

Course Learning Assistance

- **Physics Learning Center** (Monday+Wednesday 2:00-4:30 and 6:00 to 8:30 in rooms 129 and 130 Physics)
- **LEAD/tutoring** sessions (will start in 2nd/3rd week, time t.b.a.)

<http://lead.mst.edu/>

<http://studentsuccess.mst.edu/tutoring/>

- **Disability Support Services** (accommodation letters)

<http://dss.mst.edu/>

- **Testing Center**

<http://testcenter.mst.edu/>

Exam 1

Exam 1 is on Tuesday, February 14, from 5:00-6:00 PM.

According to the Student Academic Regulations “The period from 5:00 – 6:00 PM daily [is] to be designated for common exams. If a class or other required academic activity is scheduled during common exam time, the instructor of the class that conflicts with the common exam will provide accommodations for the students taking the common exam.”

- other time conflicts are the **student's responsibility** unless the student participates in a major university or intercollegiate event

Career Fair

The spring career fair is on Tuesday, February 21, 2017. There is **no conflict** between the exam and career fair.

If you have an interview or must attend a career fair activity during your recitation time on February 21, contact your **recitation instructor**.

We understand the importance of the career fair and will work with you to make sure you can participate!

Labs

Labs begin week of Jan 23!

<http://campus.mst.edu/physics/courses/2135lab/>

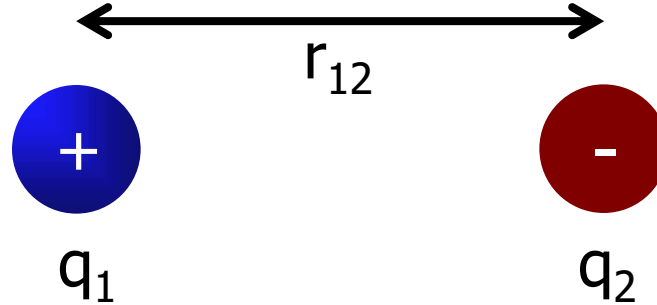
“Odd” labs (3L01, 3L03, etc.) start the week of January 23
“Even” labs (3L02, 3L04, etc.) start the week of January 30

Purchase a lab manual in the Physics office. Students not purchasing a lab manual will receive a lab grade of zero. Lab manuals are not available at the bookstore.

Review of Lecture 1, Part I

Coulomb's Law:

$$F_{12} = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r_{12}^2},$$



Coulomb's Law quantifies the force between charged particles.

Review of Lecture 1, Part II

Electric field:

- charges create electric fields
- electric field of a point charge:

magnitude: $E = k \frac{|q|}{r^2}$

direction: away from , towards 

- force felt by charge in electric field: $\vec{F} = q\vec{E}$

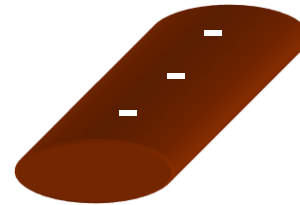
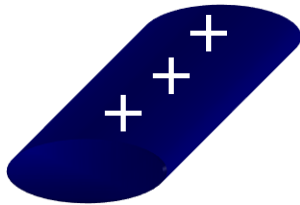
positive charge feels force **in direction of** \vec{E}

negative charge feels force in direction **opposite to** \vec{E}

Today's agenda:

Electric field due to a charge distribution.

You must be able to calculate electric field of a continuous distribution of charge.



Electric Field

Due to a Continuous Charge Distribution

Problem:

- our equations for the Coulomb force and the electric field hold for **point charges** only

Solution:

- decompose extended object into **charge elements**
- calculate electric field for each element
- sum up (**integrate**) contributions of all elements to obtain total electric field

Electric Field Due to a Continuous Charge Distribution (worked examples)

finite line of charge

general derivation: <http://www.youtube.com/watch?v=WmZ3G2DWHlg>

ring of charge

disc of charge

infinite sheet of charge

infinite line of charge

semicircle of charge

Instead of talking about electric fields of charge distributions, let's work some examples. We'll start with a "line" of charge.

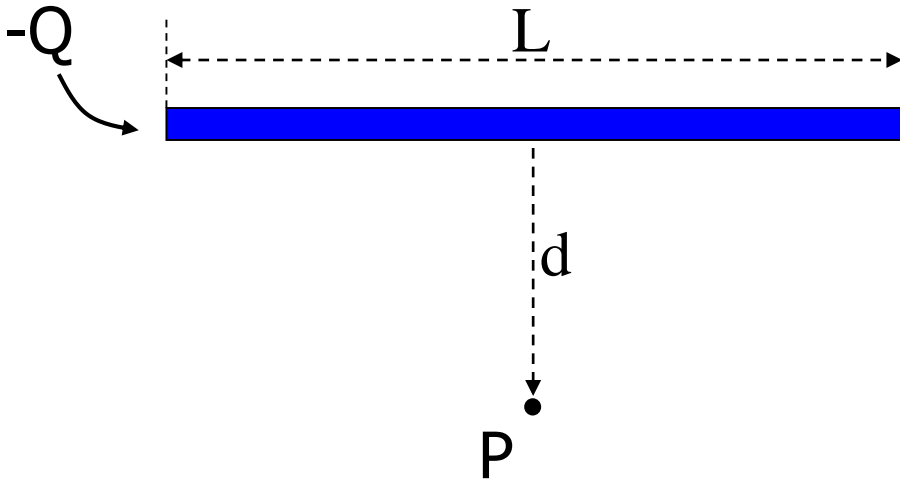
Example: A rod* of length L has a uniformly distributed total positive charge Q . Calculate the electric field at a point P located a distance d below the rod, along an axis through the left end of the rod and perpendicular to the rod.

Example: A rod* of length L has a uniformly distributed total negative charge $-Q$. Calculate the electric field at a point P located a distance d below the rod, along an axis through the center of and perpendicular to the rod.

I will work one of the above examples at the board in lecture. You should try the other for yourself.

*Assume the rod has negligible thickness.

Example: A rod of length L has a uniformly distributed total negative charge $-Q$. Calculate the electric field at a point P located a distance d below the rod, along an axis through the center of and perpendicular to the rod.



Starting equation: $E = k \frac{|q|}{r^2}$

“Legal” version of starting equation:

$$dE = k \frac{|dq|}{r^2}$$

This is “better” because it tells you how to work the problem! It also helps you avoid common vector mistakes.

You should begin electric field of charge distribution problems with this

$$d\mathbf{E} = k \frac{|dq|}{r^2}$$

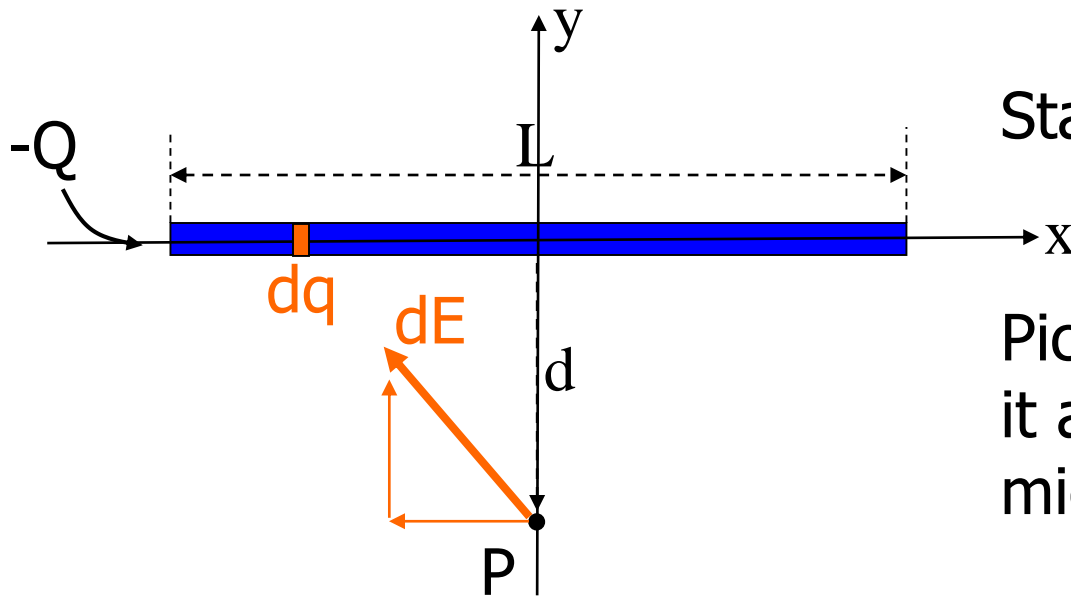
This is a “legal” version of a starting equation, so it is “official.”

because the equation “tells” you how to work the problem.

The equation says:

- (1) pick a dq of charge somewhere in the distribution
- (2) draw in your diagram the \vec{dE} due to that dq
- (3) draw the components of \vec{dE}
- (4) for each component, check for simplifications due to symmetry, then integrate over the charge distribution.

Calculate the electric field at a point P.



Starting equation: $dE = k \frac{|dq|}{r^2}$

Pick a dq (best to not put it at either end or in the middle).

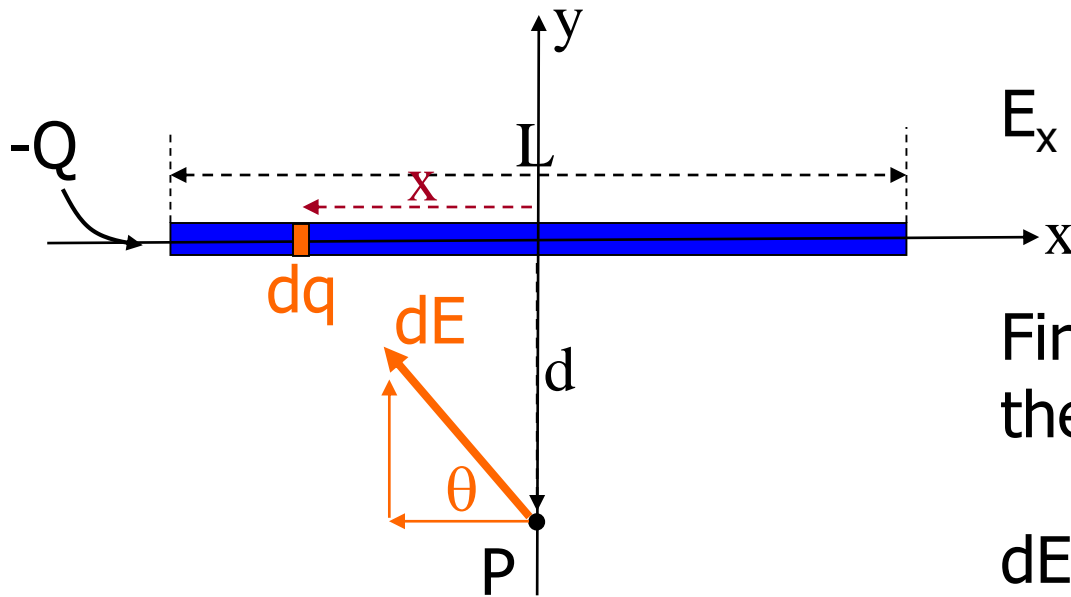
Draw the \vec{dE} due to the dq .

