

Exam 1

Exam 1 is on Tuesday, February 14, from 5:00-6:00 PM.

- **Testing Center** provides accommodations for students with special needs
- I must set up appointments one week before exam
**Deadline for submitting accommodation letters:
Monday, Feb 6 by noon**
- student must make **separate appointment** with testing center
- if you have other time conflicts, I need to know **now!**
(they are the **student's responsibility** unless the student participates in a major university or intercollegiate event)

Course Learning Assistance

- **Physics Learning Center** (Monday+Wednesday 2:00-4:30 and 6:00 to 8:30 in rooms 129 and 130 Physics)
- **LEAD/tutoring** sessions (<http://lead.mst.edu/>)

Phys 1135 (23)	Engineering Physics I	T/R	2:00 - 4:30	129-130 Physics
Phys 1135 (23)	Engineering Physics I	T/R	6:00 - 8:30	129-130 Physics
Phys 2135 (24)	Engineering Physics II	M/W	4:00 - 6:00	Sem Rm, Res Coll 1
Phys 2135 (24)	Engineering Physics II	M/W	4:00 - 6:00	TJ - G3 (North Basement)
Phys 2135 (24)	Engineering Physics II	M/W	2:00 - 4:30	129-130 Physics
Phys 2135 (24)	Engineering Physics II	M/W	6:00 - 8:30	129-130 Physics
Phys 2145	College Physics I	R	4:30 - 6:00	Sem Rm, Res Coll 1
Phys 2145	College Physics I	R	3:00 - 4:30	202 Physics
Phys 2145	College Physics I	R	6:00 - 7:30	202 Physics
Phys 2145	College Physics I	R	6:00 - 7:30	202 Physics

- **Student Success Center**, 198 Toomey Hall
Monday + Wednesday 2:00 to 4:00pm
Tuesday and Thursday 9:00 to 11:00am

Today's agenda:

Announcements.

Gauss' Law Examples.

You must be able to use Gauss' Law to calculate the electric field of a high-symmetry charge distribution.

Cultural enlightenment time.

You must be culturally enlightened by this lecture.

Conductors in electrostatic equilibrium.

You must be able to use Gauss' law to draw conclusions about the behavior of charged particles on, and electric fields in, conductors in electrostatic equilibrium.

Gauss' Law

Last time we learned that

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Gauss' Law

Always true, not always easy to apply.

and used Gauss' Law to calculate the electric field for spherically-symmetric charge distributions

Today we will calculate electric fields for charge distributions that are non-spherical but exhibit a high degree of symmetry, and then consider what Gauss' Law has to say about conductors in electrostatic equilibrium.

Example: calculate the electric field outside a long cylinder of finite radius R with a uniform volume charge density ρ spread throughout the volume of the cylinder.

To be worked at the blackboard in lecture.

“Long” cylinder with “finite” radius means neglect end effects; i.e., treat cylinder as if it were infinitely long.

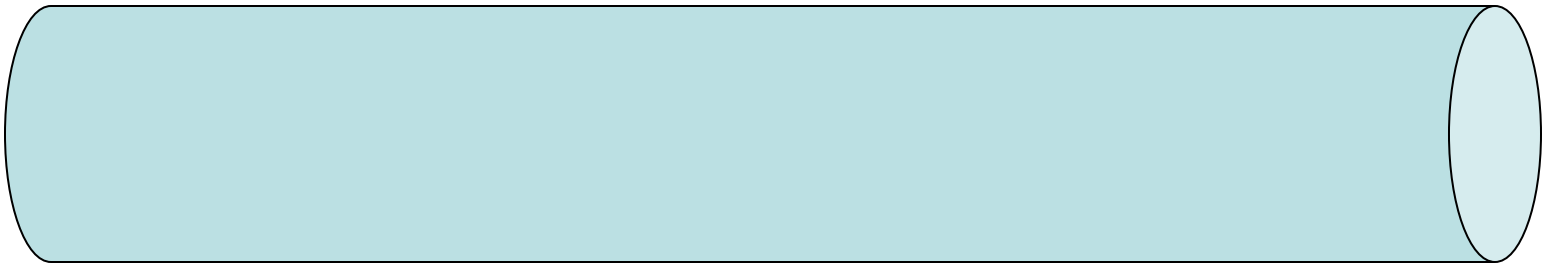
More details of the calculation shown here:

http://campus.mst.edu/physics/courses/24/Handouts/charged_cylinder.pdf

$$E = \frac{|\rho| R^2}{2\epsilon_0 r}$$

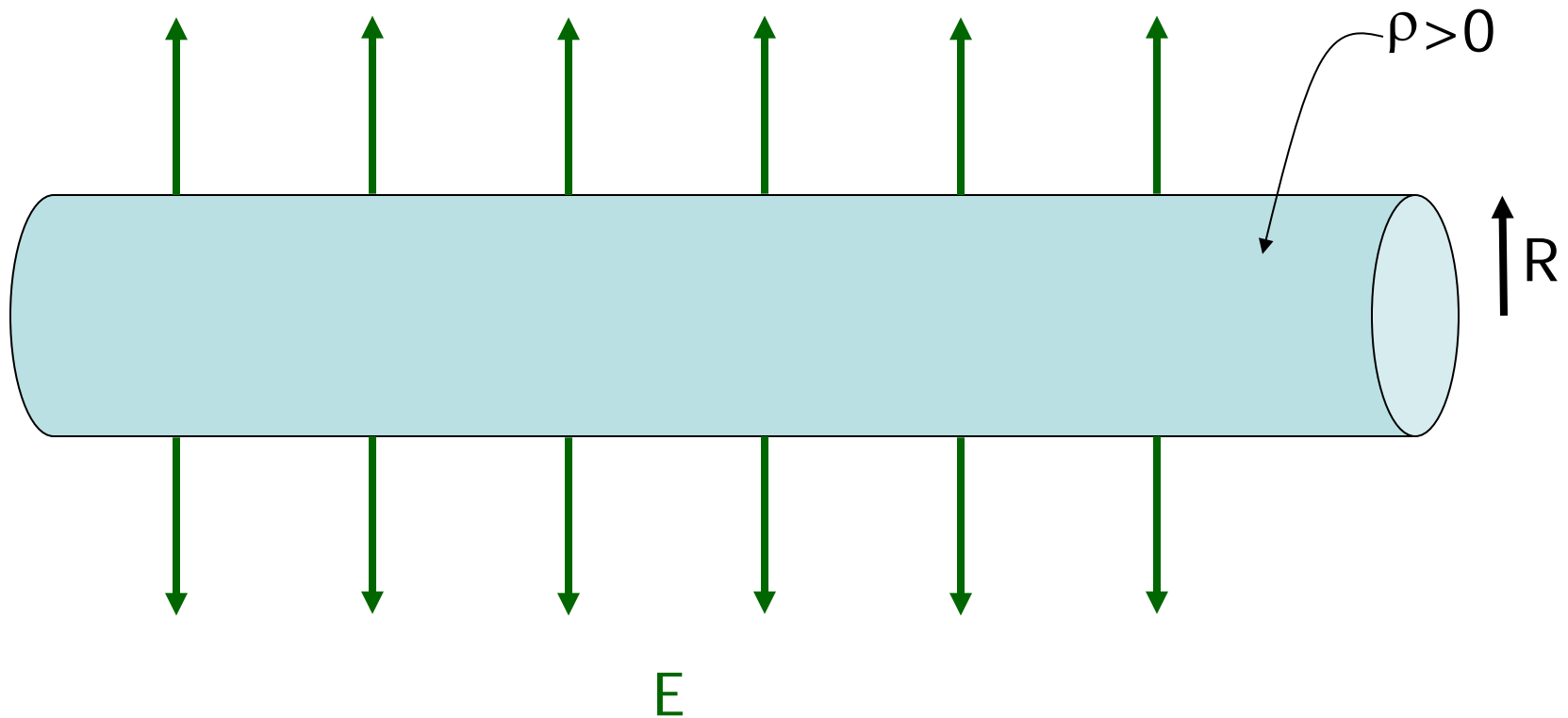
Example: calculate the electric field outside a long cylinder of finite radius R with a uniform volume charge density ρ spread throughout the volume of the cylinder.

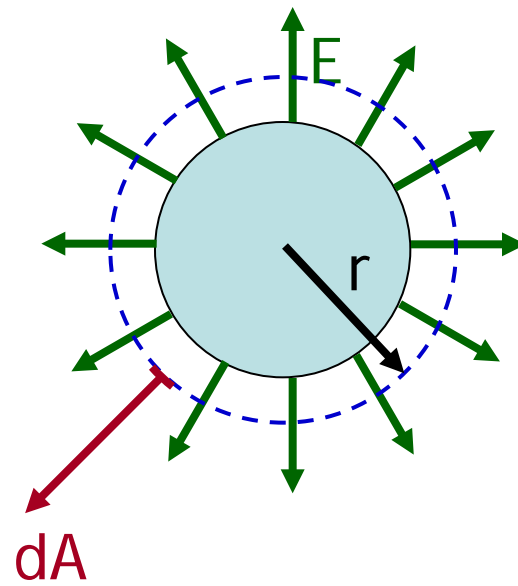
Cylinder is loooooong. I'm just showing a bit of it here.



