

Name: Solution March 22, 2019

Total Score: 120 /120

$e = 1.602 \times 10^{-19} \text{ C}$ $I = \frac{\Delta q}{\Delta t}$ $I = \frac{V}{R}$ $R = \rho \frac{L}{A}$ $P = IV = \frac{V^2}{R} = I^2 R$

series: $R_{eq} = \sum_i R_i$ parallel: $\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$ $C = \frac{Q}{V}$

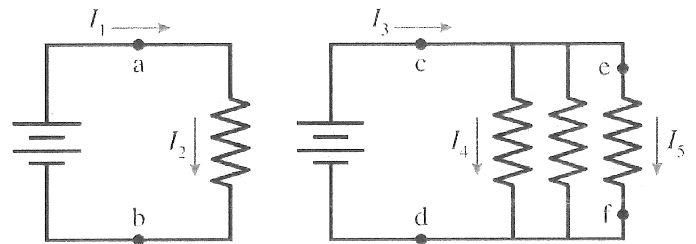
Discharging $Q = Q_0 e^{-\frac{t}{RC}}$ $I = I_0 e^{-\frac{t}{RC}}$ $V_C = V_{C0} e^{-\frac{t}{RC}}$

Charging $Q = Q_f (1 - e^{-\frac{t}{RC}})$ $I = I_0 e^{-\frac{t}{RC}}$ $V_C = V_{Cf} (1 - e^{-\frac{t}{RC}})$

D 1. (5) A wire has resistance R. What will be its resistance if it is stretched to twice its original length without changing the volume of the wire? $L \rightarrow 2L$ $A \rightarrow \frac{1}{2}A$ $R \rightarrow 4R$

- A) $\frac{1}{2}R$ B) R C) 2R D) 4R

B 2. (5) In the two circuits on the right, the batteries are identical and all resistors are identical. Which of the statements is FALSE?

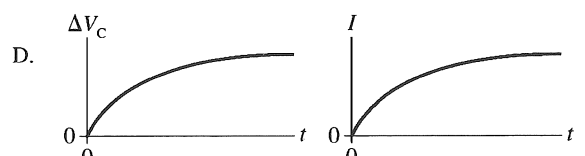
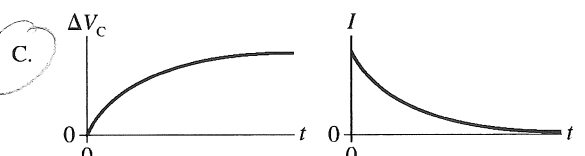
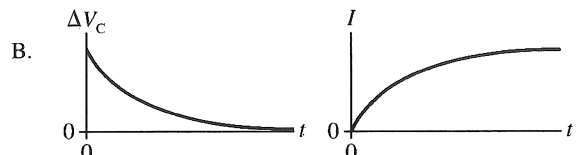
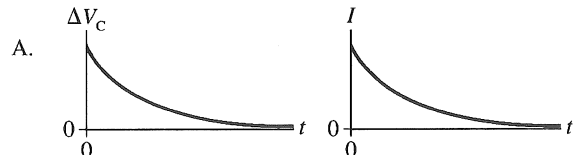


- A) $I_1 = I_2$ B) $I_1 = I_3$
 C) $I_1 = I_4$ D) $I_1 = \frac{1}{3} I_3$

D 3. (5) If the voltage across a resistor is doubled, which of the following is true? The power dissipated by the resistor _____, and the resistance _____.

- A) doubles, doubles.
 B) quadruples, doubles.
 C) doubles, remains unchanged.
D) quadruples, remains unchanged

$V \rightarrow 2V$
 $I \rightarrow 2I$
 $P \rightarrow 4P$



C 4. (5) A capacitor is charged through a resistor. Which of the graphs at the right represent best the capacitor voltage and the current?

20 /20 points for this page

5) (20) A potassium ion channel in a cell membrane is filled with intracellular fluid. The potassium ions have charge e . If the channel is open for 2ms, 1.5×10^4 ions flow through the channel. The potential difference across the channel is 70mV.

a)(5) Calculate the current through the channel.

$$I = \frac{\Delta Q}{\Delta t} = \frac{N \cdot e}{\Delta t} = \frac{1.5 \times 10^4 \cdot 1.6 \times 10^{-19} \text{ C}}{2 \times 10^{-3} \text{ s}} = 1.2 \times 10^{-12} \text{ A} = 1.2 \text{ pA}$$

b)(5) Calculate the resistance of the ion channel.

$$R = \frac{V}{I} = \frac{70 \times 10^{-3} \text{ V}}{1.2 \times 10^{-12} \text{ A}} = 5.8 \times 10^{10} \Omega$$

c) (5) The channel has the shape of a cylinder with a diameter of 0.30nm and a length of 5.0nm. Calculate the resistivity of the intracellular fluid.

$$R = \rho \frac{L}{A} \quad \rho = \frac{RA}{L} = 5.8 \times 10^{10} \Omega \cdot \frac{\frac{\pi}{4} (0.3 \times 10^{-9} \text{ m})^2}{5 \times 10^{-9} \text{ m}}$$

$$\rho = 0.82 \Omega \text{ m}$$

c)(5) Calculate the power dissipated in the channel.

$$P = IV = 1.2 \times 10^{-12} \text{ A} \cdot 70 \times 10^{-3} \text{ V} = 8.4 \times 10^{-14} \text{ W}$$

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6. a) (20 points) Find the equivalent resistance of the resistor circuit shown in the figure.

