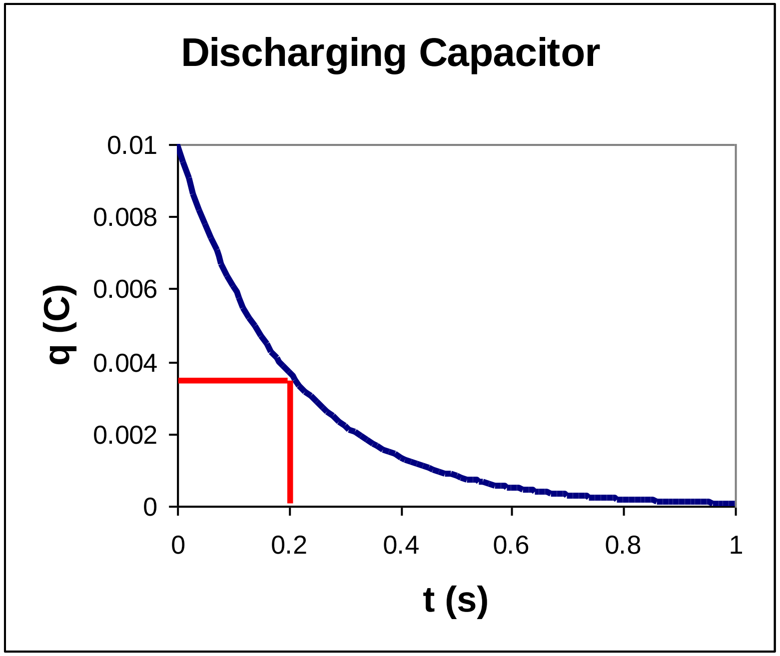


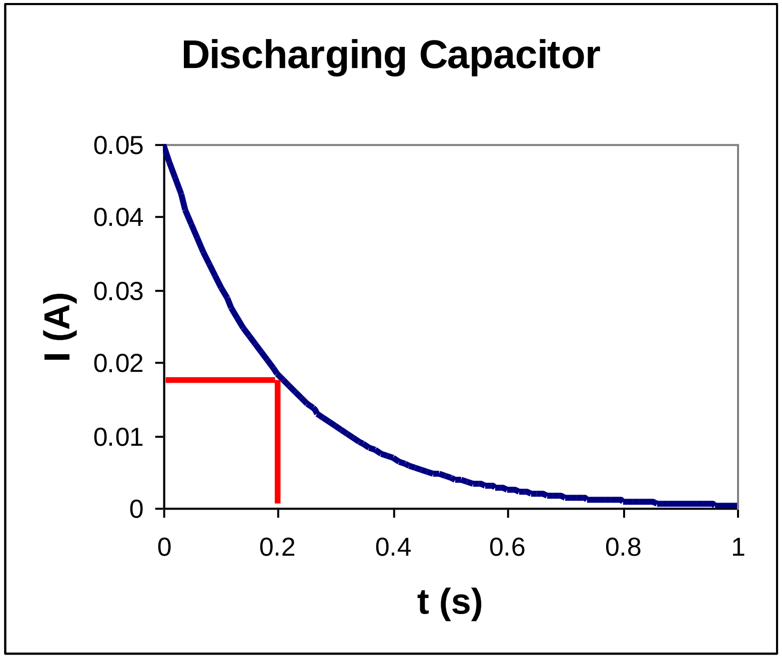


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor discharges to Qe^{-1} or 37% of its capacity…

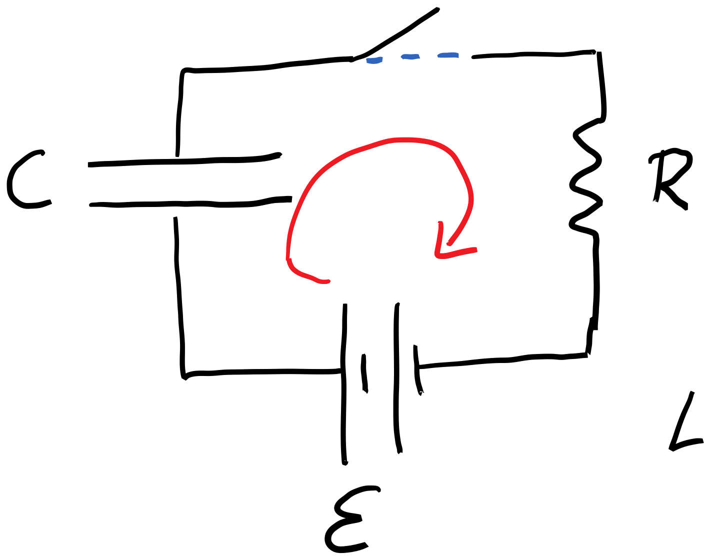
…and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