Before I draw the components, I need to define axes!

Now draw the components.

Do you see why symmetry tells me that $E_x = 0$?

Calculate the electric field at a point P.



$E_x = 0$, so calculate E_y

First, label an angle θ in the vector diagram.

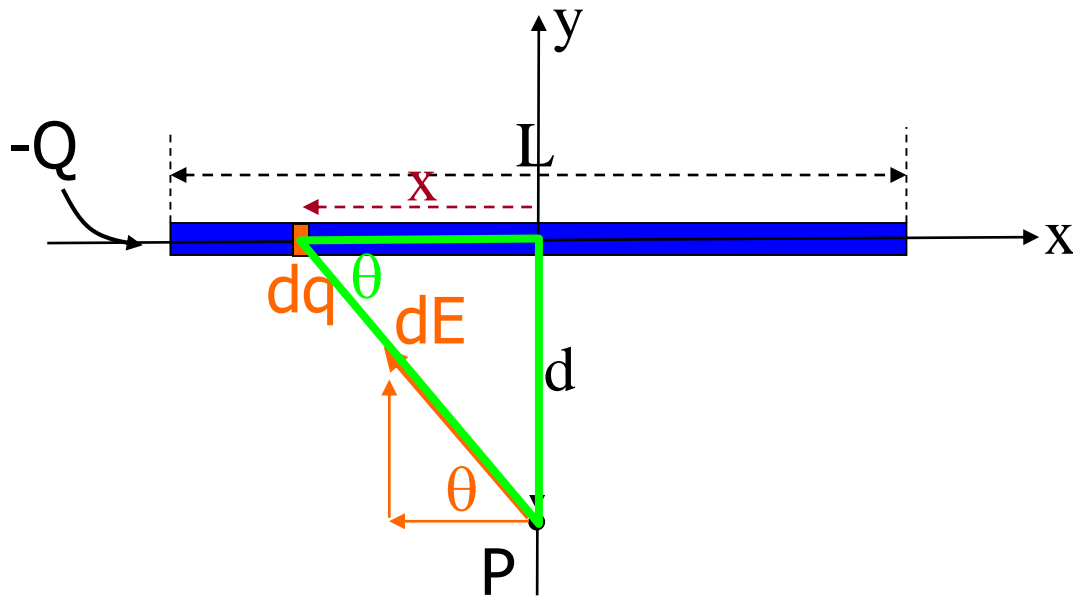
$$dE_y = +dE \sin \theta$$

yes, the + sign conveys important information

To find $\sin \theta$, we need the x-coordinate of dq . If dq is at an arbitrary position along the x-axis, what is a good name for its coordinate? That's right, we'll call it x .

The diagram is getting rather "busy," but we are almost done with it.

Calculate the electric field at a point P.

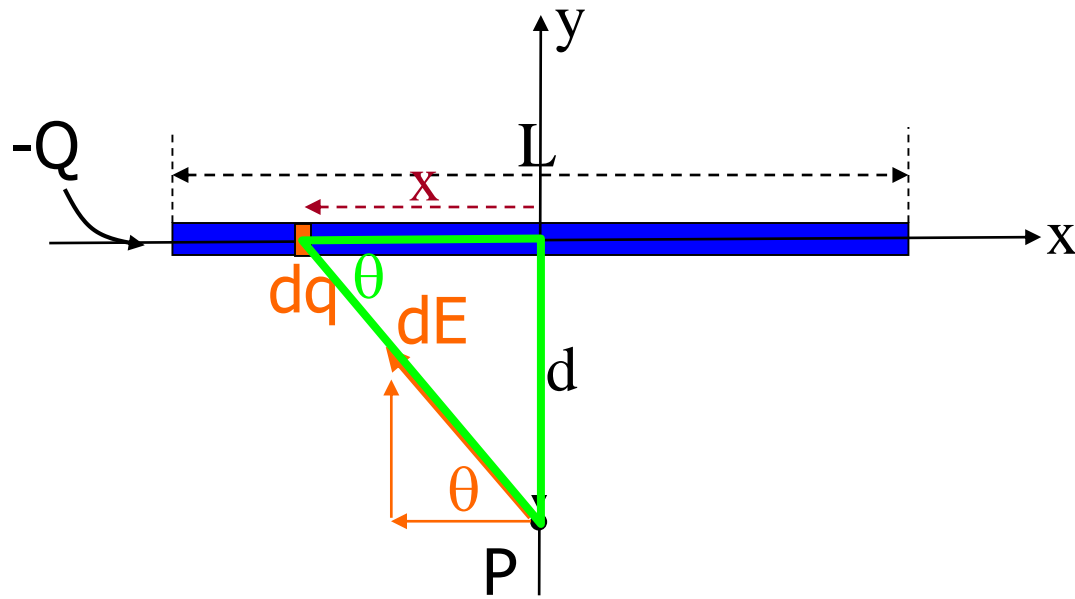


To find $\sin \theta$, look at the green triangle. The sides have length x and d , and hypotenuse r , where

$$r = \sqrt{x^2 + d^2}$$

From the green triangle, we see that $\sin \theta = d / r$.

Calculate the electric field at a point P.

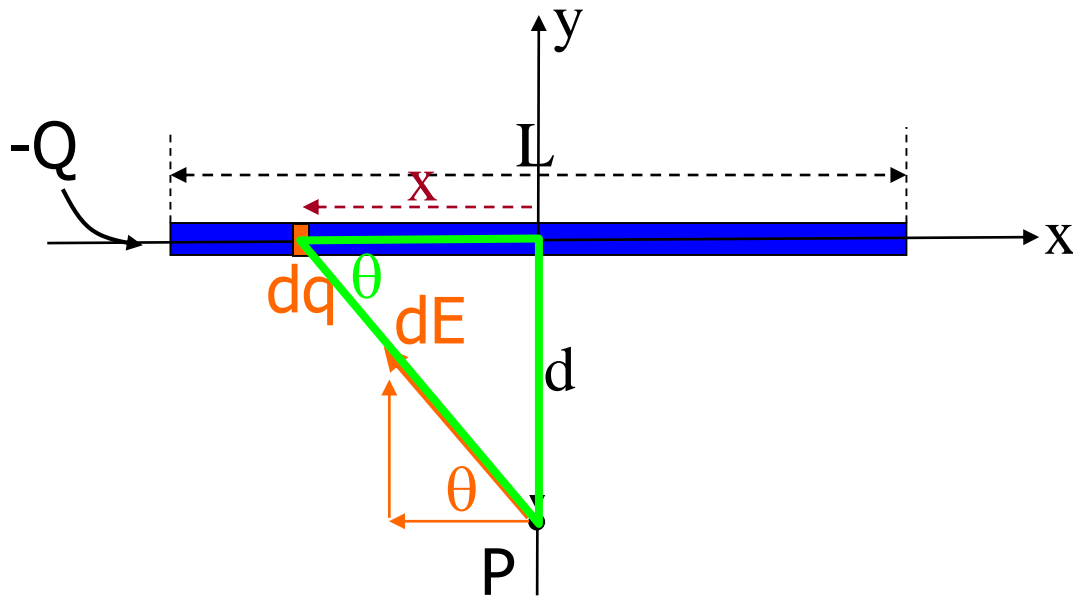


Now we start to put things together:

$$dE_y = +dE \sin \theta = +k \frac{|dq|}{r^2} \sin \theta = +k \frac{|dq|}{r^2} \frac{d}{r} = +k \frac{d|dq|}{r^3} = +kd \frac{|dq|}{(x^2 + d^2)^{3/2}}$$

To find E_y we simply integrate from one end of the rod to the other (from $-L/2$ to $L/2$).

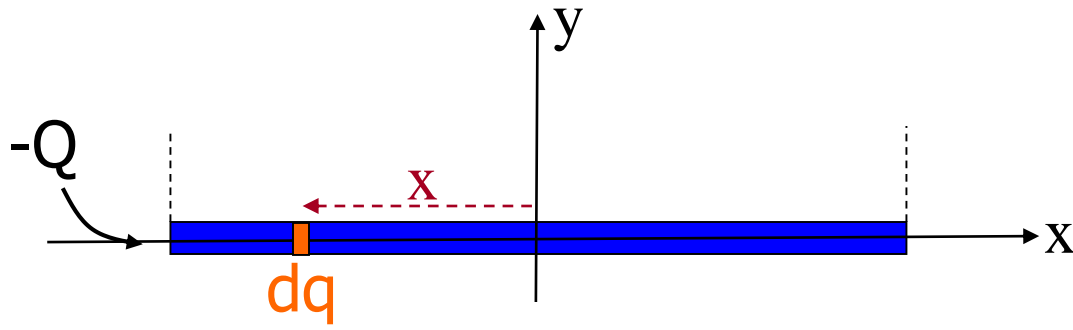
Calculate the electric field at a point P.



$$E_y = \int_{-L/2}^{L/2} k \frac{d|dq|}{(x^2 + d^2)^{3/2}}$$

But wait! We are integrating over the rod, which lies along the x -axis. Doesn't there need to be a dx somewhere?

Calculate the electric field at a point P.



