

Name: Solution March 16, 2022

Total Score: 120 /120

$e = 1.602 \times 10^{-19} \text{ C}$      $I = \frac{\Delta q}{\Delta t}$      $I = \frac{V}{R}$      $R = \rho \frac{L}{A}$      $P = IV = \frac{V^2}{R} = I^2 R$

series:  $R_{eq} = \sum_i R_i$     parallel:  $\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$      $C = \frac{Q}{V}$

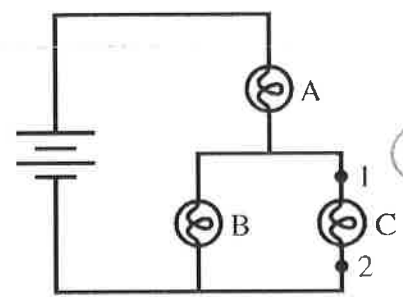
Discharging     $Q = Q_0 e^{-\frac{t}{RC}}$      $I = I_0 e^{-\frac{t}{RC}}$      $V_C = V_{C0} e^{-\frac{t}{RC}}$

Charging     $Q = Q_f (1 - e^{-\frac{t}{RC}})$      $I = I_0 e^{-\frac{t}{RC}}$      $V_C = V_{Cf} (1 - e^{-\frac{t}{RC}})$

B 1. (5) To simultaneously measure the current in a resistor and the voltage across the resistor, you must place an ammeter in \_\_\_\_\_ with the resistor and a voltmeter in \_\_\_\_\_ with the resistor.

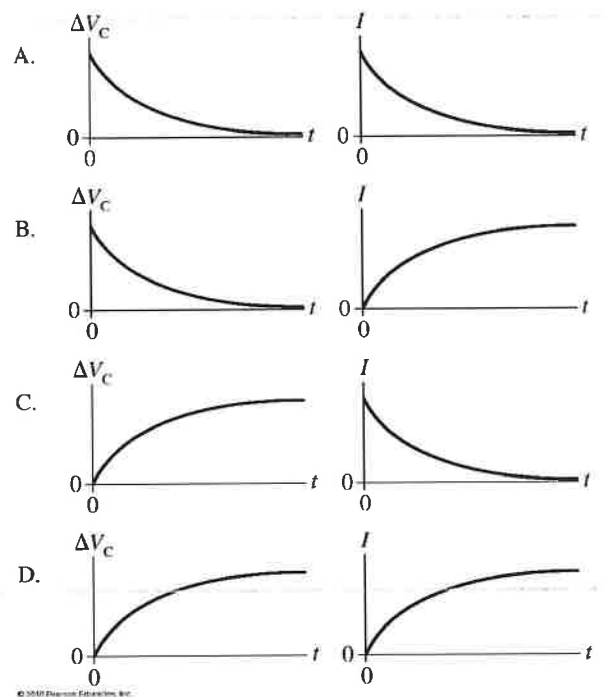
- A) series, series
- B) series, parallel
- C) parallel, series
- D) parallel, series

D 2. (5) The three bulbs in the circuit below are identical. Which of the following is true?



- A)  $V_A = V_B$
- B)  $I_A = I_B$
- C)  $V_A < V_B$
- D)  $V_A > V_B$

A 3. (5) A capacitor is discharged through a resistor. Which of the graphs at the right represent best the capacitor voltage and the current?



B 4. (5) A capacitor is discharged through a resistor. After 50 ms, the current has fallen to one third of its initial value. The circuit's time constant is approximately  
 A) 17ms    B) 45 ms    C) 55 ms    D) 150 ms

$\tau: I = 37\% I_0$   
 37% later than 33%

20/20 points for this page

5. (20) A copper wire of diameter 1.0mm and length 20cm carries a current of 3.0A. The resistivity of copper is  $1.7 \times 10^{-8} \Omega \cdot \text{m}$ .

a) (5) Calculate the resistance of the wire.

$$R = \rho \frac{L}{A} = 1.7 \times 10^{-8} \Omega \cdot \text{m} \frac{0.2 \text{ m}}{\frac{\pi}{4} (10^{-3} \text{ m})^2} = 4.3 \times 10^{-3} \Omega$$

b) (5) Calculate the potential difference between the ends of the wire.

$$I = \frac{V}{R} \quad V = IR = 0.013 \text{ V} = 13 \text{ mV}$$

c) (5) Calculate the power dissipated in the wire.

$$P = I^2 R = 0.039 \text{ W}$$

d) (5) How many electrons per second pass through the cross section of the wire?

$$I = \frac{\Delta q}{\Delta t} = \frac{Ne}{\Delta t}$$

$$N = \frac{I \Delta t}{e} = \frac{3 \text{ A} \cdot 1 \text{ s}}{1.6 \times 10^{-19} \text{ C}} = 1.875 \times 10^{19}$$