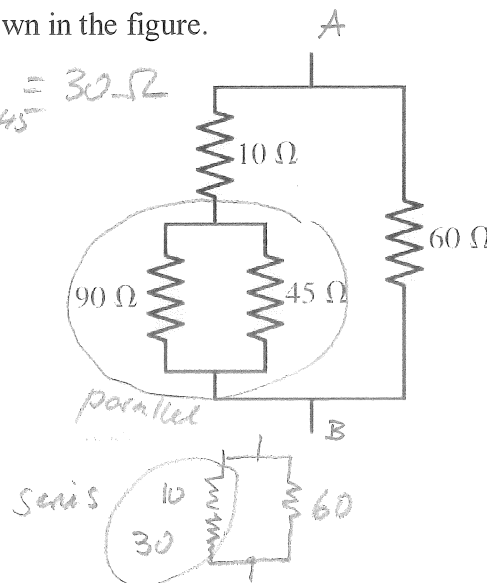
$$\frac{1}{R_{90+45}} = \frac{1}{90\Omega} + \frac{1}{45\Omega} = \frac{2+1}{90\Omega} = \frac{3}{90\Omega}$$

$$R_{90+45} = 30\Omega$$

$$R_{10+30} = 10\Omega + 30\Omega = 40\Omega$$

$$\frac{1}{R_{eq}} = \frac{1}{40\Omega} + \frac{1}{60\Omega} = \frac{3+2}{120\Omega} = \frac{5}{120\Omega} = \frac{1}{24\Omega}$$

$$R_{eq} = 24\Omega$$



b) (20) A battery provides a potential difference of 48V between points A and B. Calculate the total current through the network, the potential drop across each of the resistors, and the current through each resistor. Put the answers in the table below.

$$I_{tot} = \frac{V}{R_{eq}} = \frac{48V}{24\Omega} = 2A$$

$$60\Omega: V=48V \quad I = \frac{V}{R} = \frac{48V}{60\Omega} = 0.8A$$

$$\text{Leaves } 1.2A \text{ for other branch} \Rightarrow 10\Omega: 1.2A \quad V=IR=12V$$

$$\rightarrow \text{leaves } 36V \text{ for parallel } 90+45: \left. \begin{array}{l} I_{90} = \frac{36V}{90\Omega} = 0.4A \\ I_{45} = \frac{36V}{45\Omega} = 0.8A \end{array} \right\} 1.2A \checkmark$$

Total current:

10Ω	V= 12V	I= 1.2A
45Ω	V= 36V	I= 0.8A
90Ω	V= 36V	I= 0.4A
60Ω	V= 48V	I= 0.8A

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Charging

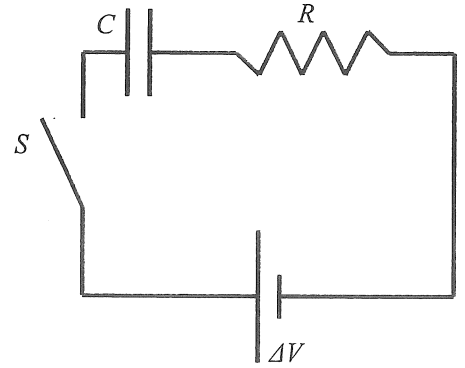
7) (40) A $4.0 \mu\text{F}$ capacitor is initially uncharged. It is connected in series with a switch, a $20 \text{ k}\Omega$ resistor, and an emf of 20 volts.

a) (5) Calculate the time constant of the circuit.

$$\tau = RC = 20 \times 10^3 \Omega \cdot 4 \times 10^{-6} \text{ F} = 80 \times 10^{-3} \text{ s}$$

b) (5) Calculate the current through the resistor immediately after the switch is closed.

$$I = \frac{\Delta V}{R} = \frac{20 \text{ V}}{20 \times 10^3 \Omega} = 10^{-3} \text{ A}$$



c) (5) Determine the charge on the capacitor a very long time ($t \rightarrow \infty$) after the switch is closed.

$$C = \frac{Q}{\Delta V} \quad Q_{\infty} = C \cdot \Delta V = 4 \times 10^{-6} \text{ F} \cdot 20 \text{ V} = 80 \times 10^{-6} \text{ C} = 80 \mu\text{C}$$

d) (10) Calculate the charge on the capacitor 20 ms after the switch is closed.

$$Q = Q_f (1 - e^{-t/RC})$$

$$Q = 80 \mu\text{C} (1 - e^{-\frac{20 \text{ ms}}{80 \text{ ms}}}) = 17.7 \mu\text{C}$$

e) (15) Calculate the time after which the current has decreased to one fourth its maximum value.

$$I = I_0 e^{-t/RC}$$

$$\frac{1}{4} I_0 = I_0 e^{-t/RC}$$

$$\frac{1}{4} = e^{-t/RC}$$

$$\ln \frac{1}{4} = -\frac{t}{RC}$$

$$t = -RC \ln \frac{1}{4}$$

$$t = RC \ln 4$$

$$t = 80 \text{ ms} \cdot \ln 4$$

$$t = 111 \text{ ms}$$

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