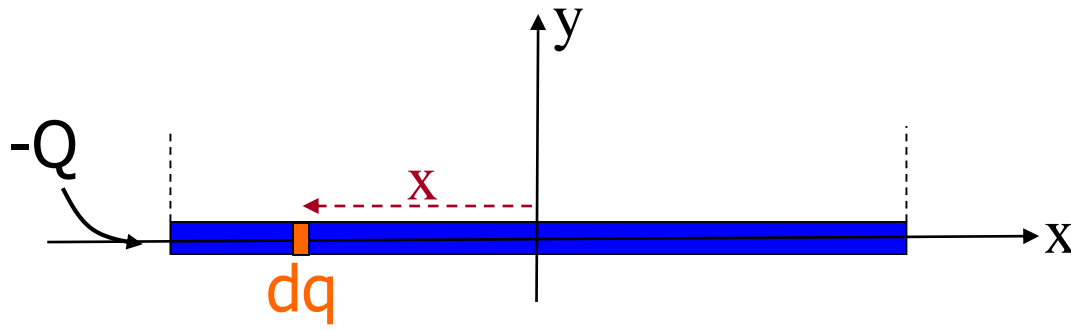
I removed un-needed "stuff" from the figure.

dq is a tiny bit of charge on the uniformly charged rod.

If the charge is uniformly distributed, then the amount of charge per length of rod is

$$\text{(linear charge density)} = \frac{\text{(charge)}}{\text{(length)}} \quad \text{or} \quad \lambda = \frac{Q}{L}$$

Calculate the electric field at a point P.



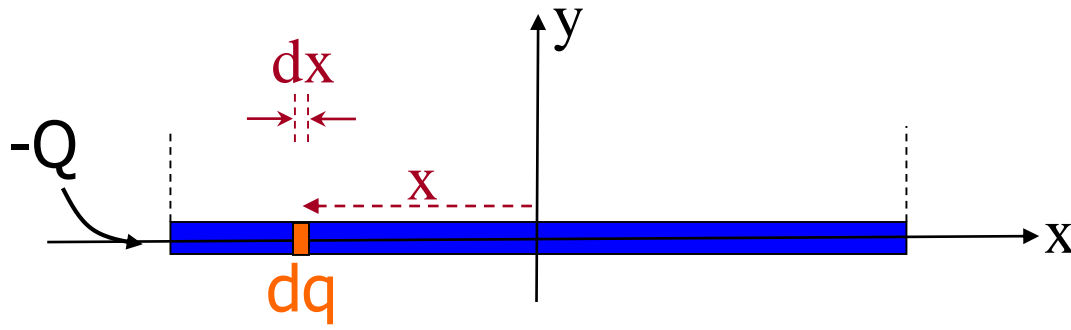
$$\lambda = \frac{Q}{L}$$

We use the symbol λ for linear charge density. You probably thought (based on Physics 1135) that λ is the symbol for wavelength. It is. But not today!

$$(\text{charge on segment of rod}) = \frac{(\text{charge})}{(\text{length})} \times (\text{length of segment of rod})$$

What would be a good name for an infinitesimal length of rod that lies along the x-axis? How about dx ?

Calculate the electric field at a point P.



Thus, $dq = \lambda dx$ and

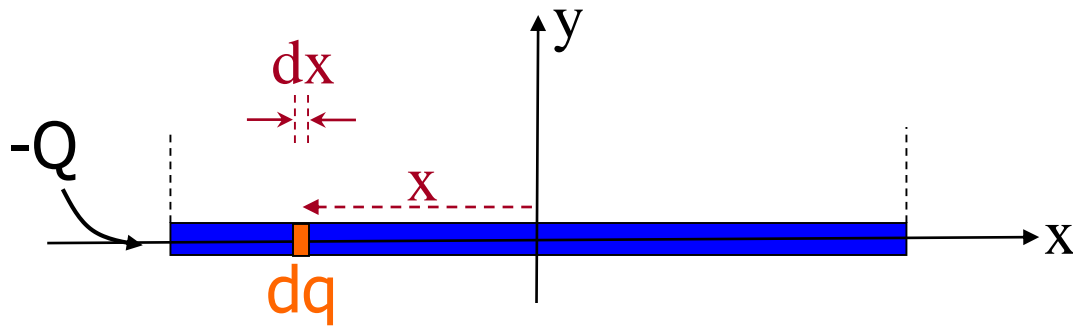
We can take λ outside the integral because the charge is uniformly distributed, so λ must be constant.

$$E_y = \int_{-L/2}^{L/2} kd \frac{|dq|}{(x^2 + d^2)^{3/2}} = \int_{-L/2}^{L/2} kd \frac{|\lambda dx|}{(x^2 + d^2)^{3/2}} = kd |\lambda| \int_{-L/2}^{L/2} \frac{dx}{(x^2 + d^2)^{3/2}}$$

$$E_y = kd \left| \frac{Q}{L} \right| \int_{-L/2}^{L/2} \frac{dx}{(x^2 + d^2)^{3/2}}$$

The physics of the problem is all done. The rest is "just" math.

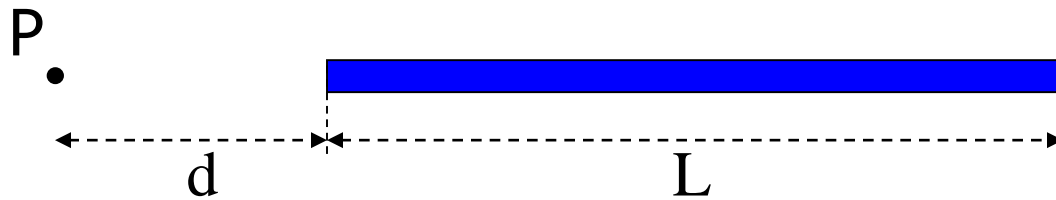
Calculate the electric field at a point P.



$$E_y = kd \left| \frac{Q}{L} \right| \int_{-L/2}^{L/2} \frac{dx}{(x^2 + d^2)^{3/2}}$$

A note on the “just” math part. We expect you to remember derivatives and integrals of simple power and trig functions, as well as exponentials. The rest you can look up; on exams we will provide tables of integrals. We would provide you with the above integral. It is not one that I could do in 5 minutes, so I would not expect you to do it.

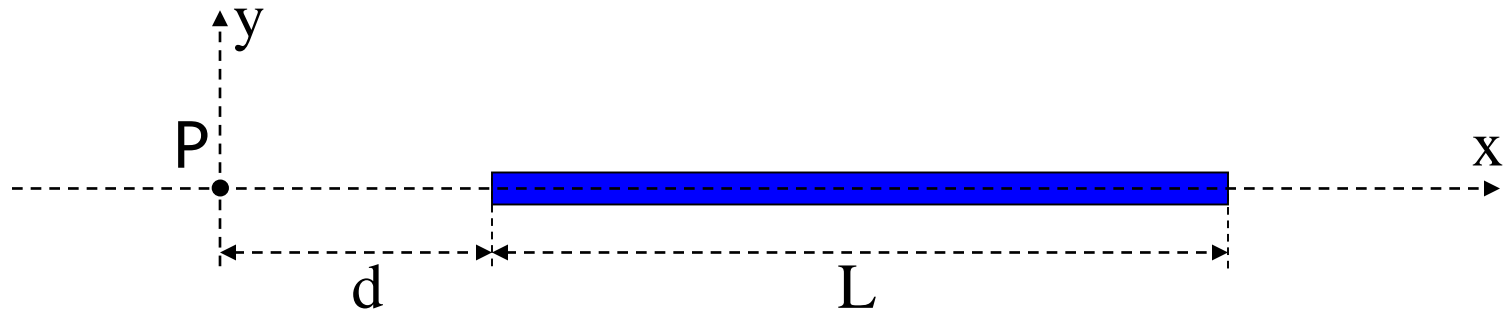
Example: A rod of length L has a uniform charge per unit length λ and a total positive charge Q . Calculate the electric field at a point P along the axis of the rod a distance d from one end.*



To be worked at the board in lecture...

*Assume the rod has negligible thickness.

Example: A rod of length L has a uniform charge per unit length λ and a total positive charge Q . Calculate the electric field at a point P along the axis of the rod a distance d from one end.



It's a good bet we will need x - and y -axes, so let's just put them in right now. Let's put the origin at P .

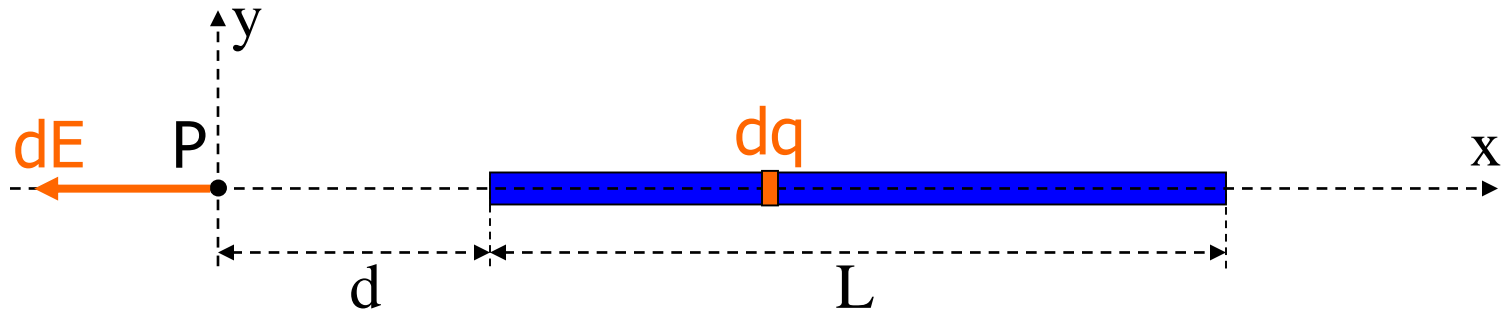
After the previous example, we realize we will need to calculate the linear charge density on the rod.

$$\lambda = \frac{Q}{L} \quad \text{and} \quad Q = \lambda L$$

Note 1: both λ and Q are given, so we can express our answer in terms of our choice of either one.

Note 2: we are told Q is positive, so we don't need the absolute value signs as in the previous example.