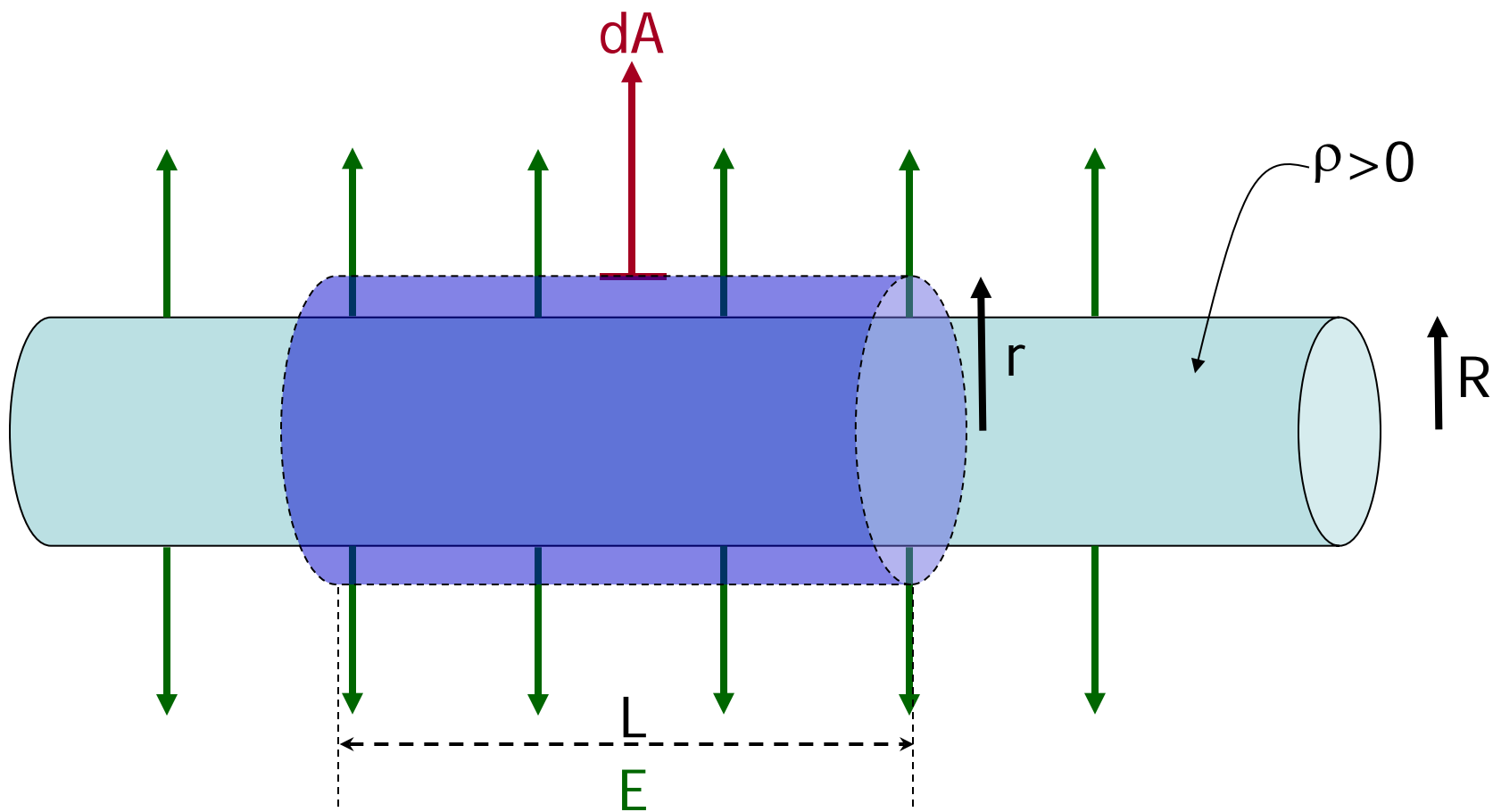
I don't even want to think of trying to use $dE = k|dq|/r^2$ for this.

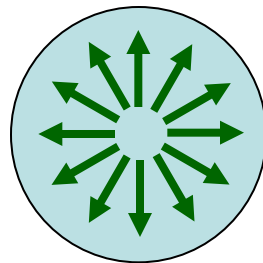
Example: calculate the electric field outside a long cylinder of finite radius R with a uniform volume charge density ρ spread throughout the volume of the cylinder.



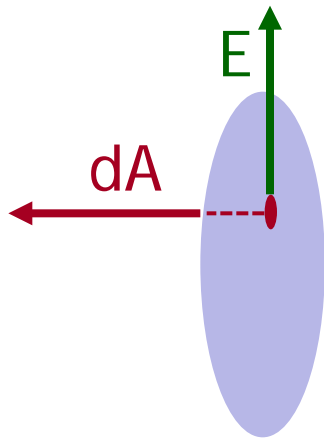


Looking down the axis of the cylinder.

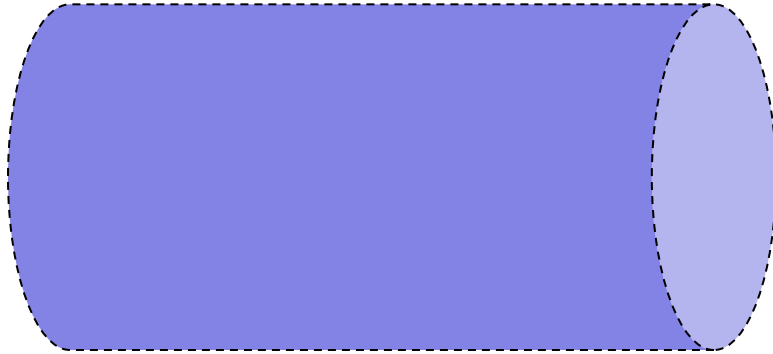




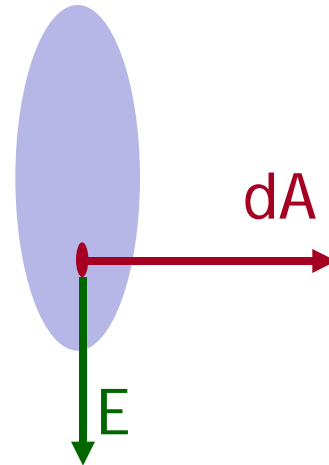
Inside the charged cylinder, by symmetry \vec{E} must be radial.

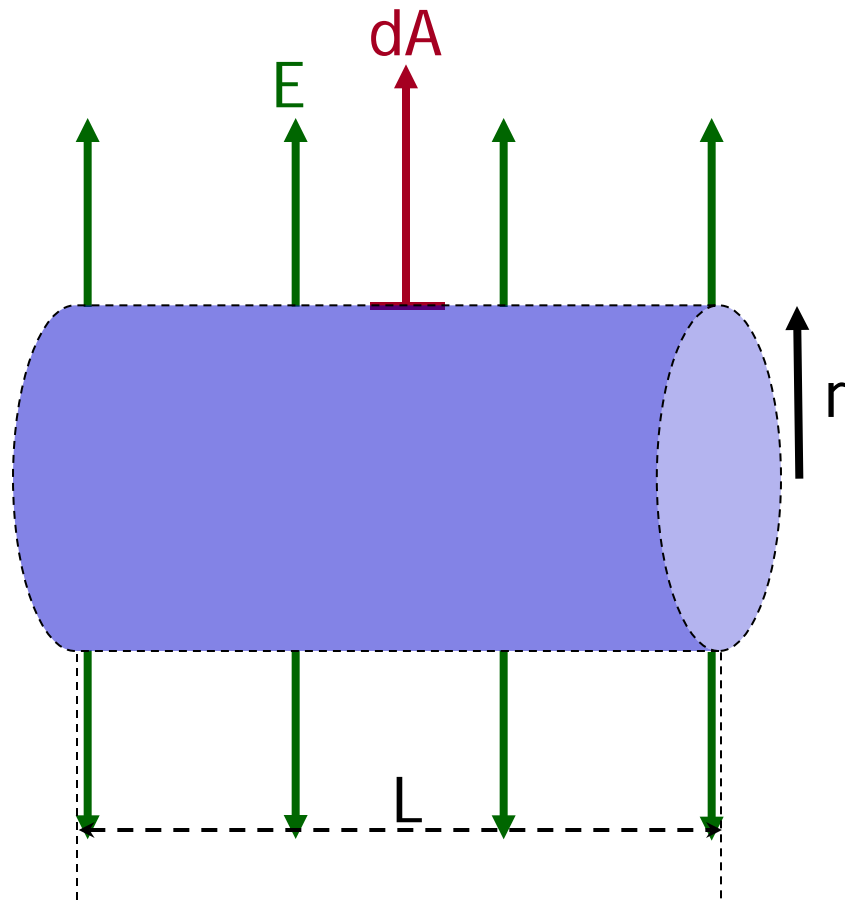


$$\vec{E} \cdot d\vec{A} = 0 \text{ because } \vec{E} \perp d\vec{A}$$



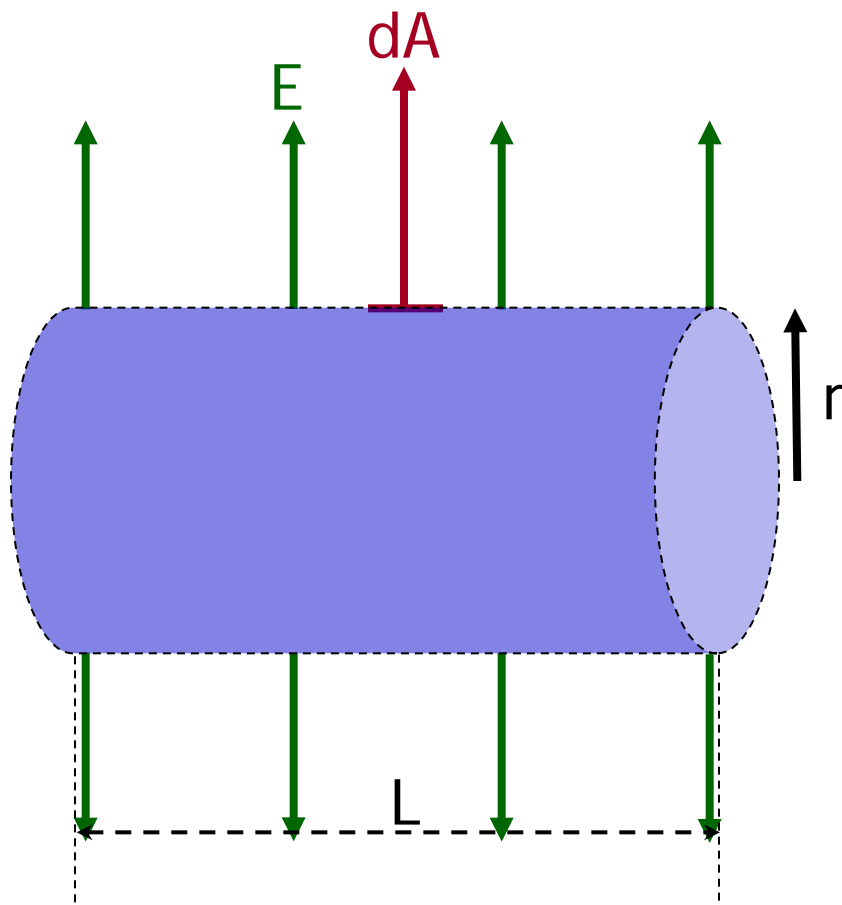
$$\vec{E} \cdot d\vec{A} = 0 \text{ because } \vec{E} \perp d\vec{A}$$