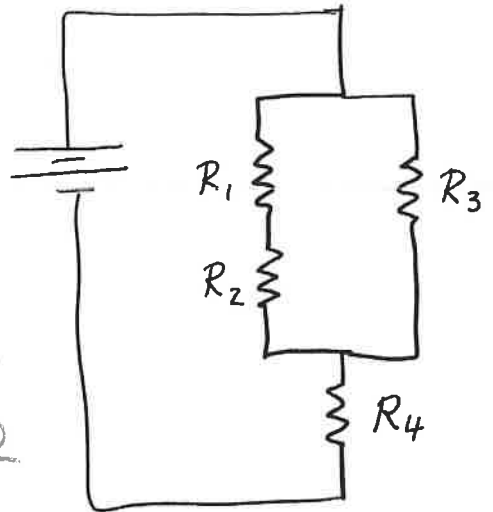
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6. a) (20) In the circuit in the figure,  $R_1=5\ \Omega$ ,  $R_2=15\ \Omega$ ,  $R_3=30\ \Omega$ , and  $R_4=8\ \Omega$ . Find the equivalent resistance of the circuit.

Series:  $R_1 + R_2 = 5\ \Omega + 15\ \Omega = 20\ \Omega$

Parallel:  $\frac{1}{R_{123}} = \frac{1}{R_{12}} + \frac{1}{R_3} = \frac{1}{20\ \Omega} + \frac{1}{30\ \Omega}$   
 $= \frac{3+2}{60\ \Omega} \quad R_{123} = 12\ \Omega$

Series:  $R_{eq} = R_{123} + R_4 = 12\ \Omega + 8\ \Omega$   
 $R_{eq} = 20\ \Omega$



b) (20) The battery provides a potential difference of 60V. Calculate the total current. Calculate the potential drop across each of the resistors and the current through each resistor. Put the answers in the table below.

$$I_{tot} = \frac{V_{bat}}{R_{eq}} = \frac{60V}{20\ \Omega} = 3A$$

All current through  $R_4 \rightarrow V_4 = R_4 I_4 = 8\ \Omega \cdot 3A = 24V$

Leaves  $60V - 24V = 36V$  for (123)

$$R_3 \text{ sees } 36V \rightarrow I_3 = \frac{V_3}{R_3} = \frac{36V}{30\ \Omega} = 1.2A$$

This leaves  $3A - 1.2A = 1.8A$  for 122

$$V_1 = I_1 R_1 = 1.8A \cdot 5\ \Omega = 9V$$

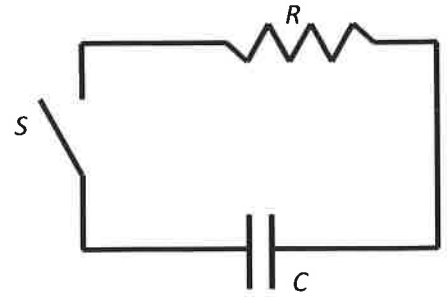
$$V_2 = I_2 R_2 = 1.8A \cdot 15\ \Omega = 27V$$

adds: 36V  
correct

$I_{total} = 3A$	
$V_1 = 9V$	$I_1 = 1.8A$
$V_2 = 27V$	$I_2 = 1.8A$
$V_3 = 36V$	$I_3 = 1.2A$
$V_4 = 24V$	$I_4 = 3A$

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7) (40) A capacitor of capacitance  $C = 3 \mu\text{F}$  has been charged so that the potential difference between its plates is 120 V. The capacitor is then connected to an  $8 \text{ k}\Omega$  resistor as shown. The switch  $S$  is closed and the capacitor begins to discharge.



a) (5) Calculate the time constant of the circuit.

$$\tau = RC = 8 \times 10^3 \Omega \cdot 3 \times 10^{-6} \text{ F} = 24 \times 10^{-3} \text{ s} = 24 \text{ ms}$$

b) (5) Determine the charge on the capacitor when the switch is closed at  $t=0$ .

$$C = \frac{Q}{\Delta V} \quad Q = C \Delta V = 3 \mu\text{F} \cdot 120 \text{ V} = 360 \mu\text{C}$$

c) (5) Calculate the current through the resistor immediately after the switch is closed.

$$I_0 = \frac{\Delta V}{R} = \frac{120 \text{ V}}{8 \times 10^3 \Omega} = 15 \text{ mA}$$

d) (10) Calculate the current 15 ms after the switch is closed.

$$I = I_0 e^{-\frac{t}{RC}} = 15 \text{ mA} e^{-\frac{15 \text{ ms}}{24 \text{ ms}}} = 8 \text{ mA}$$

e) (15) Calculate the time after which the potential difference between the capacitor plates has decreased to one fifth its maximum value.

$$\begin{aligned} \Delta V &= \Delta V_{c0} e^{-\frac{t}{RC}} \\ \frac{1}{5} \Delta V_{c0} &= \Delta V_{c0} e^{-\frac{t}{RC}} \\ \frac{1}{5} &= e^{-\frac{t}{RC}} \\ \ln \frac{1}{5} &= -\frac{t}{RC} \end{aligned}$$

$$\begin{aligned} t &= -RC \ln \frac{1}{5} = RC \ln 5 \\ t &= 38.6 \text{ ms} \end{aligned}$$

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