Example: A rod of length L has a uniform charge per unit length λ and a total positive charge Q . Calculate the electric field at a point P along the axis of the rod a distance d from one end.

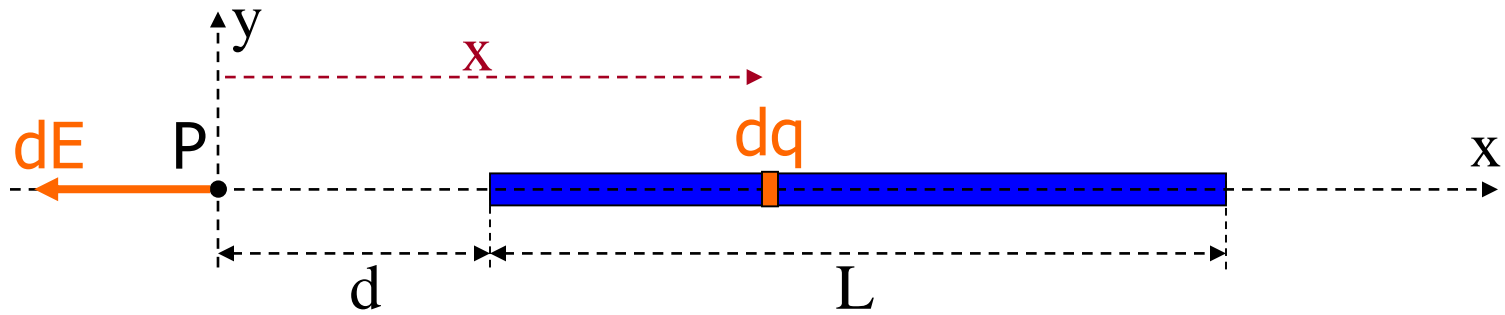


We start with our usual equation: $dE = k \frac{|dq|}{r^2}$

Remember, this is the **best** way to start a problem like this one.

The equation says pick a dq of charge, so do it!

Because dq is positive, its contribution to the electric field points away from the rod.



I could work the problem “all at once” using unit vector notation, but for now I think it’s safer to work out each component separately.

The electric field at point P has no y-component (why?).
Therefore, $E_y=0$.

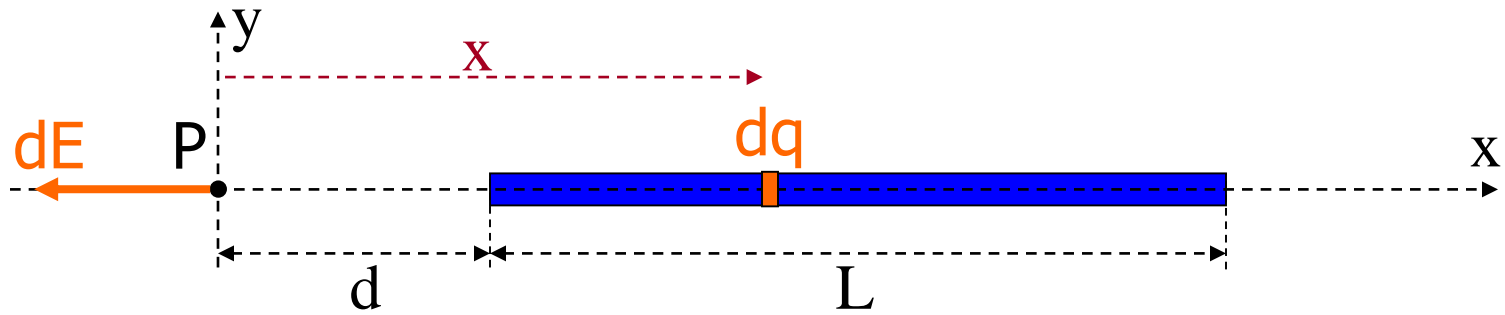
The rod has no thickness.

The infinitesimal charge dq is a distance x away from the origin.

Therefore
$$dE_x = -k \frac{|dq|}{x^2} = -k \frac{|\lambda| dx}{x^2} = -k \frac{\lambda dx}{x^2}$$

The – sign is important!

If I don’t know the sign of dq, I keep the absolute value signs but don’t know the direction of dE_x . If dq is negative, then the safest thing to do is change the – signs in the equation above to +’s, and keep the absolute value signs around dq.

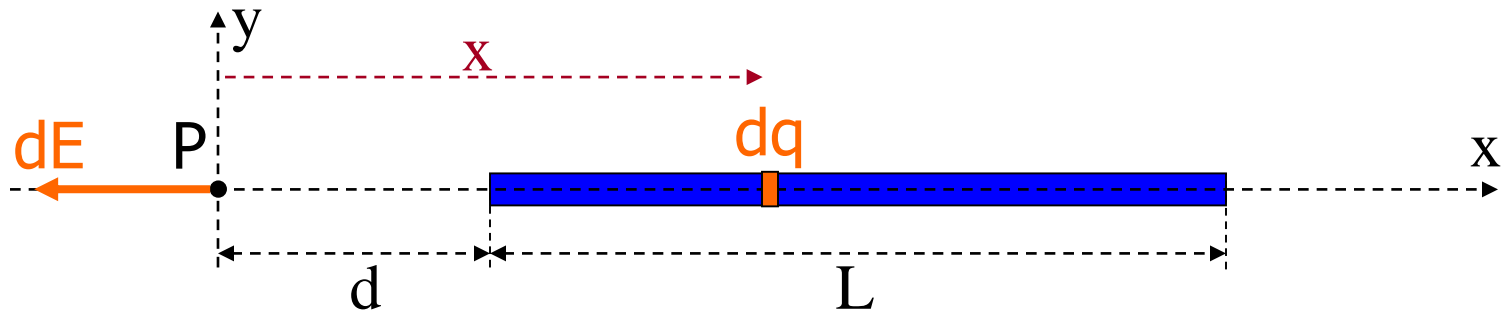


Now simply integrate over the rod.

$$E_x = \int_{\text{rod}} dE_x = \int_d^{d+L} -k \frac{\lambda dx}{x^2} = -k\lambda \int_d^{d+L} \frac{dx}{x^2}$$

$$E_x = -k\lambda \left(-\frac{1}{x} \right) \Big|_d^{d+L} = k\lambda \left(\frac{1}{d+L} - \frac{1}{d} \right) = k\lambda \left(\frac{d - (d+L)}{d(d+L)} \right)$$

$$E_x = -\frac{k\lambda L}{d(d+L)}$$



The problem asks for the electric field at point P. Let's make it easy for a potential grader by writing down our complete answer with a box around it.

Any of the boxed answers below is correct.

If a problem says "express your answer in unit vector notation," you need to do that!

$$E_y = 0 \quad E_x = -\frac{k\lambda L}{d(d+L)}$$

$$\vec{E} = -\frac{k\lambda L}{d(d+L)} \hat{i}$$

$$E = \frac{k\lambda L}{d(d+L)}, \text{ in the } -x \text{ direction}$$

On an exam, put a box around each part of an answer when you finish it, so the grader can clearly see it. You can copy parts of an answer and rewrite them together in one place so you can put a box around the whole answer at once (like I did here), but don't make a mistake copying, because you will lose points. Also, just box one answer. Do not box different versions of the same answer.

Example: calculate the electric field due to an infinite line of positive charge.

There are two approaches to the mathematics of this problem.

One approach is that of example 21.10. See notes [here](#). An alternative mathematical approach is posted [here](#). The result is

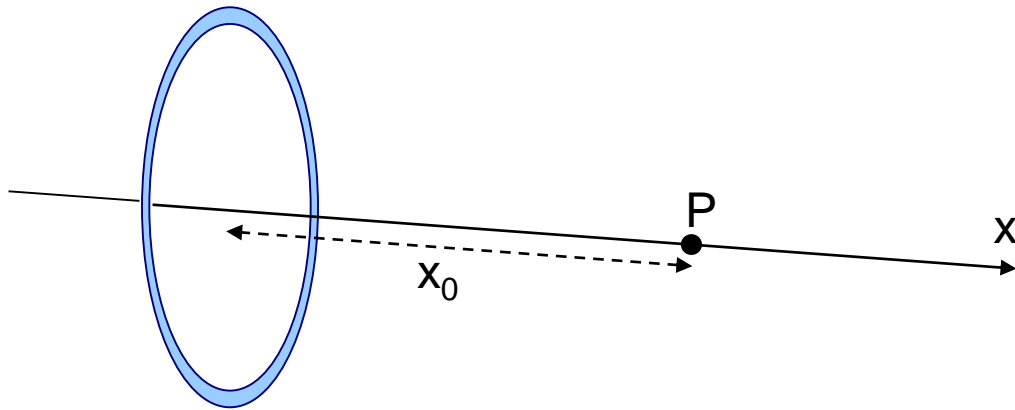
$$E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$$

This is not an "official" starting equation!

The above equation is not on your OSE sheet. In general, you may not use it as a starting equation!

If a homework problem has an infinite line of charge, you would need to repeat the derivation, unless I give you permission to use it.

Example: A ring of radius a has a uniform charge per unit length and a total positive charge Q . Calculate the electric field at a point P along the axis of the ring at a distance x_0 from its center.



To be worked at the blackboard in lecture.

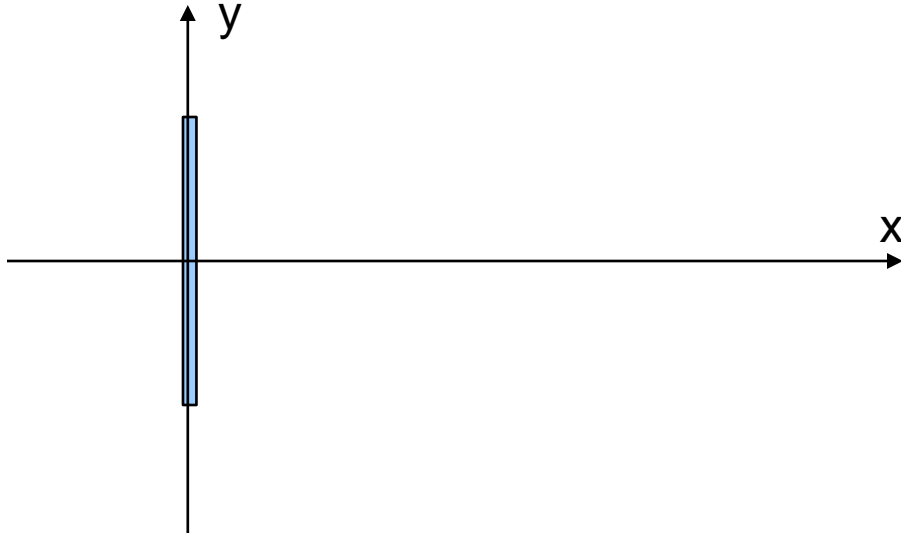
Homework hint: you must provide this derivation in your solution to any problems about rings of charge (e.g. 21.53 or 21.55, if assigned).