$$\vec{E} \cdot d\vec{A} = E dA \text{ because } \vec{E} \parallel d\vec{A}$$

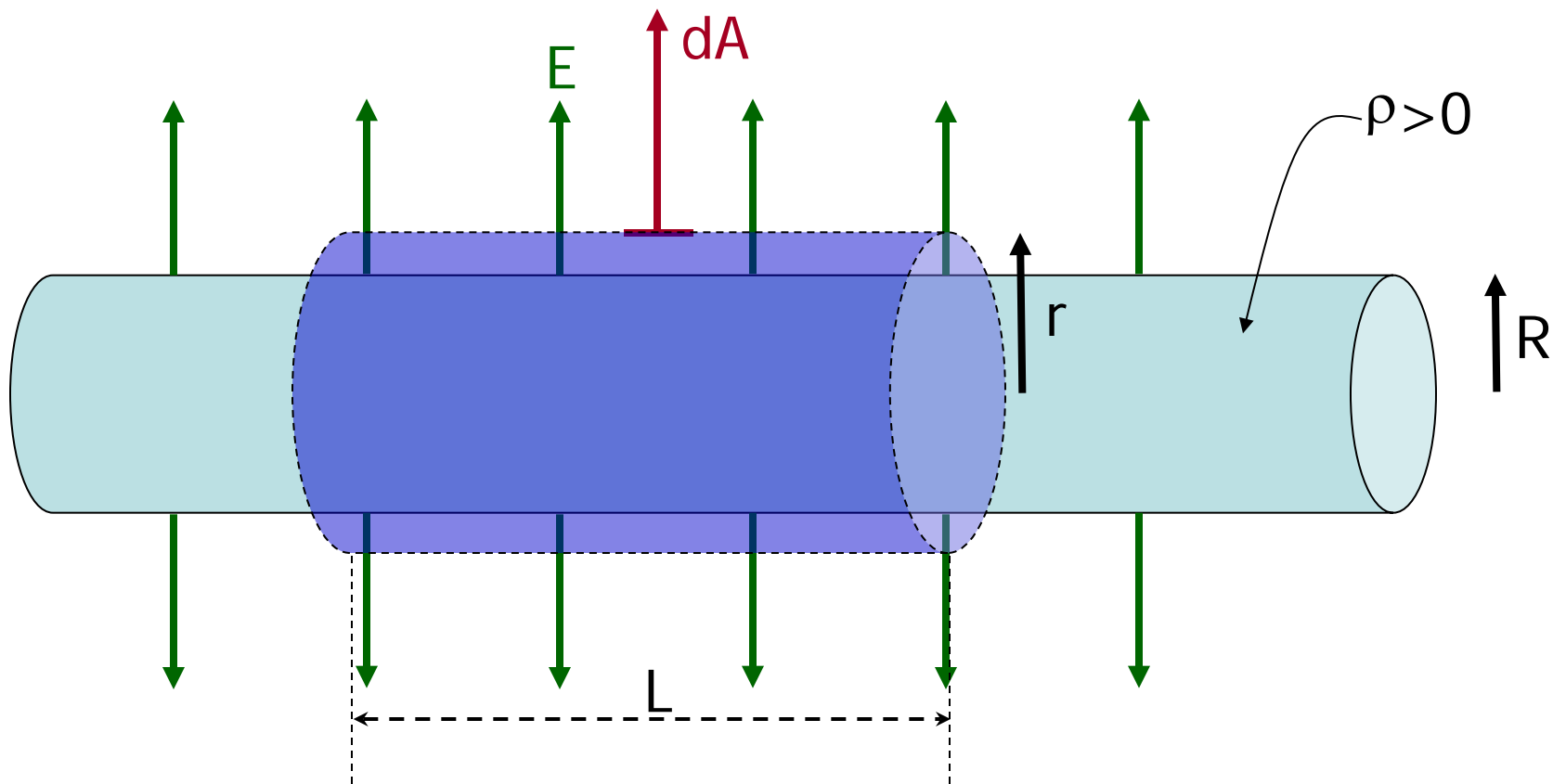
Also $|\vec{E}| = E$ must be constant at any given r .



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \int_{\text{tube}} \vec{E} \cdot d\vec{A} = \int_{\text{tube}} E \, dA = E \int_{\text{tube}} dA$$

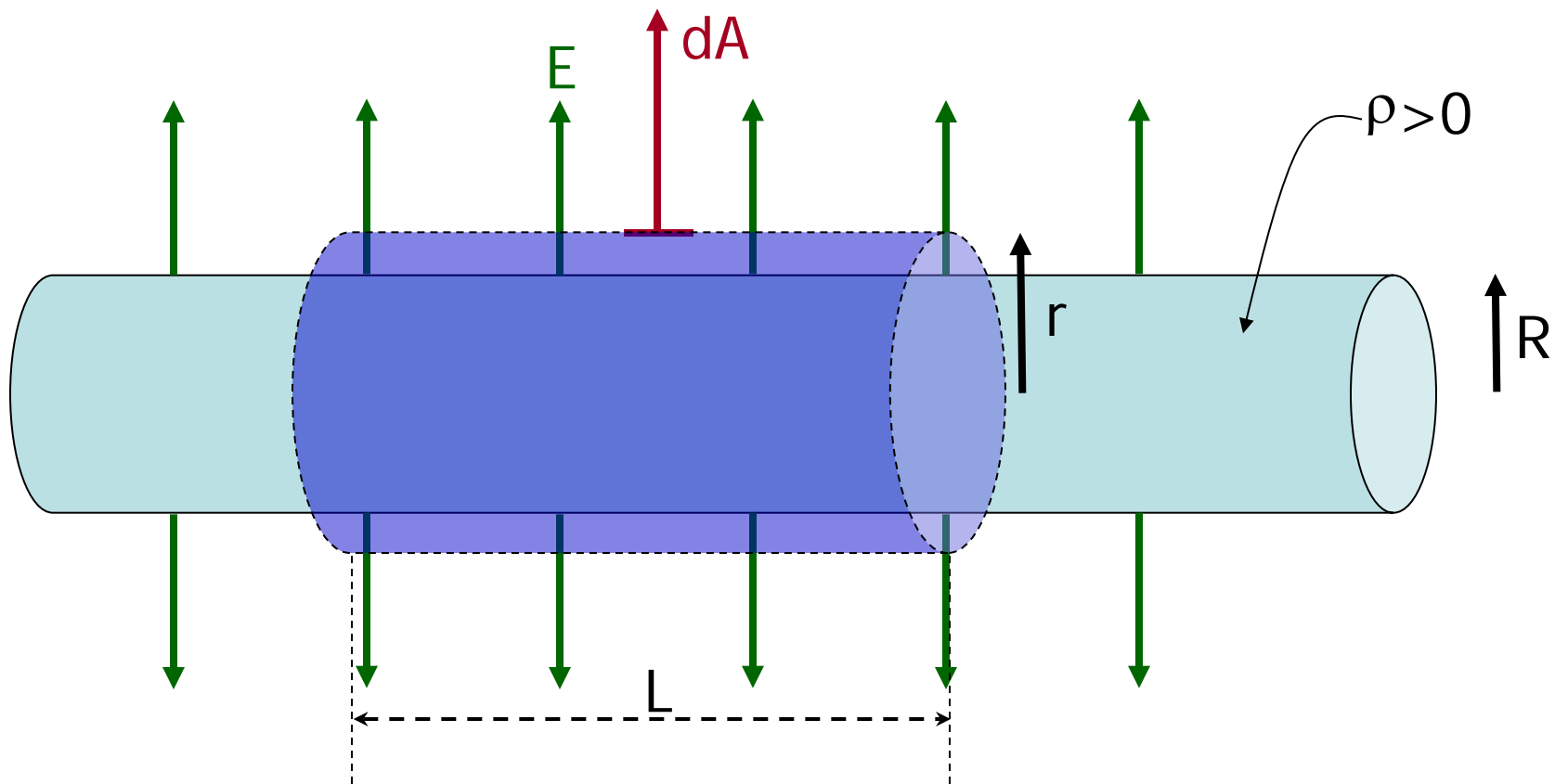
$$= E(\text{circumference of Gaussian cylinder})(\text{length of GC})$$

$$= E(2\pi r)(L)$$



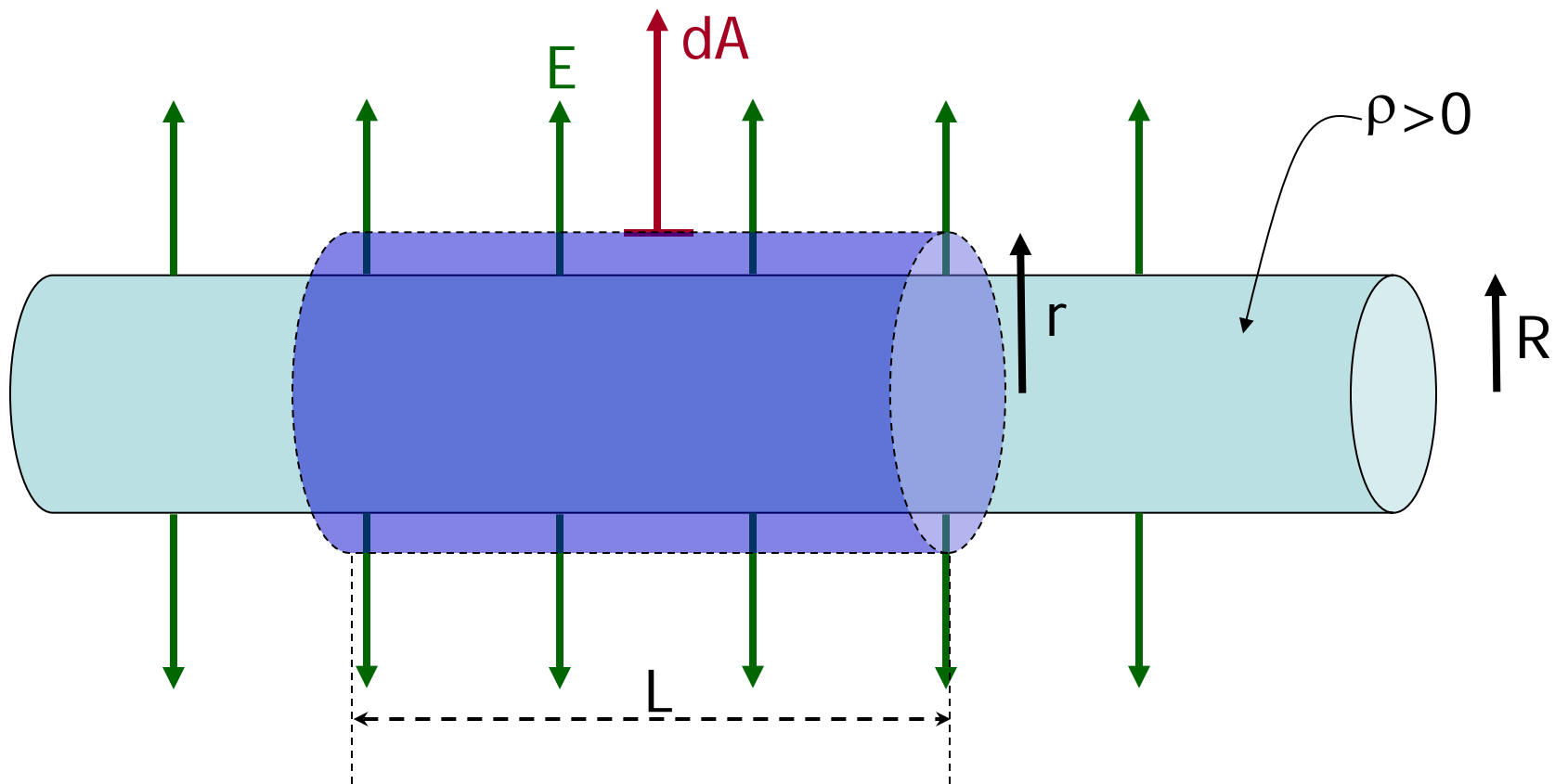
$$\oint \vec{E} \cdot d\vec{A} = E 2\pi r L = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$E 2\pi r L = \frac{\rho V_{\text{enclosed}}}{\epsilon_0} = \frac{\rho (A_{\text{base}})(\text{length})}{\epsilon_0} = \frac{\rho (\pi R^2)(L)}{\epsilon_0}$$