Charging a capacitor



switch closed at $t=0$

→ current flows

→ capacitor charges

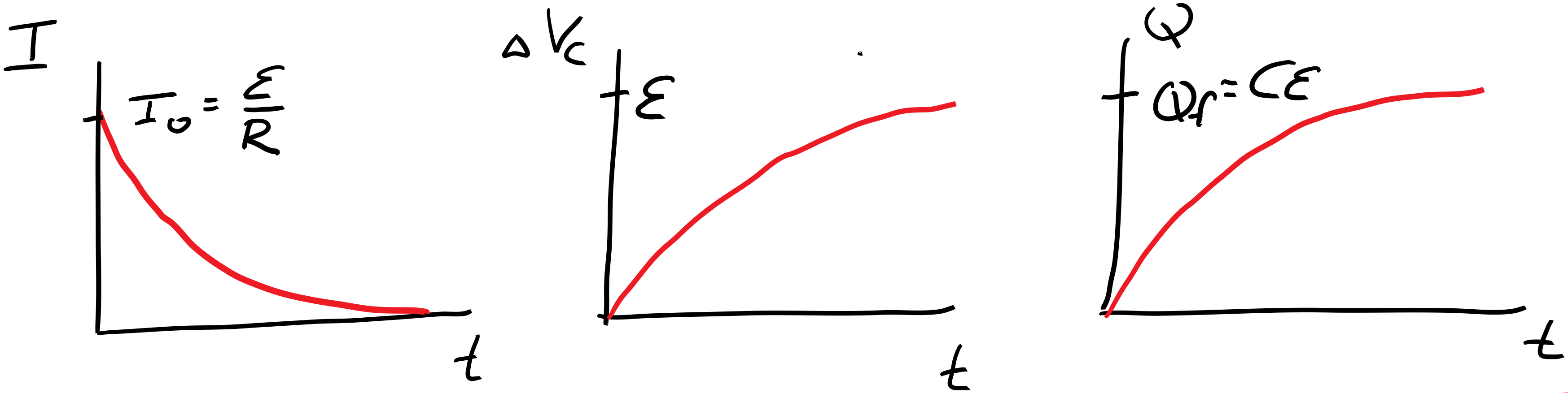
Loop rule: $\sum \Delta V_i = 0$

$$\mathcal{E} - \frac{Q}{C} - IR = 0$$

$t=0$: $Q=0$ $\quad \mathcal{E} = I_0 R \quad I_0 = \frac{\mathcal{E}}{R}$

$t \to \infty$: $Q \to Q_f = \mathcal{E} C \quad I \to 0$, "full"