Visualization here (requires Shockwave, which downloads automatically):

<http://web.mit.edu/viz/EM/visualizations/electrostatics/calculatingElectricFields/RingIntegration/RingIntegration.htm>

Example: A ring of radius a has a uniform charge per unit length and a total positive charge Q . Calculate the electric field at a point P along the axis of the ring at a distance x_0 from its center.

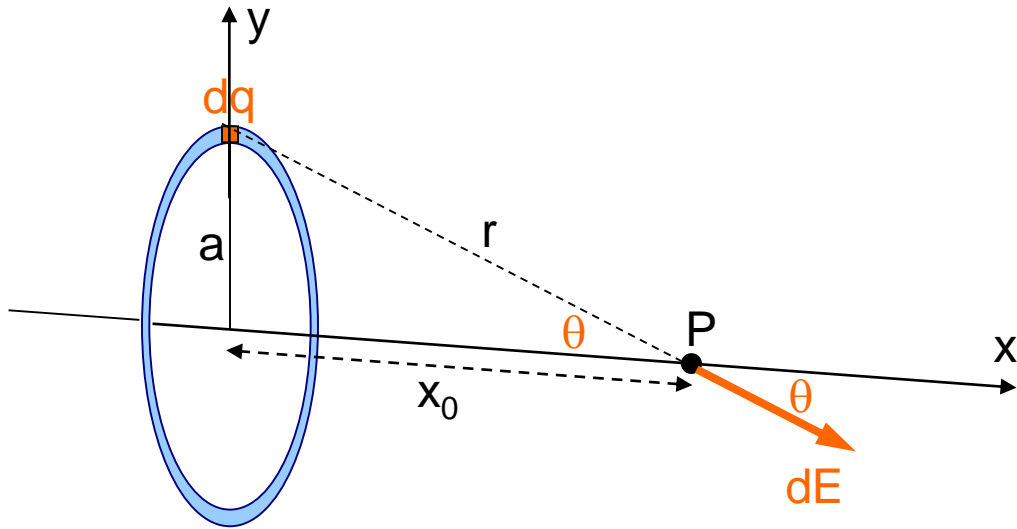
An edge-on view of the ring would look like this:



The z-axis would be coming out of the screen at you.

I will use the perspective view of the ring in my solution.

Example: A ring of radius a has a uniform charge per unit length and a total positive charge Q . Calculate the electric field at a point P along the axis of the ring at a distance x_0 from its center.



Let's add a y -axis to the figure.

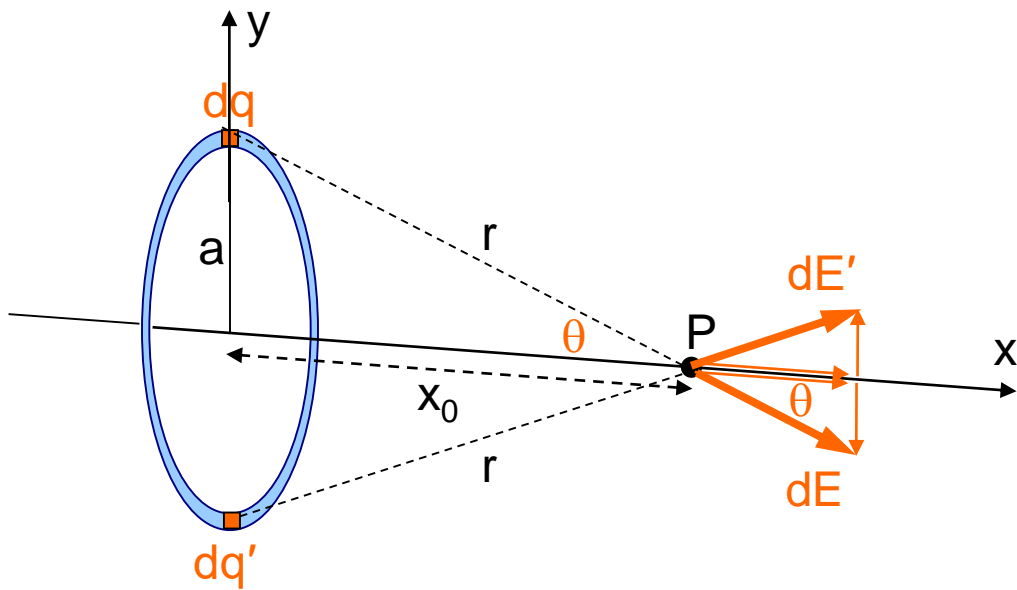
Starting equation:

$$dE = k \frac{|dq|}{r^2}$$

Pick a dq of charge. Let's put it on the y -axis for now.

Show the dE due to that positive dq .

We'll need r and θ later.



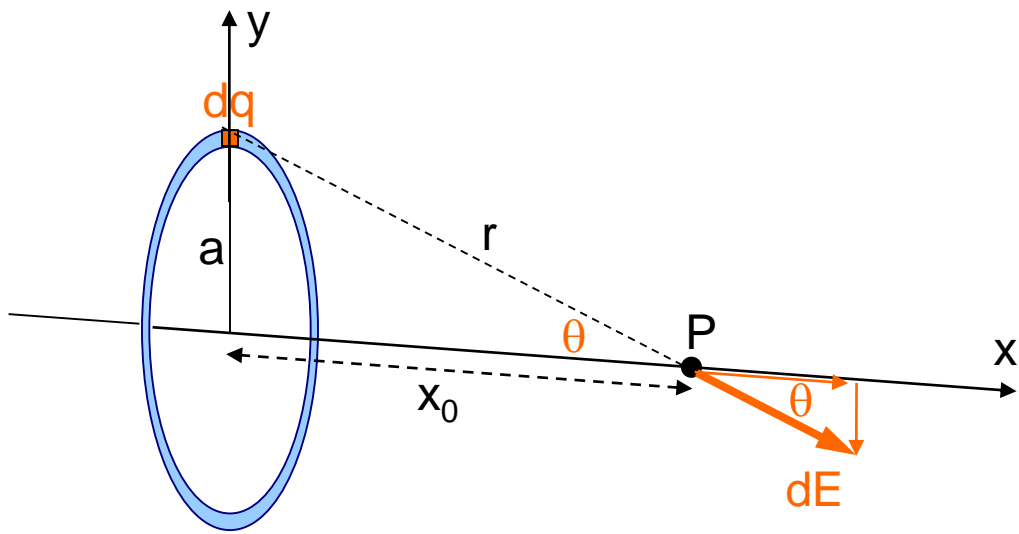
Show the x- and y-components of $d\vec{E}$. There may also be a z-component, which we'll leave out because it is difficult to draw and visualize.

Consider the dq' on the ring where it is intersected by the negative y-axis.

dq' gives rise to $d\vec{E}'$ at P. Show the components of dq' .

All points on the ring are the same distance r from point P. Also, x_0 and θ are the same for all points on the ring.

The y-components cancel pairwise! Same for the z-components (not shown). $E_y = E_z = 0$.



Back to our OSE...

$$dE = k \frac{|dq|}{r^2}$$

$$dE_x = +dE \cos \theta$$

From the diagram: $r = \sqrt{x_0^2 + a^2}$ $\cos \theta = \frac{x_0}{r}$

For a given x_0 , r is a constant for all points on the ring.

Only works because all E_x are in same direction.

$$E_x = \int_{\text{ring}} dE_x = \int_{\text{ring}} \left(k \frac{|dq|}{r^2} \right) \frac{x_0}{r} = k \frac{x_0}{r^3} \int_{\text{ring}} d|q| = k \frac{x_0}{r^3} |Q| = \frac{kx_0 |Q|}{(x_0^2 + a^2)^{3/2}}$$

$$E_x = \int_{\text{ring}} dE_x = \int_{\text{ring}} \left(k \frac{|dq|}{r^2} \right) \frac{x_0}{r} = k \frac{x_0}{r^3} \int_{\text{ring}} d|q| = k \frac{x_0}{r^3} |Q| = \frac{kx_0 |Q|}{(x_0^2 + a^2)^{3/2}}$$

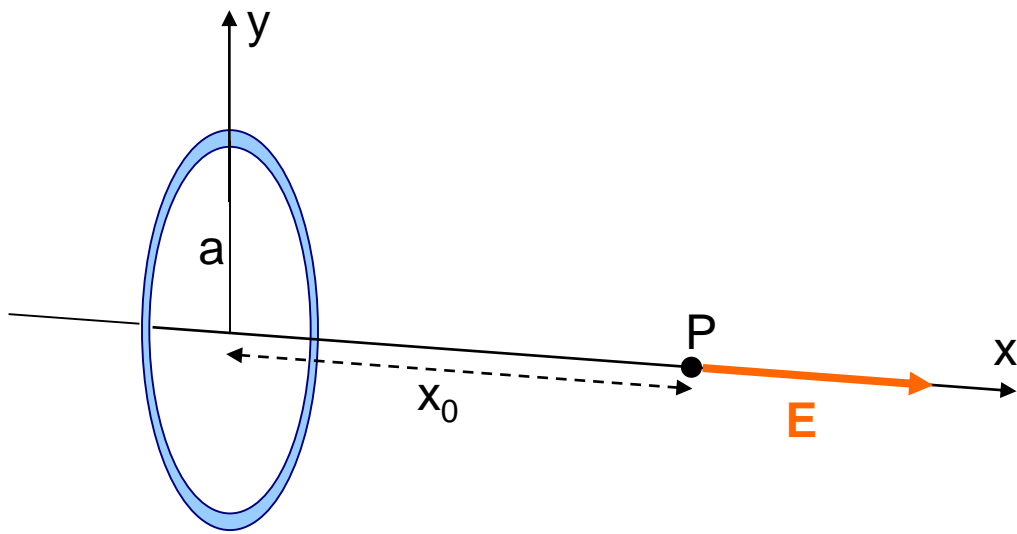
Some of you are wondering why all the absolute value signs.