$$E \cdot 2\pi r L = \frac{\rho(\pi R^2)(L)}{\epsilon_0}$$

$$E = \frac{\rho\pi R^2}{2\pi\epsilon_0 r} = \frac{\rho R^2}{2\epsilon_0 r}$$



For positive ρ : $\vec{E} = \frac{\rho R^2}{2\epsilon_0 r}$, radially out

In general: $E = \frac{|\rho| R^2}{2\epsilon_0 r}$

Why does this vary as $1/r$ instead of $1/r^2$?

For a solid cylinder...

$$\text{Charge per volume is } \rho = \frac{Q}{\pi R^2 L}$$

$$\text{Charge per length is } \lambda = \frac{Q}{L}$$

$$\text{So } \rho = \frac{Q}{\pi R^2 L} = \frac{1}{\pi R^2} \frac{Q}{L} = \frac{\lambda}{\pi R^2}$$

$$\text{And } E = \frac{|\rho| R^2}{2\epsilon_0 r} = \frac{\left| \frac{\lambda}{\pi R^2} \right| R^2}{2\epsilon_0 r} = \frac{|\lambda|}{2\pi\epsilon_0 r}$$

Example: use Gauss' Law to calculate the electric field due to a long line of charge, with linear charge density λ .

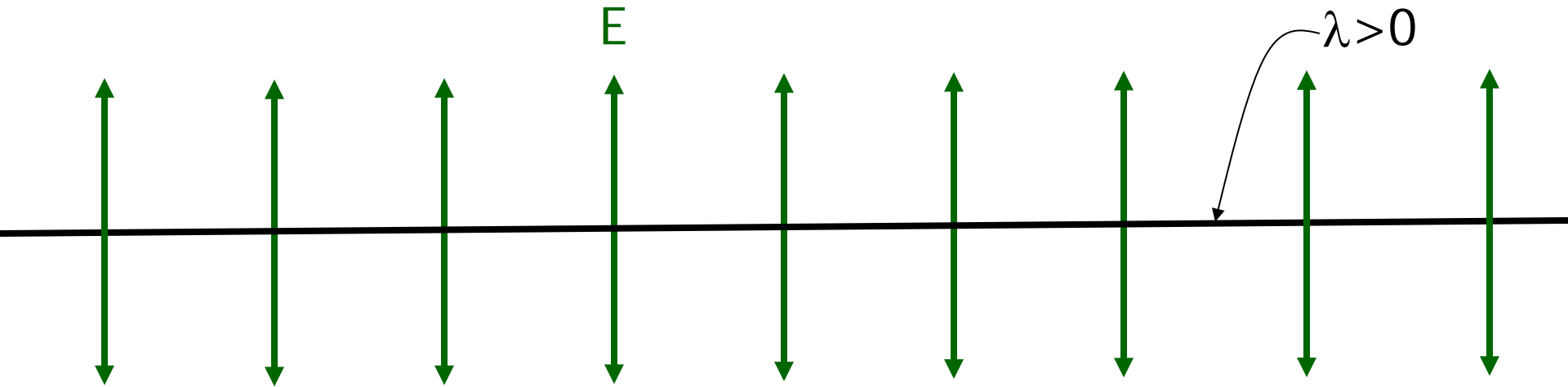
This is easy using Gauss' Law (remember what a pain it was in the previous chapter). **Study the examples in this lecture and others in your text!**

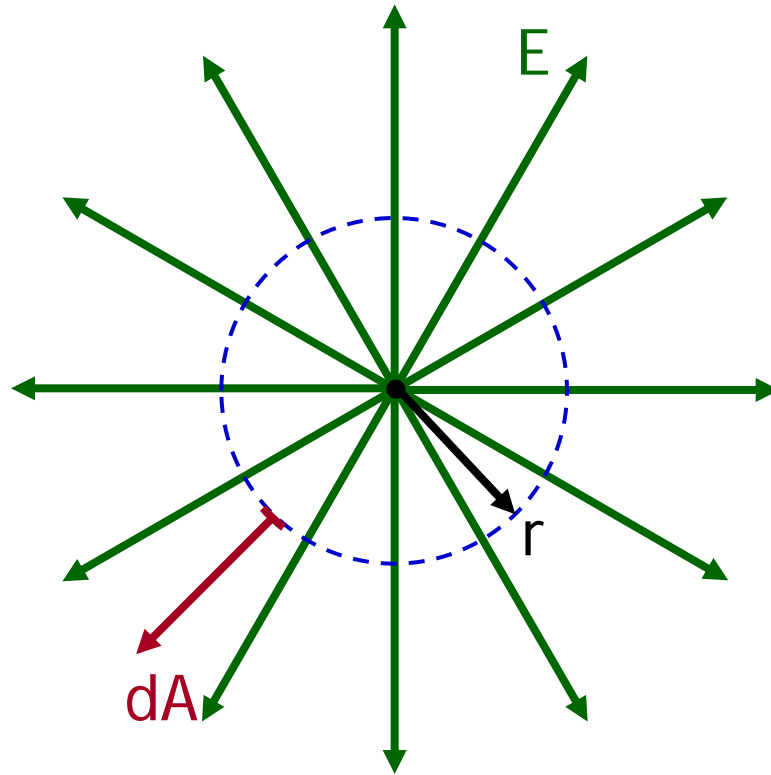
To be worked at the blackboard in lecture.

