

Name: Solution February 19, 2022

Total Score: 120 /120

Constants: electron mass  $m_e = 9.11 \times 10^{-31}$  kg

proton mass  $m_p = 1.67 \times 10^{-27}$  kg

$e = 1.602 \times 10^{-19}$  C

$\epsilon_0 = 8.85 \times 10^{-12}$  C<sup>2</sup>/N•m<sup>2</sup>

$k = 9.0 \times 10^9$  N•m<sup>2</sup>/C<sup>2</sup>

$$F = k \frac{|q_1 q_2|}{r^2}$$

$$E = k \frac{q}{r^2}$$

$$\vec{F} = q\vec{E}$$

$$U = k \frac{q_1 q_2}{r}$$

$$V = k \frac{q}{r}$$

$$U = qV$$

$$C = \frac{Q}{\Delta V}$$

$$C = \kappa \epsilon_0 \frac{A}{d}$$

$$\Delta V = Ed$$

$$U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \frac{Q^2}{C}$$

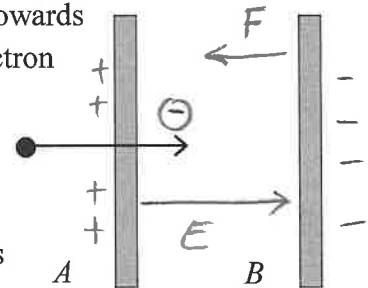
$$K = \frac{1}{2} m v^2$$

$$\Delta K = -q \Delta V \quad \text{parallel: } C_{eq} = \sum_i C_i$$

$$\text{series: } \frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$

C 1.(5) An electron travels through a small hole in plate A and then towards plate B. A uniform electric field between the plates **slows down** the electron without deflecting it. What is the direction of the electric field?

- A) up      B) down      C) right      D) left

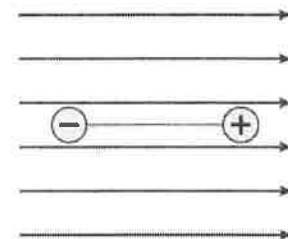


C 2. (5) As the electron in question 1 goes from plate A to plate B, its potential energy \_\_\_\_\_ and it moves towards \_\_\_\_\_ electric potential.

- A) increases, higher      B) decreases, higher  
C) increases, lower      D) decreases, lower

E 3. (5) An electric dipole is placed in a uniform electric field that is directed to the right, as shown in the figure. Which is true?

- A) The dipole moves to the right.  
 B) The dipole moves to the left.  
 C) The dipole rotates clockwise.  
 D) The dipole rotates counterclockwise.  
E) The dipole remains motionless.



B 4.(5) Which of the following is true?

- A) Equipotentials are parallel to the electric field vector.  
 B) The surface of a conductor is an equipotential surface.  
 C) The electric field vector points towards higher potential.  
 D) The electric field is smaller where the equipotentials are close together and larger where they are spaced further apart

20/20 points for this page

5. (20) A parallel plate capacitor consists of two square plates of side length 6.0cm that are spaced 2.0mm apart with air between the plates. Each plate stores a charge of 2.0 nC.

a) (5) Calculate the capacitance.

$$C = \kappa \epsilon_0 \frac{A}{d} = 1 \cdot 8.85 \times 10^{-12} \frac{\text{C}^2 \text{N}}{\text{Nm}^2} \cdot \frac{(0.06 \text{ m})^2}{2 \times 10^{-3} \text{ m}} = 1.59 \times 10^{-11} \text{ F} = 16 \text{ pF}$$

b)(5) Calculate the potential difference between the plates.

$$C = \frac{Q}{\Delta V} \quad \Delta V = \frac{Q}{C} = \frac{2 \times 10^{-9} \text{ C}}{1.6 \times 10^{-11} \text{ F}} = 125 \text{ V}$$

c) (5) Calculate the electric field between the plates.