You don't really need them in this example, because Q is positive.

When I draw the dE_x and dE_y in the diagram, the sign of Q determines the directions of the components.

Because I used the sign of Q to determine the directions of the components in my diagram, I don't want to accidentally use the sign again later and get the wrong direction in my final answer; hence the absolute value signs, for safety.

If x_0 is negative, then E_x points along the $-x$ direction, as it should, so I don't want to put absolute value signs around the x_0 in the answer.



Back to our example...

$$E_x = \frac{kx_0 |Q|}{(x_0^2 + a^2)^{3/2}}$$

$$E_y = E_z = 0$$

Also "legal" answers:

$$\vec{E} = \frac{kx_0 |Q|}{(x_0^2 + a^2)^{3/2}} \hat{i}$$

$$E = \frac{kx_0 |Q|}{(x_0^2 + a^2)^{3/2}}, \text{ away from the center}$$

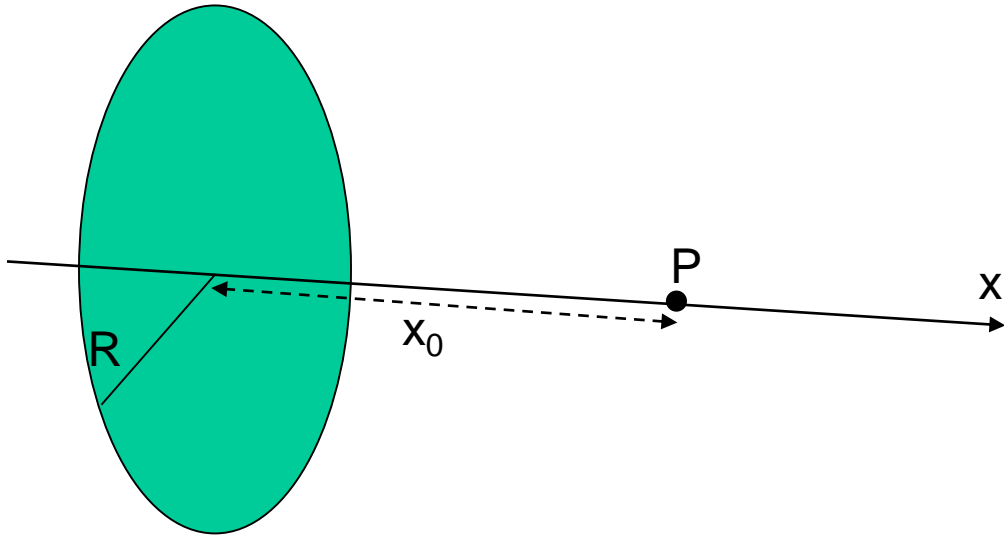
These equations are only valid for P along the x-axis!

Awesome Youtube Derivation: <http://www.youtube.com/watch?v=80mM3kSTZcE>

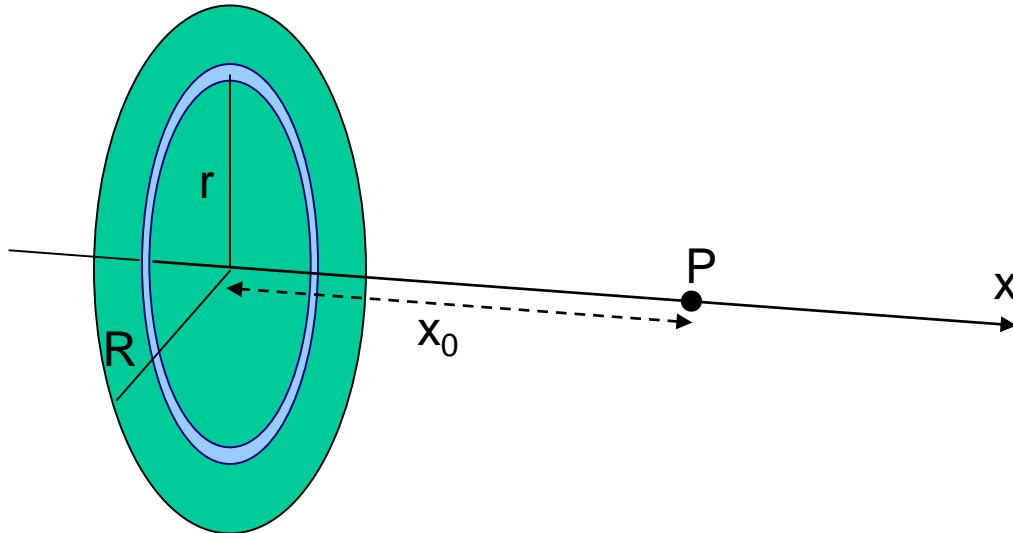
(he leaves out a factor of a in several steps, but finds it in the end).

What would be different if Q were negative? If P were on the negative x-axis?

Example: A disc of radius R has a uniform charge per unit area σ . Calculate the electric field at a point P along the central axis of the disc at a distance x_0 from its center.

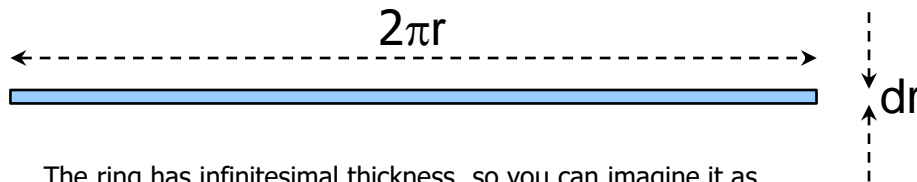


Example: A disc of radius R has a uniform charge per unit area σ . Calculate the electric field at a point P along the central axis of the disc at a distance x_0 from its center.



The disc is made of concentric rings.

Imagine taking a ring and cutting it so you can lay it out along a line.

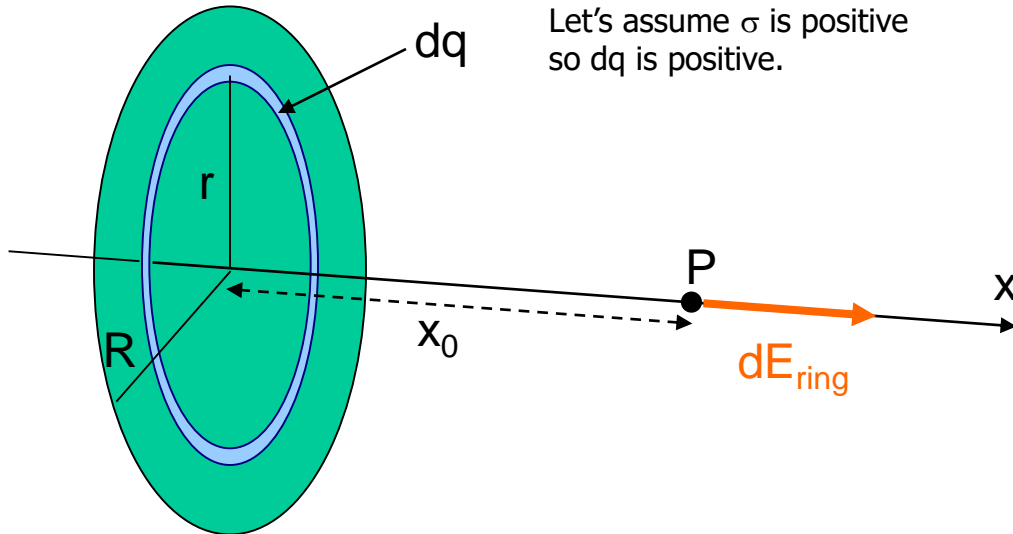


The ring has infinitesimal thickness, so you can imagine it as a rectangular strip.

The length is $2\pi r$, the thickness is dr , so the area of a ring at a radius r is $2\pi r dr$.

Caution! In the previous example, the radius of the ring was R . Here the radius of the disc is R , and the rings it is made of have (variable) radius r .

Example: A disc of radius R has a uniform charge per unit area σ . Calculate the electric field at a point P along the central axis of the disc at a distance x_0 from its center.



The charge on each ring is $dq = \sigma(2\pi r dr)$.

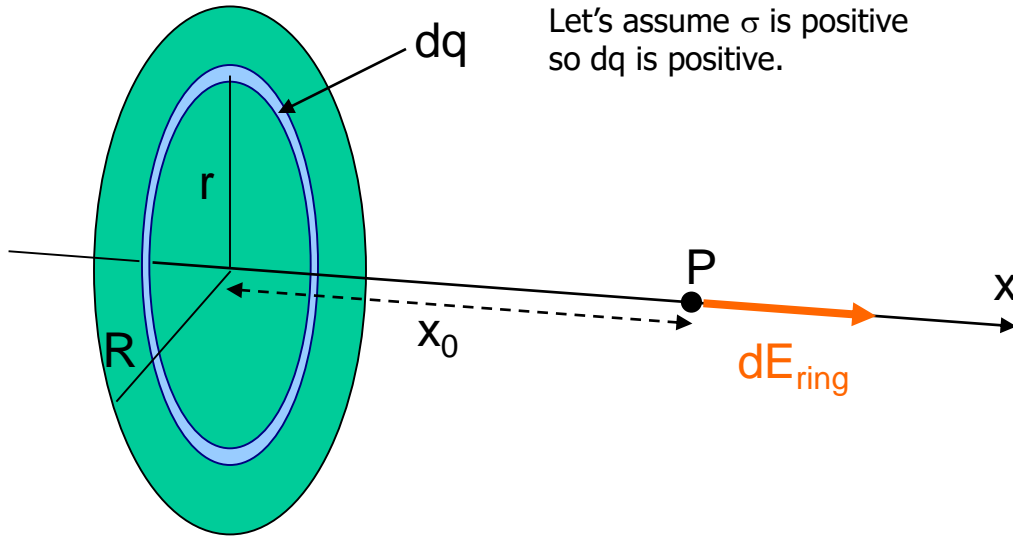
(charge on ring) = (charge per area) \times (area)