$$E = \frac{|\lambda|}{2\pi\epsilon_0 r}$$

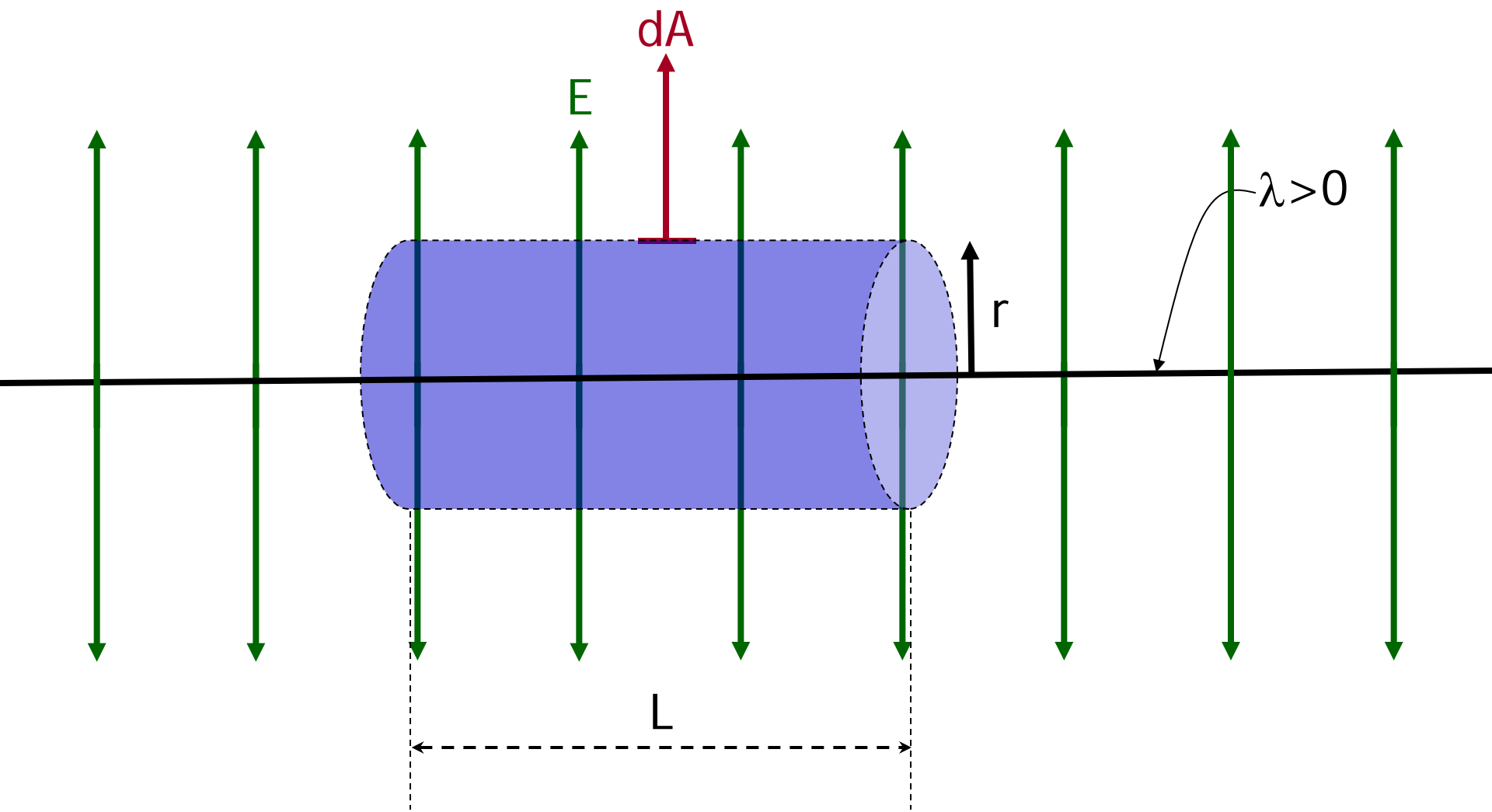
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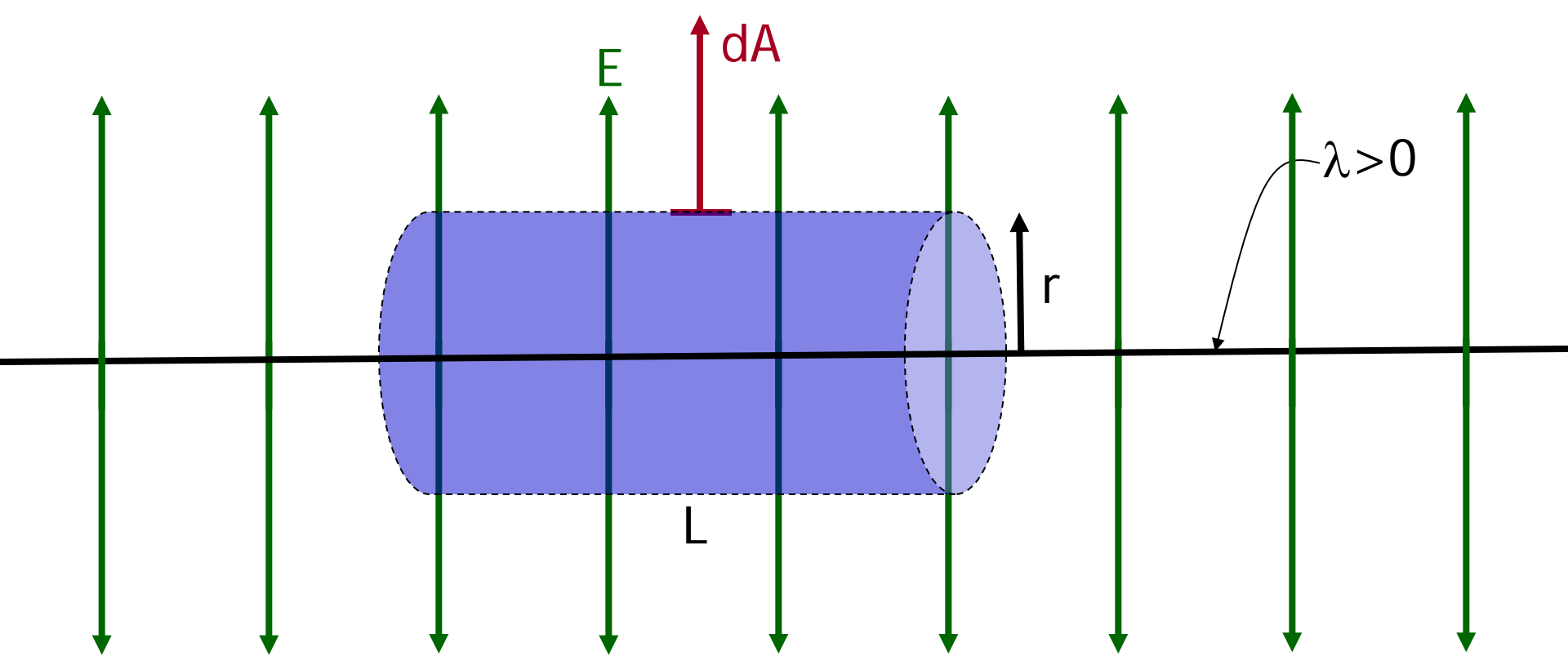
Line is loooooong.





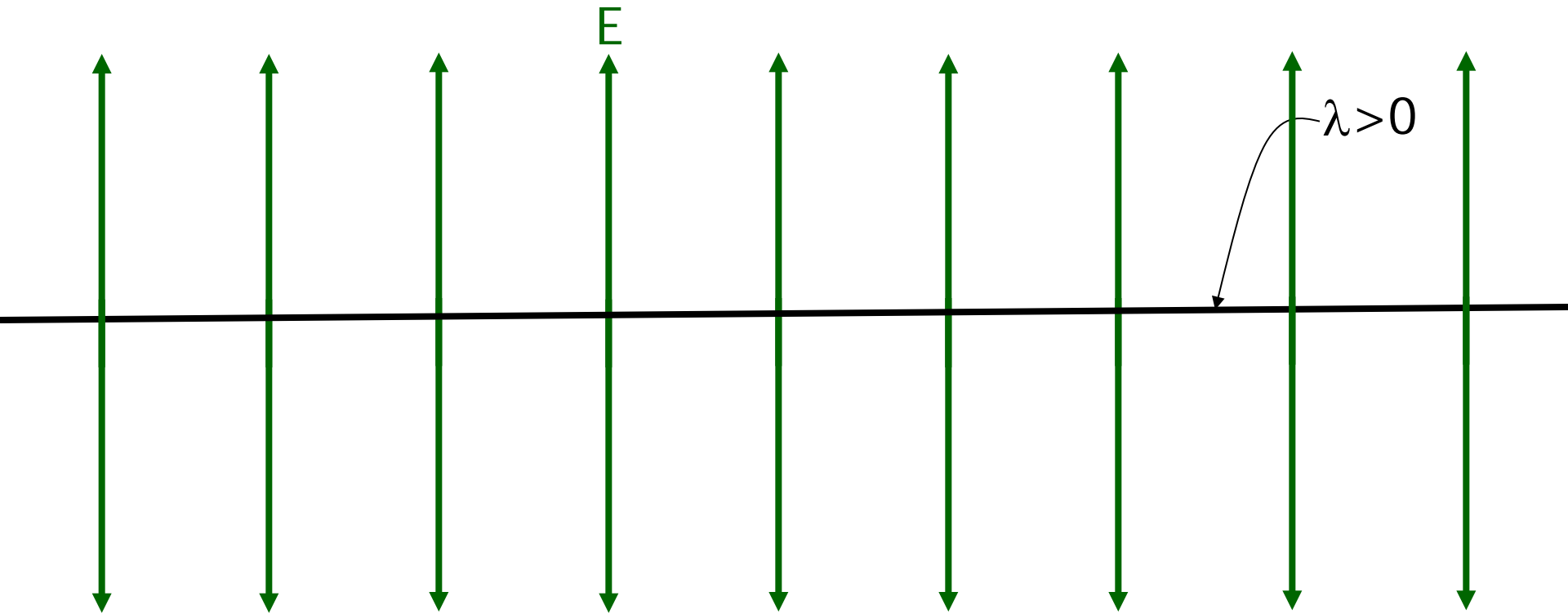
Looking down the line.





$$\oint \vec{E} \cdot d\vec{A} = E(2\pi r)(L) = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$



For positive λ : $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}$, radially out

In general: $E = \frac{|\lambda|}{2\pi\epsilon_0 r}$

Same as outside a solid cylinder!

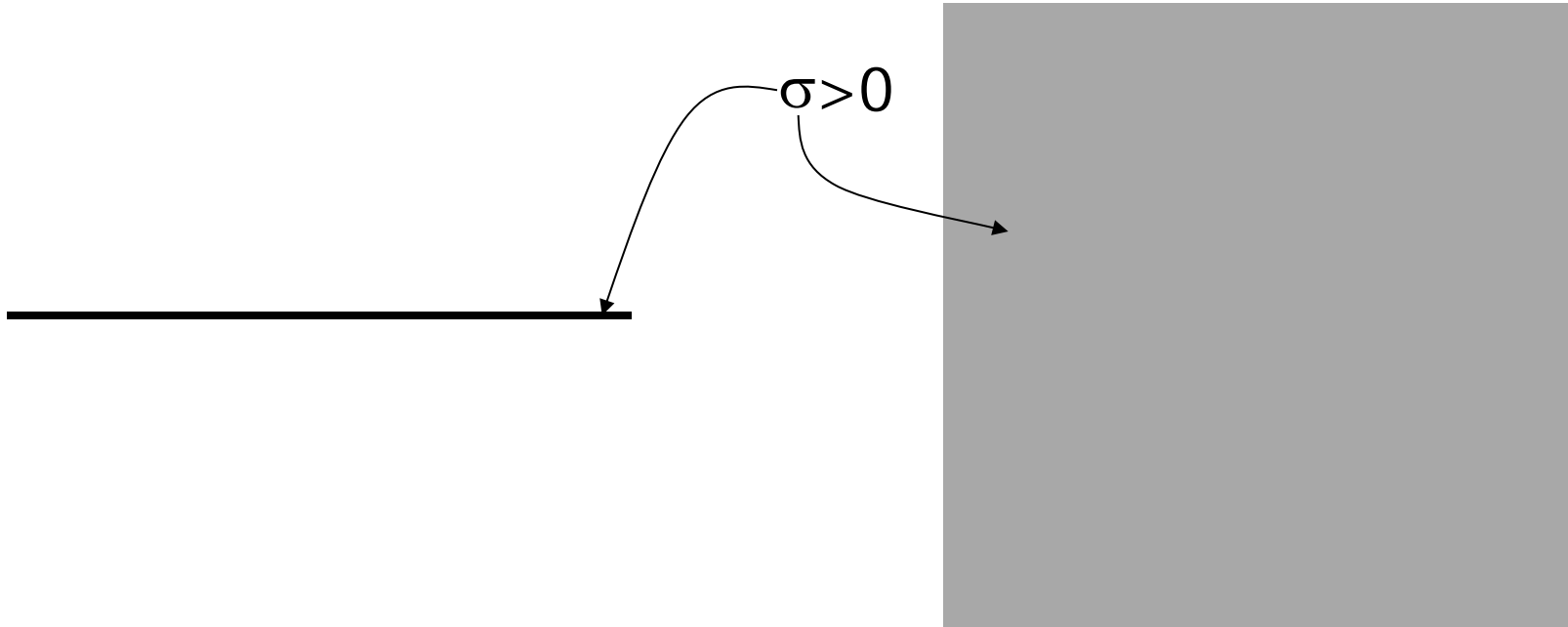
Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ .

This is easy using Gauss' Law (remember what a pain it was in the previous chapter). **Study the examples in this lecture and others in your text!**

To be worked at the blackboard in lecture.

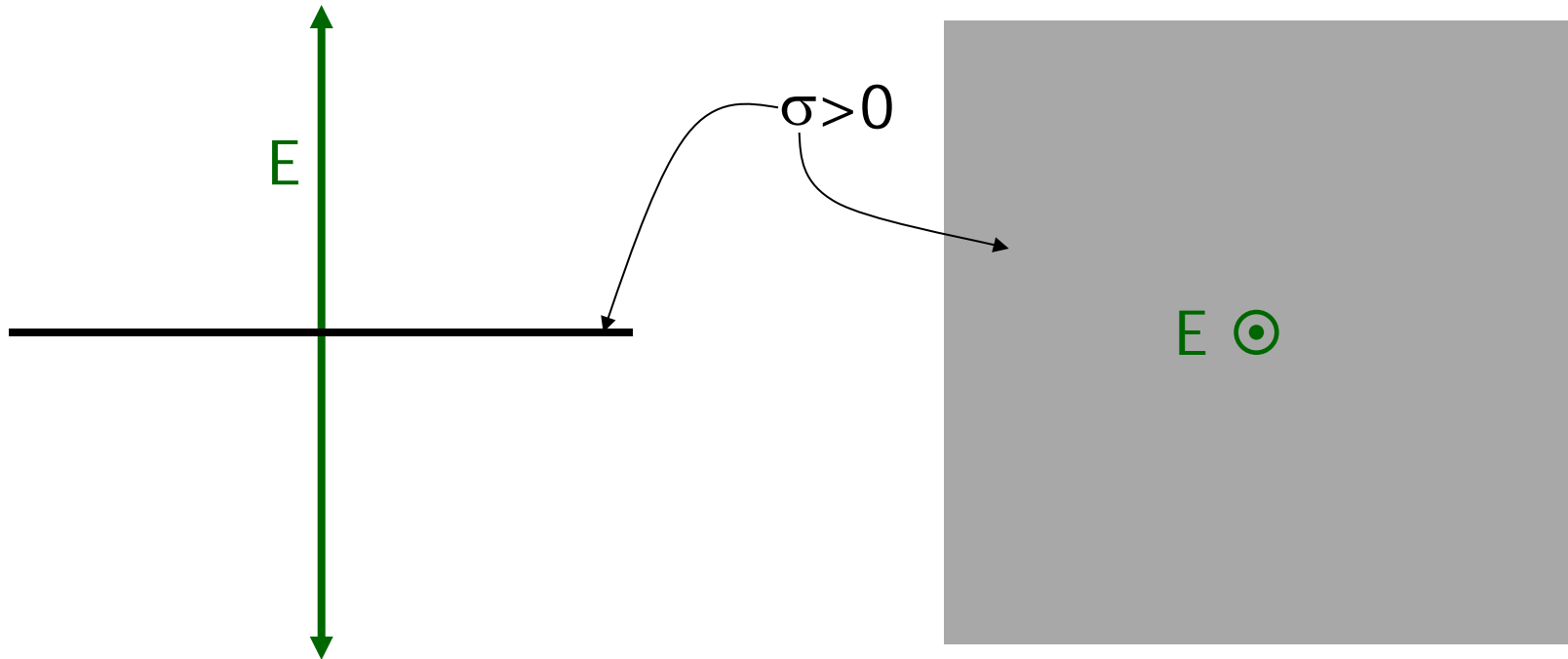
$$E_{\text{sheet}} = \frac{|\sigma|}{2\epsilon_0}.$$

Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ .

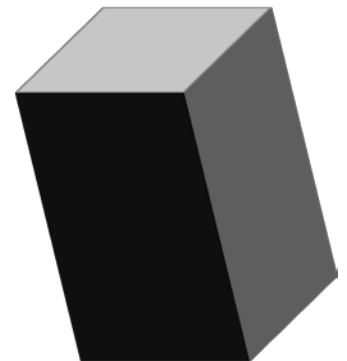


Two views of sheet of charge; side view looking edge on, and top view looking down. Sheet extends infinitely far in two dimensions.

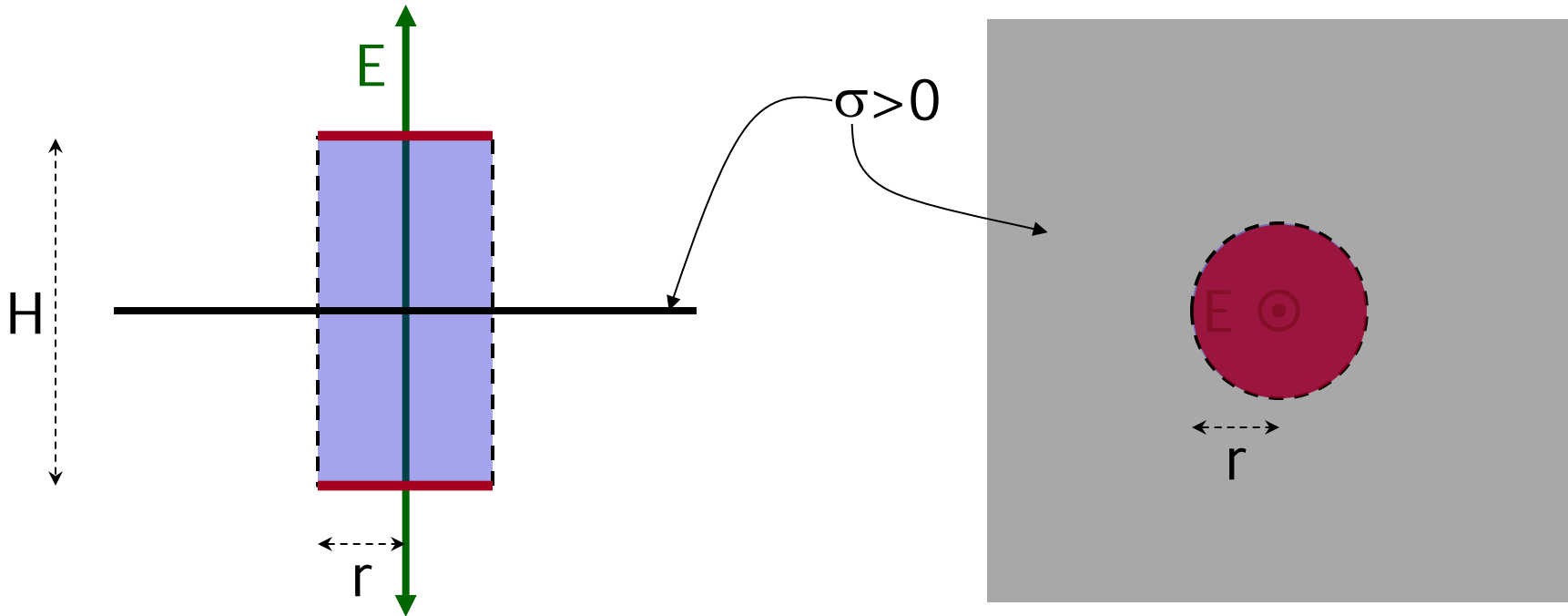
Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ .



For this electric field symmetry, we usually use a "pillbox" (cylinder shape) for our Gaussian surface. In the views above, it will look like a rectangle and a circle. You could also use a rectangular box.



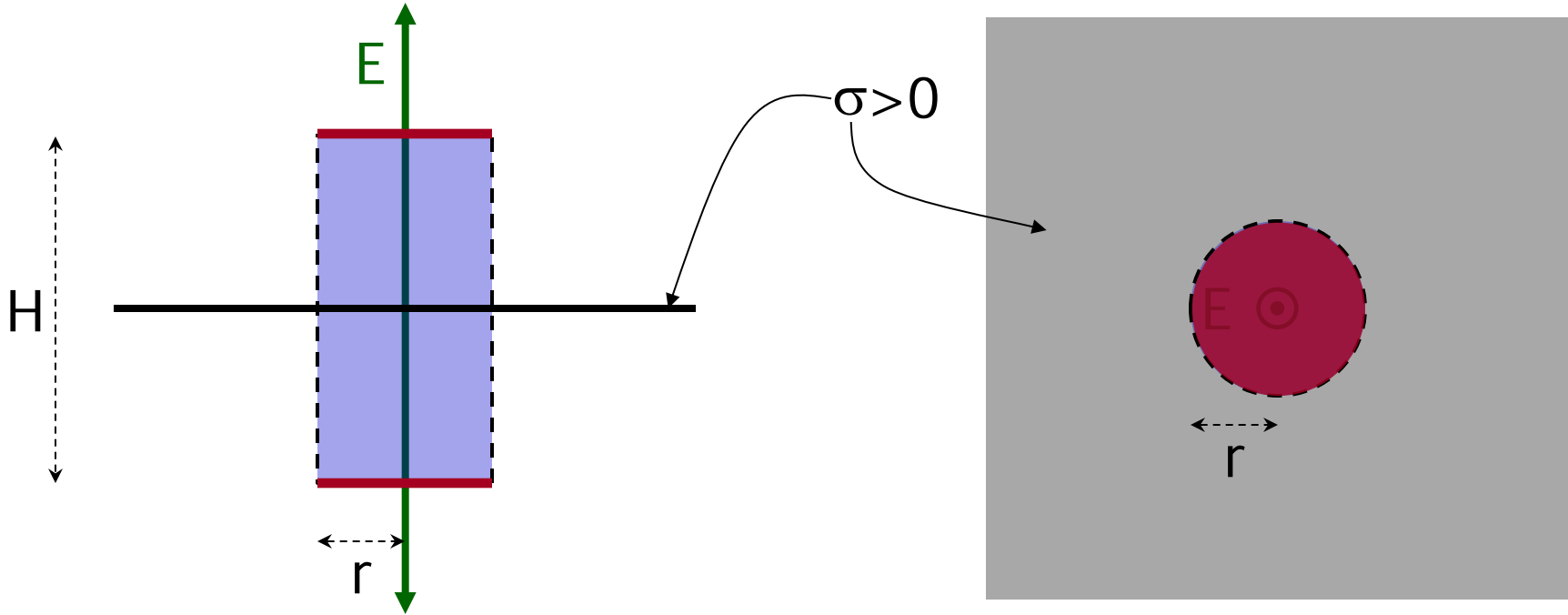
Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ .



$$\oint \vec{E} \cdot d\vec{A} = 2[E(\pi r^2)] = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$Q_{\text{enclosed}} = \sigma(\pi r^2)$$

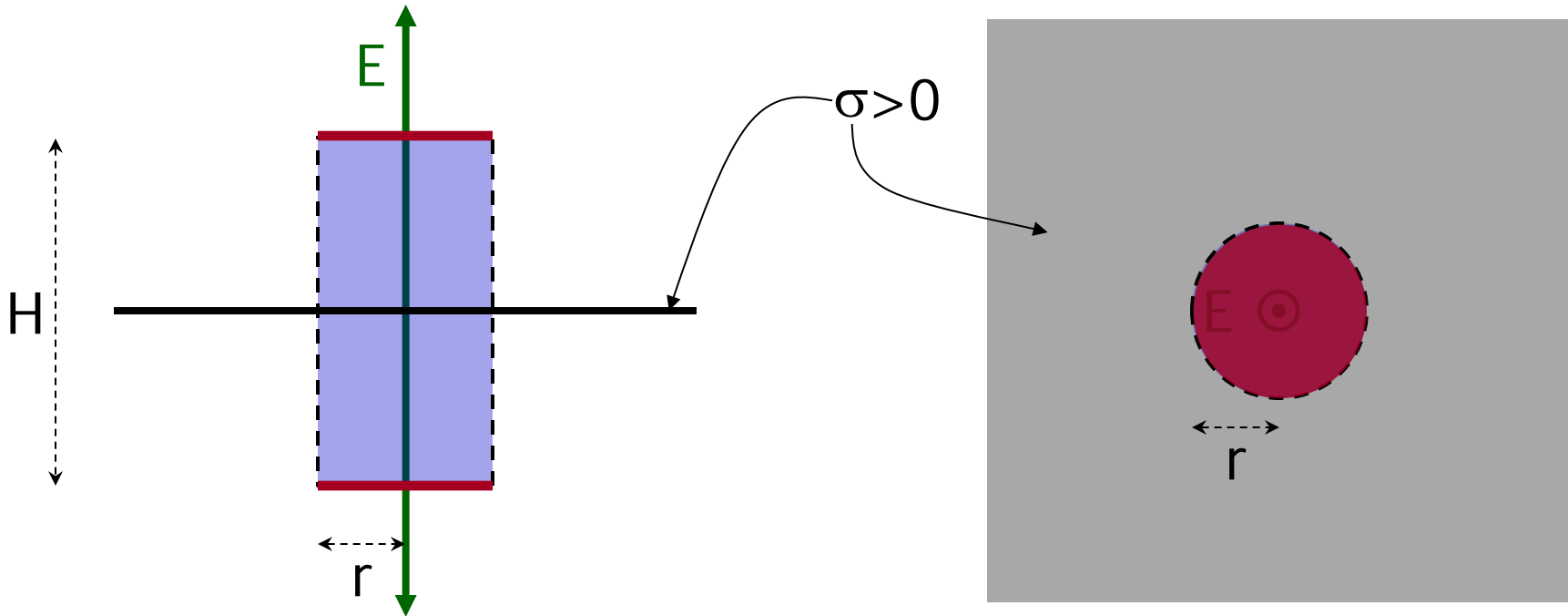
Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ .



$$2[E(\pi r^2)] = \frac{\sigma(\pi r^2)}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ .



For positive σ : $\vec{E} = \frac{\sigma}{2\epsilon_0}$, away from the sheet

In general: $E = \frac{|\sigma|}{2\epsilon_0}$

That sure was easier than the derivation back in Lecture 2!

Gauss' Law works well for three kinds of symmetry:

Charge Symmetry

spherical
cylindrical
planar

Gaussian Surface

concentric sphere
coaxial cylinder
pillbox

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Cultural enlightenment time.

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Conductors in electrostatic equilibrium.

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Cultural Enlightenment* Time

The top 5 reasons why we make you learn Gauss' Law:

5. You can solve difficult (high-symmetry) problems with it.
4. It's good for you. It's fun! What more can you ask!
3. It's easy. Smart physicists go for the easy solutions.
2. If **I** had to learn it, **you** do too.

And the number one reason...

...will take a couple of slides to present

*This will not be tested on the exam.

You have learned the integral form of Gauss' Law:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

This equation can also be written in differential form:


$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

 a 3-dimensional derivative operator

Now you can see we are on the trail of something Really Big...

This will not be tested on the exam.

The Missouri S&T Society of Physics Students T-Shirt!



And God said...

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

...and there was light.

This will not be tested on the exam.

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You must be able to use Gauss' Law to calculate the electric field of a high-symmetry charge distribution.

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Conductors in electrostatic equilibrium.

You must be able to use Gauss' law to draw conclusions about the behavior of charged particles on, and electric fields in, conductors in electrostatic equilibrium.

Conductors in Electrostatic Equilibrium

Homework hints buried in the next 3 slides!

Electrostatic equilibrium means there is no **net** motion of the charges inside the conductor.

The electric field inside the conductor must be zero.

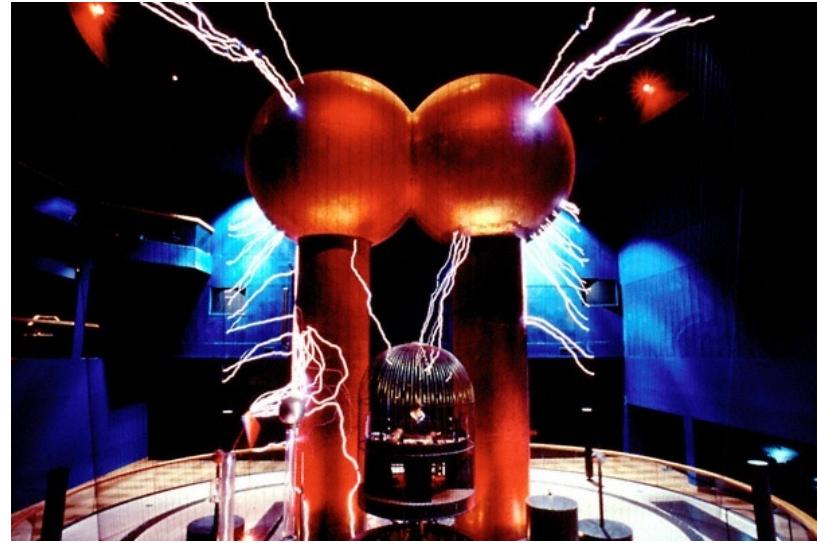
If this were not the case, charges would accelerate.

Any excess charge must reside on the outside surface of the conductor.

Apply Gauss' law to a Gaussian surface just inside the conductor surface. The electric field is zero, so the net charge inside the Gaussian surface is zero. Any excess charge must go outside the Gaussian surface, and on the conductor surface.

The electric field just outside a charged conductor must be perpendicular to the conductor's surface.

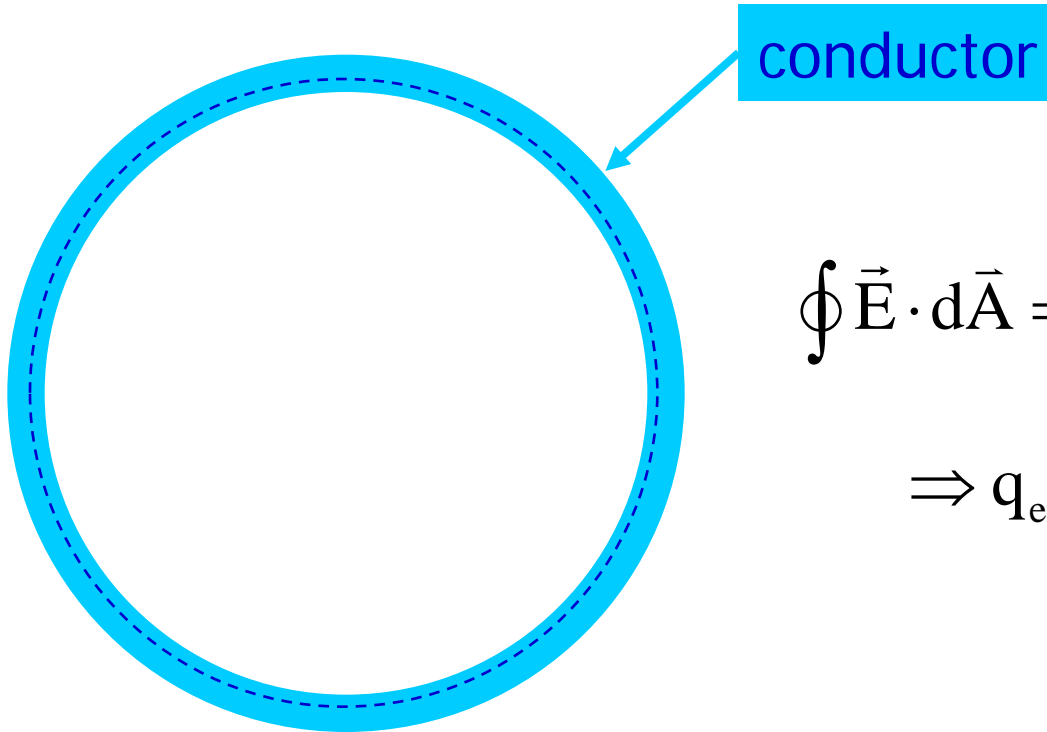
Otherwise, the component of the electric field parallel to the surface would cause charges to accelerate.



The magnitude of the electric field just outside a charged conductor is equal to $|\sigma|/\epsilon_0$, where $|\sigma|$ is the magnitude of the local surface charge density.

A simple application Gauss' Law. Different from infinite sheet of charge because E is zero inside the conductor.

If there is an empty nonconducting cavity inside a conductor, Gauss' Law tells us there is no net charge on the interior surface of the conductor.