Charging a capacitor



$$I = I_0 e^{-\frac{t}{RC}}$$

$$\Delta V_c = \varepsilon\left(1 - e^{-t/RC}\right)$$

$$Q = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$$

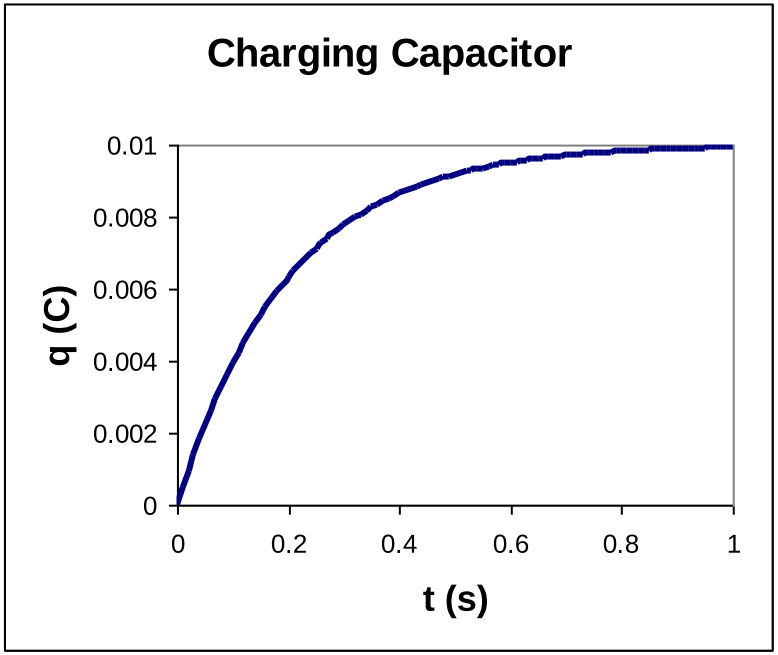
$\tau = RC$ time constant

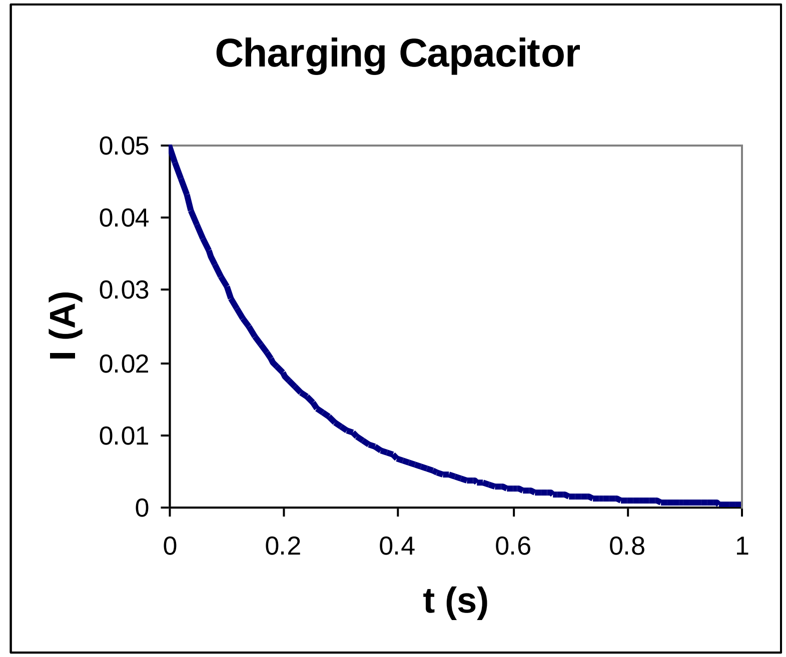
Charging a capacitor:

Thanks to Dr. Pringle for the use of these slides

$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

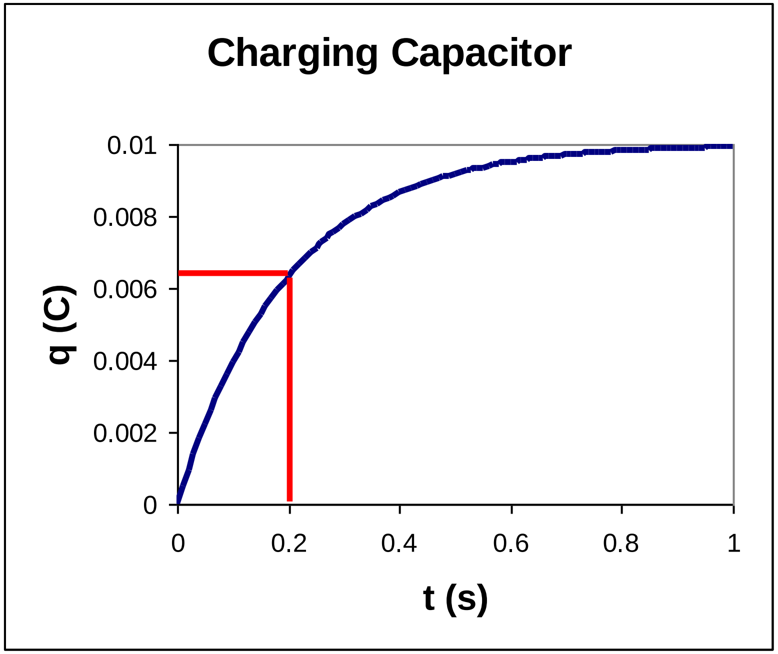


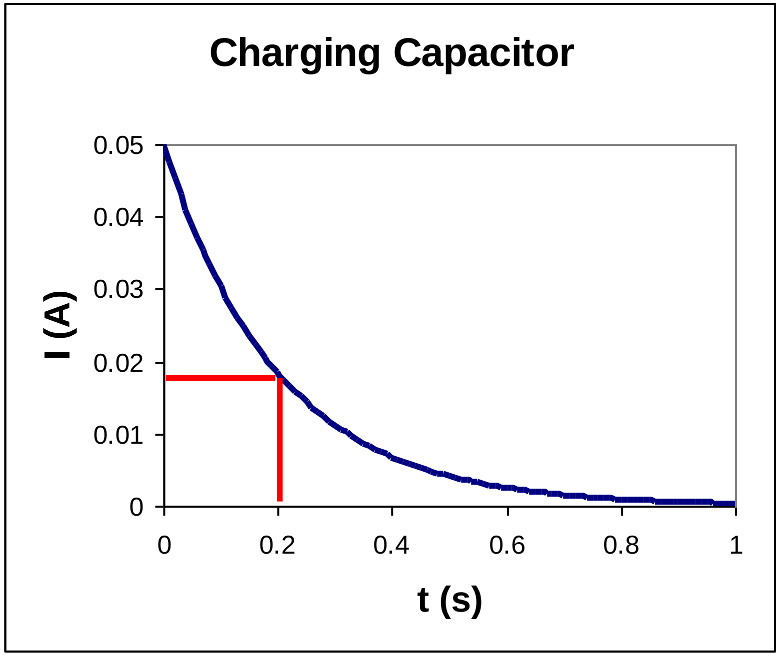


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor charges to $Q(1-e^{-1})$ or 63% of its capacity…

…and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

τ=RC is called the **time constant** of the RC circuit