We previously derived an equation for the electric field of this ring. We'll call it dE_{ring} here, because the ring is an infinitesimal part of the entire disc.

$$dE_{\text{ring}} = \frac{kx_0 |dq_{\text{ring}}|}{(x_0^2 + r^2)^{3/2}}$$

[Skip to slide 47 for result.](#)

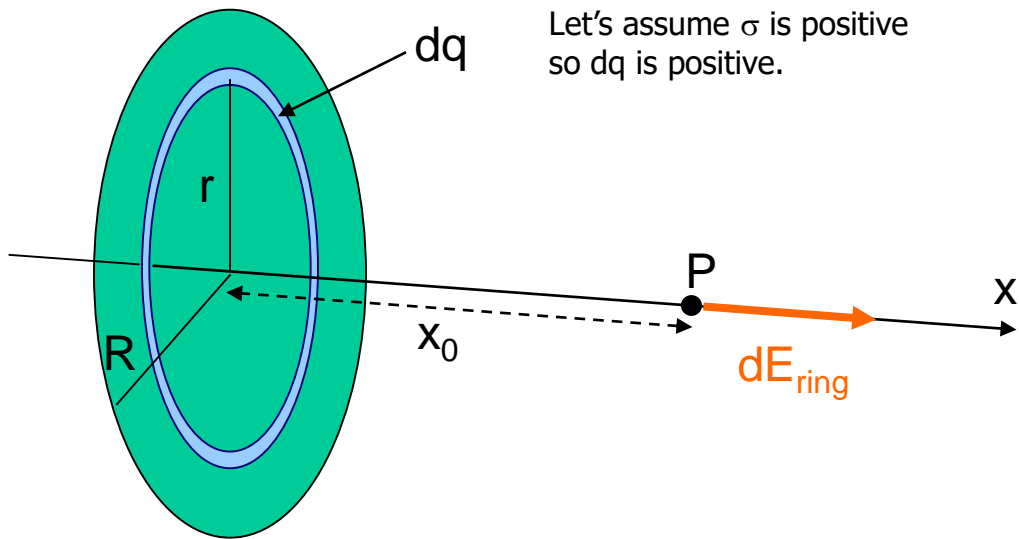
Example: A disc of radius R has a uniform charge per unit area σ . Calculate the electric field at a point P along the central axis of the disc at a distance x_0 from its center.



$$dE_{\text{ring}} = \frac{kx_0 |dq_{\text{ring}}|}{(x_0^2 + r^2)^{3/2}}$$

$$= \frac{kx_0 \sigma (2\pi r dr)}{(x_0^2 + r^2)^{3/2}}$$

$$E_{\text{disc}} = \int_{\text{disc}} dE_{\text{ring}} = \int_{\text{disc}} \frac{kx_0 \sigma 2\pi r dr}{(x_0^2 + r^2)^{3/2}} = kx_0 \pi \sigma \int_0^R \frac{2r dr}{(x_0^2 + r^2)^{3/2}}$$

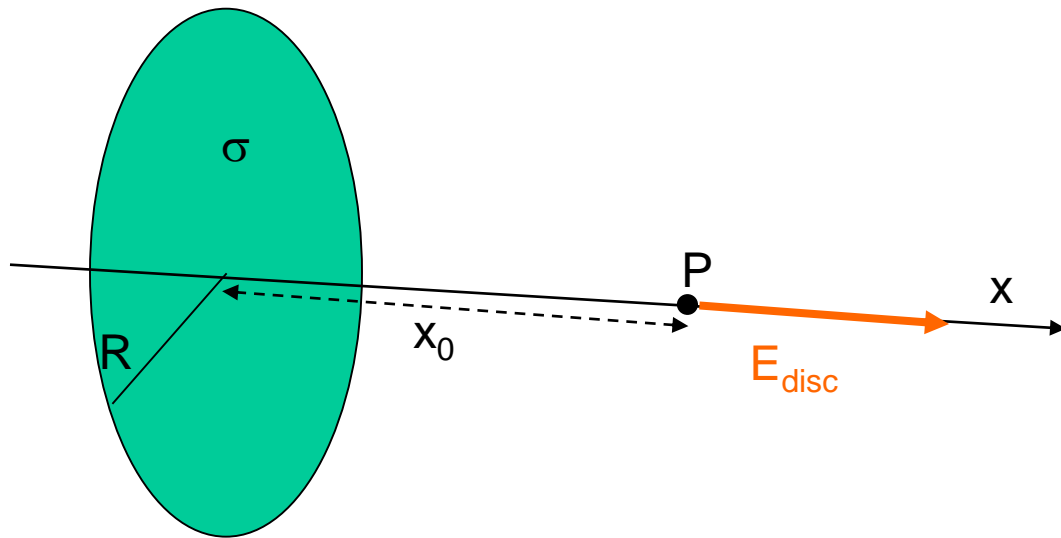


$$E_{\text{disc}} = kx_0 \pi \sigma \int_0^R \frac{2r \, dr}{(x_0^2 + r^2)^{3/2}}$$

You know how to integrate this. The integrand is just (stuff)^{-3/2} d(stuff)

$$E_{\text{disc}} = kx_0 \pi \sigma \left[\frac{(x_0^2 + r^2)^{-1/2}}{-1/2} \right]_0^R = 2k\pi\sigma \left(\frac{x_0}{|x_0|} - \frac{x_0}{(x_0^2 + R^2)^{1/2}} \right)$$

Kind of nasty looking, isn't it.



As usual, there are several ways to write the answer.

$$E_x = 2k\pi\sigma \left(\frac{x_0}{|x_0|} - \frac{x_0}{(x_0^2 + R^2)^{1/2}} \right) \quad E_y = E_z = 0$$

$$\vec{E} = 2k\pi\sigma \left(\frac{x_0}{|x_0|} - \frac{x_0}{(x_0^2 + R^2)^{1/2}} \right) \hat{i}$$

Or you could give the magnitude and direction.

Example: Calculate the electric field at a distance x_0 from an infinite plane sheet with a uniform charge density σ .

An infinite sheet is “the same as” disc of infinite radius.

$$E_{\text{sheet}} = \lim_{R \rightarrow \infty} \left[2k\pi\sigma \left(\frac{x_0}{|x_0|} - \frac{x_0}{(x_0^2 + R^2)^{1/2}} \right) \right]$$

Take the limit and use $k = \frac{1}{4\pi\epsilon_0}$ to get

$$E_{\text{sheet}} = \frac{|\sigma|}{2\epsilon_0}.$$

This is the magnitude of E . The direction is away from a positively-charged sheet, or towards a negatively-charged sheet.

Example: Calculate the electric field at a distance x_0 from an infinite plane sheet with a uniform charge density σ .

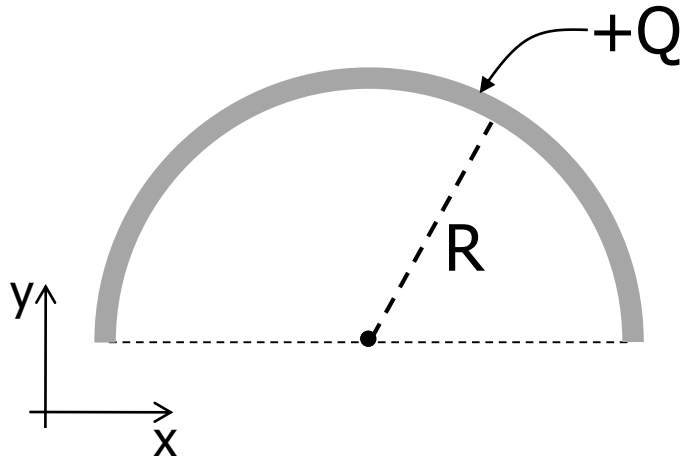
$$E_{\text{sheet}} = \frac{|\sigma|}{2\epsilon_0}.$$

Interesting...does not depend on distance from the sheet. Does that make sense?

This is your fourth Official Starting Equation, and the only one from all of today's lecture!

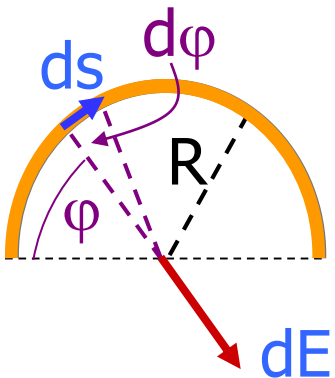
I've been Really Nice and put this on your starting equation sheet. You don't have to derive it for your homework!

Example: calculate the electric field at “center” of semicircular line of uniformly-distributed positive charge, oriented as shown.

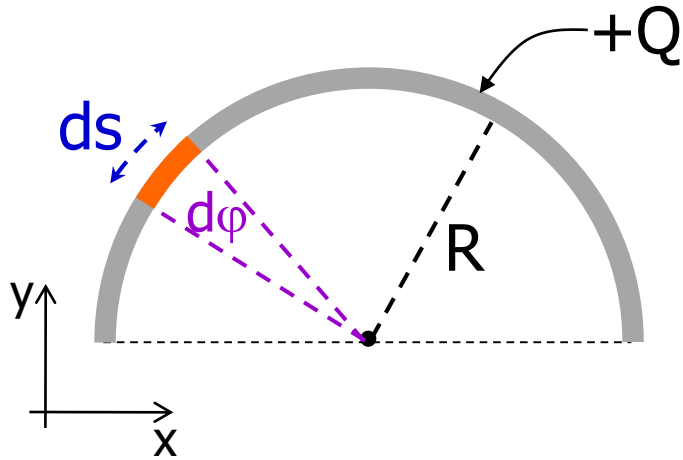


To be worked at the blackboard in lecture.

You don't have to follow the steps in the exact order I present here. Just let the problem tell you what to. You may do things in a different order; that's probably OK.



Example: calculate the electric field at “center” of semicircular line of uniformly-distributed positive charge, oriented as shown.