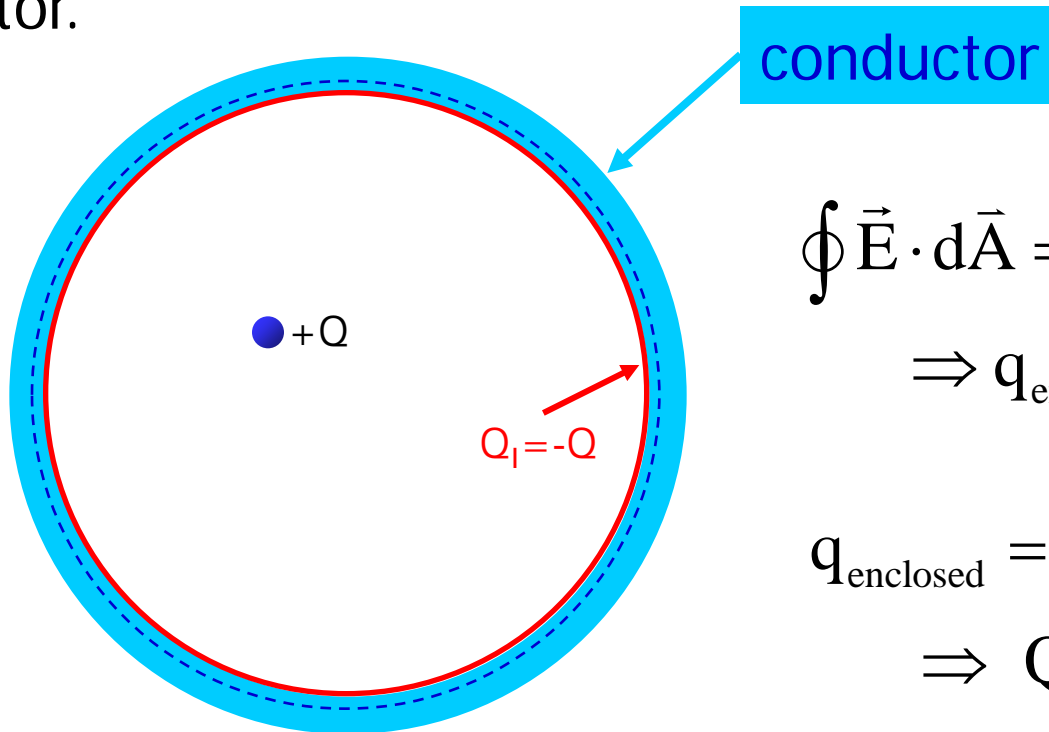


$$\oint \vec{E} \cdot d\vec{A} = \oint 0 \cdot dA = 0$$

$$\Rightarrow q_{\text{enclosed}} = 0$$

Construct a Gaussian surface that includes the inner surface of the conductor. The electric field at the Gaussian surface is zero, so no electric flux passes through the Gaussian surface. Gauss' Law says the charge inside must be zero. **Any excess charge must lie on the outer surface!** The conductor does not have to be symmetric, as shown.

If there is a nonconducting cavity inside a conductor, with a charge inside the cavity, Gauss' Law tells us there is an equal and opposite induced charge on the interior surface of the conductor.



$$\oint \vec{E} \cdot d\vec{A} = \oint 0 \cdot dA = 0$$

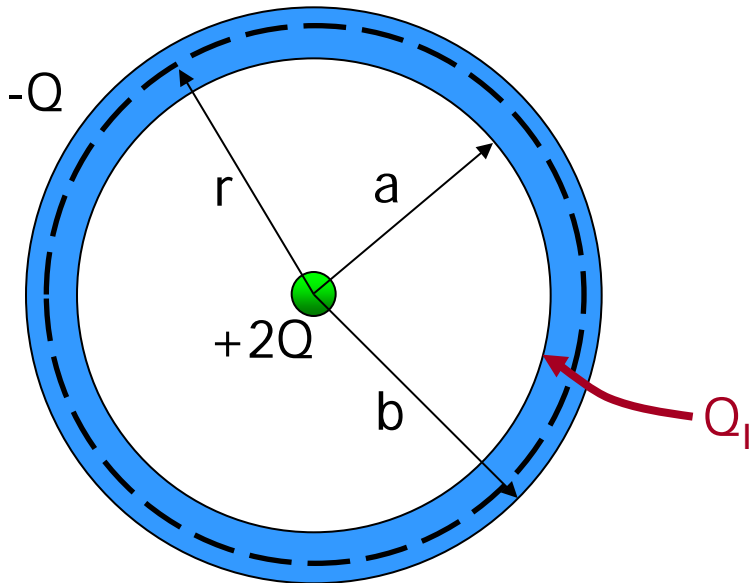
$$\Rightarrow q_{\text{enclosed}} = 0$$

$$q_{\text{enclosed}} = 0 = +Q + Q_I$$

$$\Rightarrow Q_I = -Q$$

Construct a Gaussian surface that includes the inner surface of the conductor. The electric field at the Gaussian surface is zero, so no electric flux passes through the Gaussian surface. Gauss' Law says the charge inside must be zero. There must be a $-Q$ on the inner surface. **If the net charge on the conductor is not $-Q$, any additional charge must lie on the outer surface!** The conductor does not have to be symmetric.

Example: a conducting spherical shell of inner radius a and outer radius b with a net charge $-Q$ is centered on point charge $+2Q$. Use Gauss's law to show that there is a charge of $-2Q$ on the inner surface of the shell, and a charge of $+Q$ on the outer surface of the shell.



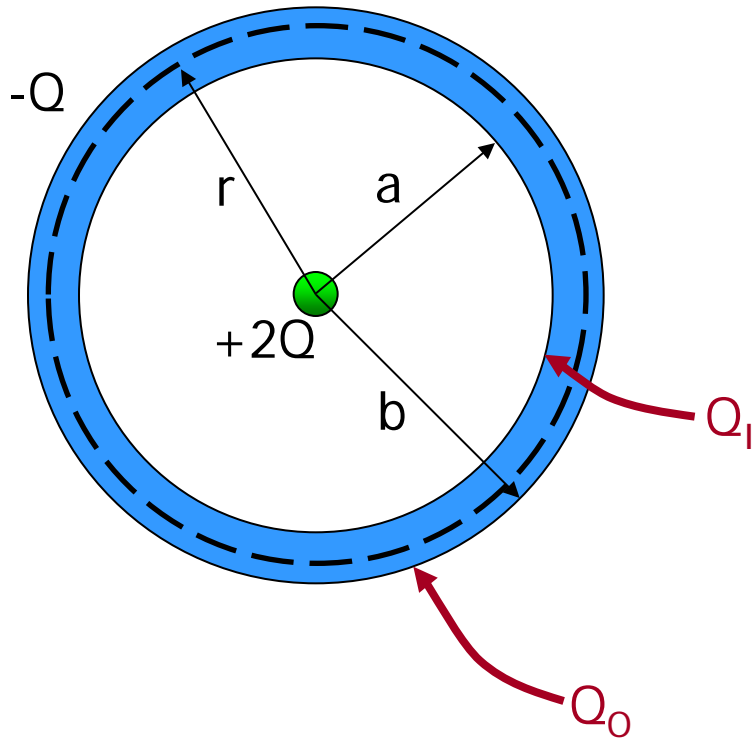
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$E=0$ inside the conductor!

Let r be infinitesimally greater than a .

$$0 = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{Q_I + 2Q}{\epsilon_0} \Rightarrow Q_I = -2Q$$

Example: a conducting spherical shell of inner radius a and outer radius b with a net charge $-Q$ is centered on point charge $+2Q$. Use Gauss's law to show that there is a charge of $-2Q$ on the inner surface of the shell, and a charge of $+Q$ on the outer surface of the shell.



$$Q_I = -2Q$$

From Gauss' Law we know that excess* charge on a conductor lies on surfaces.

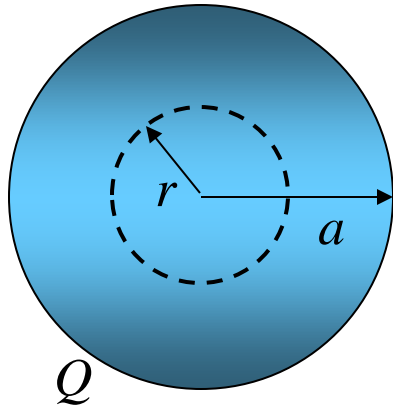
Electric charge is conserved:

$$Q_{\text{shell}} = -Q = Q_I + Q_O = -2Q + Q_O$$

$$-Q = -2Q + Q_O \Rightarrow Q_O = +Q$$

*excess=beyond that required for electrical neutrality

Example: an insulating sphere of radius a has a uniform charge density ρ and a total positive charge Q . Calculate the electric field at a point inside the sphere.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0} = \frac{\rho V_{\text{enclosed}}}{\epsilon_0}$$

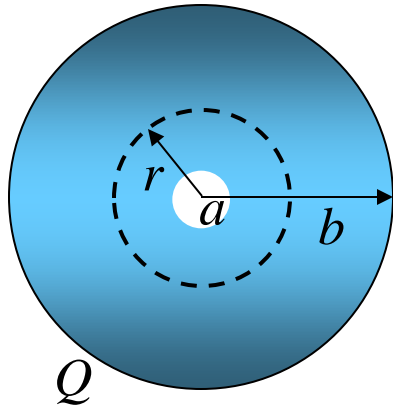
$$E(4\pi r^2) = \frac{\rho \left(\frac{4}{3} \pi r^3 \right)}{\epsilon_0}$$

This object in this example is not a conductor. See Dr. Waddill's [lecture](#) on Gauss' Law from a few years ago. Click on the word "lecture" in the previous sentence to view/download the lecture.

Here is the address for you to copy and paste into a web browser, in case the link in the above paragraph doesn't work:

http://campus.mst.edu/physics/courses/24/Handouts/Lec_03.ppt

Example: an insulating spherical shell of inner radius a and outer radius b has a uniform charge density ρ . Calculate the electric field at a point inside the sphere.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

$$q_{\text{enclosed}} = \rho V_{\text{enclosed}} = \rho \left(\frac{4}{3} \pi r^3 - \frac{4}{3} \pi a^3 \right)$$

Also see exam 1, Fall 2015, problem 7, for a related example.

Calculate the electric field at a point outside the sphere.

$$q_{\text{enclosed}} = \rho V_{\text{enclosed}} = \rho \left(\frac{4}{3} \pi b^3 - \frac{4}{3} \pi a^3 \right)$$

Demo: Professor Tries to Avoid Debilitating Electrical Shock While Demonstrating Van de Graaff Generator