$$\Delta V = E d \quad E = \frac{\Delta V}{d} = \frac{125 \text{ V}}{2 \times 10^{-3} \text{ m}} = 6.25 \times 10^4 \frac{\text{V}}{\text{m}}$$

d) (5) The capacitor is **disconnected** from the battery, and the plates are moved further apart. Which of these quantities **increases**? Circle the correct answer.

charge  
same

capacitance  
↓

potential difference

electric field  
same

$$\Delta V = \frac{Q}{C}$$

6. (20) The potential difference between two plates of a parallel plate capacitor equals 3,000 V. An electron is launched from the negative plate with a speed of  $1.5 \times 10^7$  m/s.

a) Derive a **symbolic answer** in terms of system parameters and calculate a **numerical value** for the speed with which the electron strikes the positive plate.

$$\Delta K = -q \Delta V$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = -(-e) \Delta V$$

$$\frac{1}{2} m v_f^2 = \frac{1}{2} m v_i^2 + e \Delta V$$

$$v_f = \sqrt{v_i^2 + \frac{2e \Delta V}{m}}$$

$$v_f = \sqrt{\left(1.5 \times 10^7 \frac{\text{m}}{\text{s}}\right)^2 + \frac{2 \cdot 1.6 \times 10^{-19} \text{ C} \cdot 3000 \text{ V}}{9 \times 10^{-31} \text{ kg}}}$$

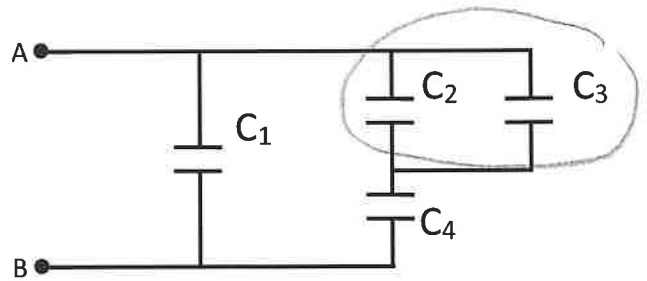
$$v_f = 3.6 \times 10^7 \frac{\text{m}}{\text{s}}$$

b) (5) During this process, what is the electron's change in kinetic energy in electron volt?

$$3000 \text{ eV}$$

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7. (20) For the capacitor circuit in the figure,  
 $C_1 = 6 \text{ pF}$ ,  $C_2 = 10 \text{ pF}$ ,  $C_3 = 30 \text{ pF}$ , and  $C_4 = 40 \text{ pF}$ .  
 a)(15) Find the equivalent capacitance.



Parallel

$$C_{23} = C_2 + C_3 = 10 \text{ pF} + 30 \text{ pF}$$

$$C_{23} = 40 \text{ pF}$$

Series:

$$\frac{1}{C_{234}} = \frac{1}{C_{23}} + \frac{1}{C_4}$$

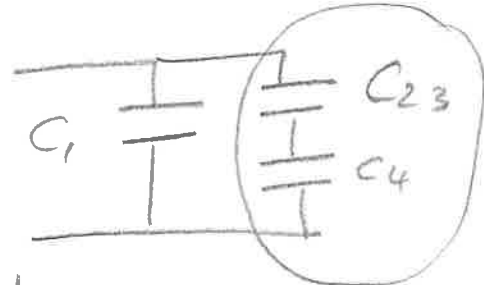
$$= \frac{1}{40 \text{ pF}} + \frac{1}{40 \text{ pF}} = \frac{2}{40 \text{ pF}} = \frac{1}{20 \text{ pF}}$$

$$C_{234} = 20 \text{ pF}$$

Parallel:

$$C_{eq} = C_1 + C_{234} = 6 \text{ pF} + 20 \text{ pF}$$

$$C_{eq} = 26 \text{ pF}$$



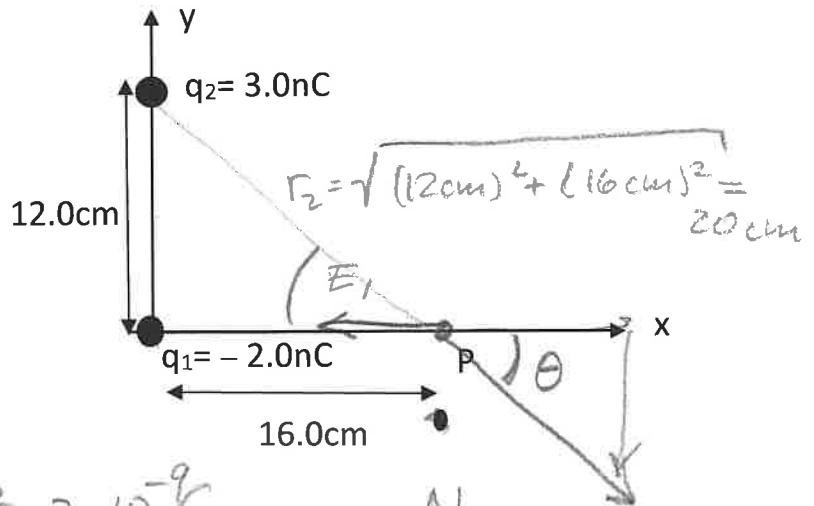
b) (5) If the applied voltage between points A and B is  $V_{ab} = 20 \text{ V}$ , find the total charge on the system.

$$C_{eq} = \frac{Q_{net}}{\Delta V}$$

$$\begin{aligned} Q_{net} &= C_{eq} \Delta V \\ &= 26 \text{ pF} \cdot 20 \text{ V} \\ &= 520 \times 10^{-12} \text{ C} \\ &= 0.52 \text{ nC} \end{aligned}$$

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8. (40) Two charges,  $q_1 = -2.0 \text{ nC}$  and  $q_2 = +3.0 \text{ nC}$ , are located as shown in the figure. Charge  $q_1$  is at the origin. Charge  $q_2$  is located on the y-axis at  $y = 12.0 \text{ cm}$ . Point P is on the x-axis at  $x = 16.0 \text{ cm}$ .



a)(5) At point P, draw the electric field vectors created by each of the charges.

b)(10) Calculate the magnitudes of the electric fields created by each of the charges at point P.

$$E_1 = k \frac{|q_1|}{r_1^2} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \cdot \frac{2 \times 10^{-9} \text{C}}{(0.16 \text{m})^2} = \underline{703 \frac{\text{N}}{\text{C}}}$$

$$E_2 = k \frac{|q_2|}{r_2^2} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \cdot \frac{3 \times 10^{-9}}{(0.2 \text{m})^2} = \underline{675 \frac{\text{N}}{\text{C}}}$$

$$\tan \theta = \frac{12 \text{cm}}{16 \text{cm}} = \frac{3}{4}$$

$$\theta = \underline{37^\circ}$$

c)(10) Calculate the x- and y- components of the net electric field at point P.

$$E_{1x} = -E_1 = -703 \text{ N/C}$$

$$E_{2x} = E_2 \cos \theta = 540 \text{ N/C}$$

$$E_{1y} = 0$$

$$E_{2y} = -E_2 \sin \theta = -405 \text{ N/C}$$

$$E_{\text{net}x} = E_{1x} + E_{2x} = \underline{-163 \text{ N/C}}$$

$$E_{\text{net}y} = E_{1y} + E_{2y}$$

$$= \underline{-405 \text{ N/C}}$$

d)(5) Calculate the magnitude of the net electric field at point P.

$$E_{\text{net}} = \sqrt{E_{\text{net}x}^2 + E_{\text{net}y}^2} = \sqrt{(163 \frac{\text{N}}{\text{C}})^2 + (405 \frac{\text{N}}{\text{C}})^2} = \underline{436 \frac{\text{N}}{\text{C}}}$$

e)(10) Calculate the electric potential at point P.

$$V = k \frac{q_1}{r_1} + k \frac{q_2}{r_2} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \left( -\frac{2 \times 10^{-9} \text{C}}{0.16 \text{m}} + \frac{3 \times 10^{-9} \text{C}}{0.2 \text{m}} \right)$$

$$V = \underline{22.5 \text{ V}}$$

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