Start with our usual OSE.

$$dE = k \frac{|dq|}{r^2}$$

Pick an infinitesimal dq of charge.

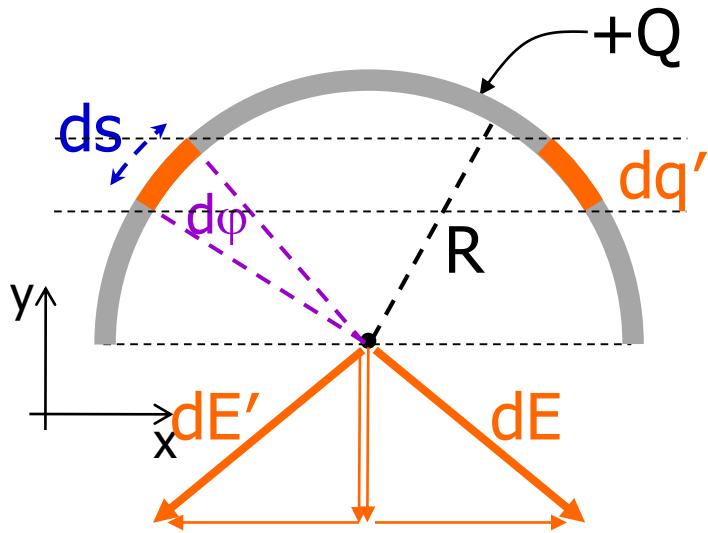
dq subtends an arc length ds, and an angle dφ.

What is the charge dq?

$$dq = (\text{charge per length of arc}) \times (\text{length of the arc})$$

$$dq = \lambda ds$$

Example: calculate the electric field at “center” of semicircular line of uniformly-distributed positive charge, oriented as shown.



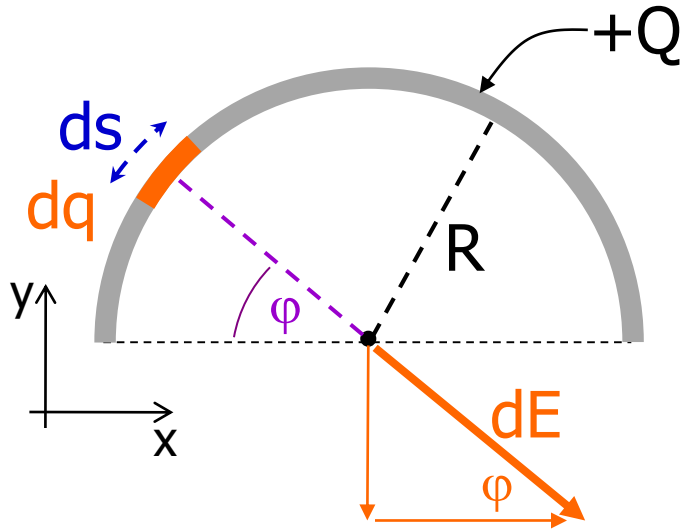
Draw the dE due to the dq , and show its components.

Do you see any helpful symmetry?

Pick a dq' horizontally across the arc from dq . The x -components of dq and dq' will cancel. Because of this symmetry, $E_x = 0$

Each dE_y points downward so E_y will be negative.

Example: calculate the electric field at “center” of semicircular line of uniformly-distributed positive charge, oriented as shown.

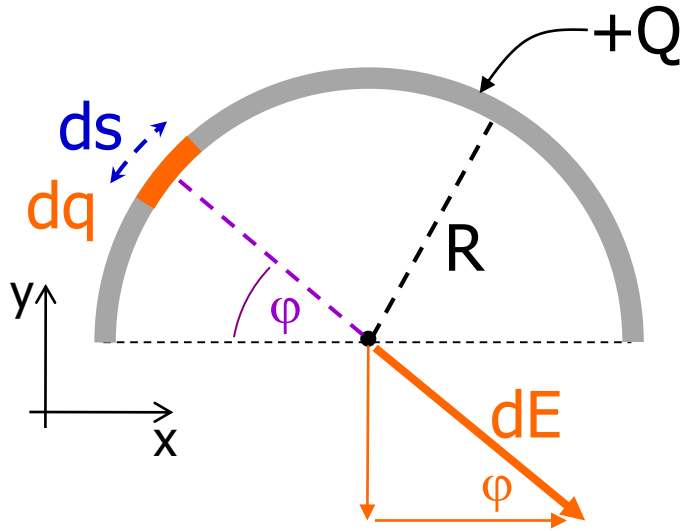


Recall that dq and ds are infinitesimal. dq is located at an angle φ along the semicircle from the negative y -axis.

φ is also one of the angles in the vector triangle.

$$dE_y = -dE \sin \varphi$$

Example: calculate the electric field at “center” of semicircular line of uniformly-distributed positive charge, oriented as shown.

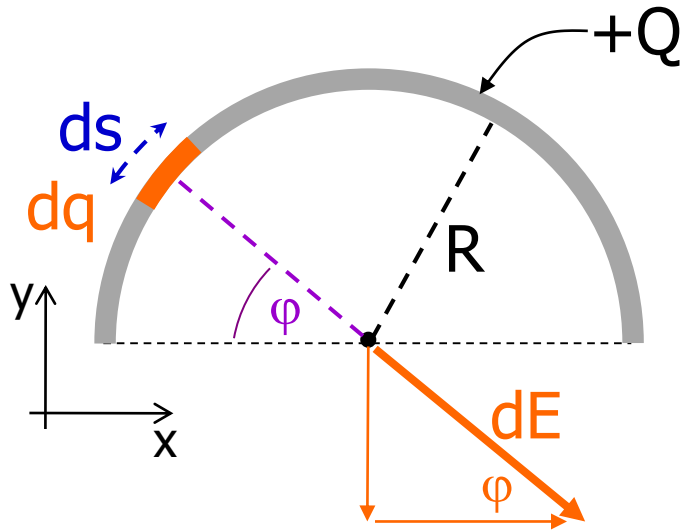


An arc of a circle has a length equal to the circle radius times the angle subtended (in radians):

$$ds = R \, d\phi$$

Also,

$$\lambda = \frac{(\text{charge on arc})}{(\text{length of arc})} = \frac{Q}{\frac{1}{2}(2\pi R)} = \frac{Q}{\pi R}$$



Let's summarize what we have done so far.

$$dE = k \frac{|dq|}{r^2}$$

$$dq = \lambda ds$$

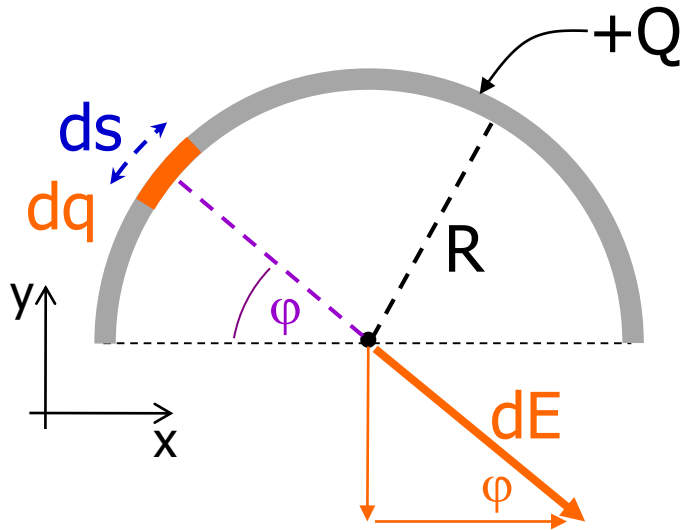
$$\lambda = \frac{Q}{\pi R}$$

$$ds = R d\phi$$

Every dq is a distance R away from the arc center: $r = R$

$$dE = k \frac{|\lambda ds|}{R^2} = k \frac{\left| \left(\frac{Q}{\pi R} \right) (R d\phi) \right|}{R^2} = k \frac{Q d\phi}{\pi R^2}$$

$$dE_y = -dE \sin \phi$$



$$dE_y = -\frac{kQ}{\pi R^2} \sin \phi$$

$$E_y = \int_{\text{arc}} dE_y = \int_{\text{arc}} \left(-\frac{kQ}{\pi R^2} \sin \phi \right)$$

$$E_y = -\frac{kQ}{\pi R^2} \int_{\text{arc}} \sin \phi \, d\phi = -\frac{kQ}{\pi R^2} \int_0^\pi \sin \phi \, d\phi = +\frac{kQ}{\pi R^2} \cos \phi \Big|_0^\pi$$

$$E_y = +\frac{kQ}{\pi R^2} [\cos \pi - \cos 0] = +\frac{kQ}{\pi R^2} [(-1) - (1)] = -\frac{2kQ}{\pi R^2}$$

Help!

We covered a lot of material in a brief time.

If you want to explore a slightly different presentation of this at your leisure, try the MIT Open Courseware site:

<http://ocw.mit.edu/courses/physics/>.

Homework Hints (may not apply every semester)

Your starting equations so far are:

$$F_{12} = k \frac{|q_1 q_2|}{r_{12}^2}$$

$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

$$E = k \frac{|q|}{r^2}$$

$$E_{\text{sheet}} = \frac{|\sigma|}{2\epsilon_0}$$

(plus Physics 2135 starting equations).

This is a “legal variation” (use it for charge distributions):

$$dE = k \frac{|dq|}{r^2}$$

You can remove the absolute value signs if you know that dq is always positive.

Homework Hints (may not apply every semester)

Suppose you have to evaluate this integral in your homework...

$$\int_0^a \frac{dx}{(a+r-x)^2}$$

Let $u=(a+r-x)^2$. Then $du=-dx$ and $u=a+r$ when $x=0$, $u=r$ when $x=a$.

Or look it up if you have tables that contain it.

Homework Hints (may not apply every semester)

The integrals below are in appendix B of your text.

$$\int \frac{dx}{(x^2 + a^2)^{3/2}}$$

$$\int \frac{x dx}{(x^2 + a^2)^{3/2}}$$

Your recitation instructor will supply you with needed integrals.

The above integrals may or may not be needed this semester.

Learning Center Today

2:00-4:30, 6:00-8:30
Rooms 129/130 Physics

